OPTIONAL PURE / VECTORS & MATRICES

Vector Product

Questions by topic 13 questions · 111 marks

EXAM BOARDS Edexcel FP1 9FM0/3A · OCR A Y545 · AQA 7367 · OCR A H245 · OCR MEI H645 CIE 9231

Questions

Question 1	*6 marks*	2
Question 2	*7 marks*	3
Question 3	*8 marks*	4
Question 4	*9 marks*	5
Question 5	*9 marks*	6
Question 6	*9 marks*	7
Question 7	*7 marks*	8
Question 8	*7 marks*	9
Question 9	*5 marks*	10
Question 10	*7 marks*	11
Question 11	*10 marks*	12
Question 12	*9 marks*	13
Question 13	*18 marks*	14

1. **(a)** The vectors

$$\mathbf{p} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \quad \text{and} \quad \mathbf{q} = \begin{pmatrix} 2 \\ m \\ -1 \end{pmatrix}$$

span a plane with normal vector

$$\mathbf{n} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

Given that $\mathbf{q} \times \mathbf{p}$ is in the same direction as $\mathbf{n}$, find m and the positive constant λ such that

$$\mathbf{q} \times \mathbf{p} = \lambda \mathbf{n}$$ **[3]**

(b) Using $\mathbf{q} \times \mathbf{p}$, determine the acute angle between $\mathbf{p}$ and $\mathbf{q}$. **[3]**

2. Relative to the origin O, the points A, B and C have position vectors

$$\mathbf{a} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{c} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$$

(a) Find, in its simplest exact form, the area of triangle ABC. **[4]**

(b) By calculating each side, show that

$$(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} \neq \mathbf{a} \times (\mathbf{b} \times \mathbf{c})$$ **[3]**

3. You are given the fixed points $A(1, 0, 1)$ and $C(4, 0, 2)$, and the variable point $B(t, 2, 5 - t)$, where t is a real parameter.

(a) Using only the geometrical properties of the vector product, explain why the statement "$\overrightarrow{AB} \times \overrightarrow{AC} = \mathbf{0}$" is false for all values of t. **[2]**

(b) (i) Use the vector product to find an expression, in terms of t, for the area of triangle ABC. **[4]**

(ii) Hence determine the value of t for which the area of triangle ABC is a minimum. **[2]**

4. With respect to a fixed origin O, the points A and B have coordinates $(1, 1, 0)$ and $(p, 3, 2)$ respectively, where p is a constant.

For each of the following, determine the possible values of p for which

(a) $\overrightarrow{AB}$ makes an angle of $60°$ with the positive y-axis. **[3]**

(b) $\overrightarrow{OA} \times \overrightarrow{AB}$ is parallel to

$$\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$$

[3]

(c) the area of triangle OAB is $\frac{3}{2}$. **[3]**

5. **(a)** Let

$$\mathbf{p} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \mathbf{q} = \begin{pmatrix} x \\ 0 \\ 1 \end{pmatrix}$$

The area of the parallelogram formed by $\mathbf{p}$ and $\mathbf{q}$ is $\sqrt{2}$.
It is also given that $\mathbf{p} \times \mathbf{q}$ is perpendicular to

$$\begin{pmatrix} 1 \\ 3 \\ m \end{pmatrix}$$

Find x and m. **[6]**

(b) Hence, using $|\mathbf{p} \times \mathbf{q}|$, determine the acute angle between $\mathbf{p}$ and $\mathbf{q}$. **[3]**

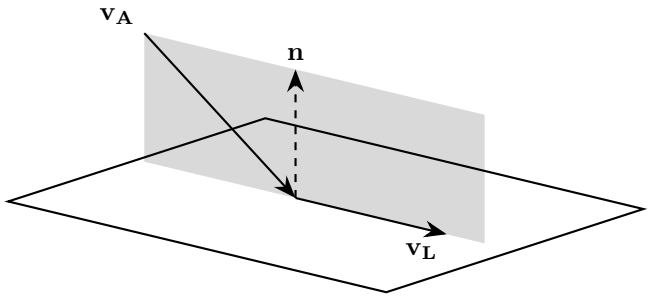
6. The point P has coordinates $P(2, -1, 1)$ and the midpoint of PR is $M(1, 0, 2)$. The vector $\overrightarrow{PQ}$ is given by

$$\overrightarrow{PQ} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

(a) Use a vector product to show that the area of triangle PQR is $\sqrt{14}$. **[5]**

(b) Find a vector equation of the plane containing P, Q and R in the form $\mathbf{r} \cdot \mathbf{n} = k$. **[2]**

(c) Hence find the exact distance of the plane from the origin. **[2]**



7. A small aircraft is landing in a field.
In a model for the landing, the aircraft travels in different straight lines before and after it lands, as shown in the diagram above.
The vector $\mathbf{v_A}$ is in the direction of travel of the aircraft as it approaches the field, and the vector $\mathbf{v_L}$ is in the direction of travel of the aircraft after it lands.
With respect to a fixed origin, the field is modelled as the plane with vector equation

$$\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + s \begin{pmatrix} 10 \\ 0 \\ -1 \end{pmatrix} + t \begin{pmatrix} 0 \\ 20 \\ -1 \end{pmatrix}$$

and

$$\mathbf{v_A} = \begin{pmatrix} 4 \\ -1 \\ -2 \end{pmatrix}$$

(a) Show that the vector $\mathbf{n} = 2\mathbf{i} + \mathbf{j} + 20\mathbf{k}$ is normal to the field. **[2]**

(b) Show that

$$\mathbf{n} \times \mathbf{v_A} = \lambda \begin{pmatrix} 3 \\ 14 \\ -1 \end{pmatrix}$$

where λ is a constant to be determined. **[2]**

When the aircraft lands, it remains in contact with the field and travels in the direction $\mathbf{v_L}$.
The vector $\mathbf{v_L}$ lies in the same plane as both $\mathbf{v_A}$ and $\mathbf{n}$, as shown in the diagram above.

(c) Determine a vector which has the same direction as $\mathbf{v_L}$. **[3]**

8. The points P, Q and R have position vectors $\mathbf{p} = 2\mathbf{i} - 2\mathbf{k}$, $\mathbf{q} = 3\mathbf{j} + 3\mathbf{k}$ and $\mathbf{r} = t\mathbf{i} - \mathbf{j} + \mathbf{k}$ respectively, relative to the origin O.

Determine the value(s) of t in each of the following cases.

(a) The line OR is parallel to a normal to the plane OPQ. **[3]**

(b) The volume of tetrahedron $OPQR$ is 9. **[4]**

9. The vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are such that

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \qquad \mathbf{b} \times \mathbf{c} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} \qquad \text{and} \qquad \mathbf{c} \times \mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$$

Work out $(\mathbf{a} + 2\mathbf{b} - \mathbf{c}) \times (3\mathbf{a} - \mathbf{b} + \mathbf{c})$. **[5]**

10. Points P, Q and R have position vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ respectively, relative to origin O.

It is given that $(\mathbf{q}-\mathbf{p})\times(\mathbf{r}-\mathbf{q})=2\mathbf{p}$ and that $|\mathbf{p}|=5$.

(a) Determine each of the following as either a single vector or a scalar quantity.

(i) $(\mathbf{r}-\mathbf{q})\times(\mathbf{q}-\mathbf{p})$ **[1]**

(ii) $\mathbf{p}\times\big((\mathbf{q}-\mathbf{p})\times(\mathbf{r}-\mathbf{q})\big)$ **[2]**

(iii) $\mathbf{p}\cdot\big((\mathbf{q}-\mathbf{p})\times(\mathbf{r}-\mathbf{q})\big)$ **[2]**

(b) Describe a geometrical relationship between the points O, P, Q and R which can be deduced from

(i) the statement $(\mathbf{q}-\mathbf{p})\times(\mathbf{r}-\mathbf{q})=2\mathbf{p}$. **[1]**

(ii) the result of part (a)(iii). **[1]**

11. The points A, B and C lie on the plane Π.

The position vectors of A and B are $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = 5\mathbf{i} + \mathbf{j}$ respectively.

The centroid G of triangle ABC has position vector $4\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

(a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$. **[4]**

(b) Find an equation for Π in the form $\mathbf{r} \cdot \mathbf{n} = p$. **[2]**

The point D has position vector $\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$.

(c) Determine the volume of the tetrahedron $ABCD$. **[4]**

12. The lines L_1, L_2 and L_3 are given by

$$L_1: \quad \mathbf{r} = \begin{pmatrix} 8 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$$

$$L_2: \quad x = 9 - 4\mu,\ y = -2 + 3\mu,\ z = -1 + 3\mu$$

$$L_3: \quad \frac{x-3}{1} = \frac{y+6}{-5} = \frac{z-1}{-2}$$

The pairwise intersections of these lines are the vertices of a triangle.

Find the area of this triangle. **[9]**

13. With respect to a fixed origin O the point A has coordinates $(9, 10, -1)$ and the line ℓ is the intersection of the planes $x - z = -2$ and $x + 2y = 5$.

The points B and C lie on ℓ such that $AB = AC = 15$.

Given that A does not lie on ℓ and that the z coordinate of B is negative

(a) determine the coordinates of B and the coordinates of C. **[6]**

(b) Hence determine a Cartesian equation of the plane containing the points A, B and C. **[4]**

The point D has coordinates $(8, 4, \alpha)$, where α is a constant.

Given that the volume of the tetrahedron $ABCD$ is 156

(c) determine the possible values of α. **[4]**

Given that $\alpha > 0$

(d) determine the shortest distance between the line ℓ and the line passing through the points A and D, giving your answer to 2 significant figures. **[4]**