OPTIONAL PURE / VECTORS & MATRICES

Shortest Distances

Questions by topic 10 questions · 112 marks

EXAM BOARDS Edexcel FP1 9FM0/3A · Edexcel 9FM0 · AQA 7367 · OCR A H245 · OCR MEI H645 CIE 9231

Instructions

Edexcel (9FM0): the vector (cross) product is Further Pure 1 content, not Core Pure. Questions 4 and 9 can be skipped; everything else can be done by scalar (dot) product methods, even where the solutions use the vector product.

Questions

Question 1	*5 marks*	2
Question 2	*8 marks*	3
Question 3	*13 marks*	4
Question 4	*18 marks*	5
Question 5	*8 marks*	6
Question 6	*10 marks*	7
Question 7	*8 marks*	8
Question 8	*17 marks*	9
Question 9	*9 marks*	10
Question 10	*16 marks*	11

1. The line ℓ has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

The point A has coordinates $(4, 1, 3)$.

(a) Find the coordinates of the point N on ℓ which is closest to A. **[4]**

(b) Hence find the shortest distance from A to ℓ. **[1]**

2. The planes Π_1 and Π_2 have equations $2x - y + 2z = 1$ and $4x - 2y + 4z = 11$ respectively.

(a) Show that Π_1 and Π_2 are parallel, and find the distance between them. **[3]**

The lines ℓ_1 and ℓ_2 have equations

$$\ell_1 : \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$$

$$\ell_2 : \mathbf{r} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ -4 \\ 4 \end{pmatrix}$$

(b) Show that ℓ_1 and ℓ_2 are parallel, and find the distance between them. **[5]**

3. The plane Π has equation $3x + y + 2z = -4$. The point P has coordinates $(5, -5, 3)$.

(a) Calculate the shortest distance from P to Π. **[2]**

The line L has vector equation

$$\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$

(b) Verify that P lies on L. **[2]**

(c) Find the coordinates of the point where L meets Π. **[3]**

(d) Determine the acute angle between L and Π. **[3]**

(e) Use the results of parts (b), (c) and (d) to verify your answer to part (a). **[3]**

4. The points C and D have coordinates $(0, 0, 2)$ and $(3, 3, -1)$ respectively.

The planes P_1 and P_2 have equations $x + y + z = 3$ and $2x - y = 1$ respectively.

(a) Find the acute angle between the line CD and the plane P_1. **[4]**

(b) Show that the line CD meets P_1 and P_2 in a common point, and find its coordinates. **[5]**

(c) (i) Find $(\mathbf{i} + \mathbf{j} + \mathbf{k}) \times (2\mathbf{i} - \mathbf{j})$. **[2]**

(ii) Hence find the acute angle between the planes P_1 and P_2. **[3]**

(iii) Find the shortest distance between the point C and the line of intersection of the planes P_1 and P_2. **[4]**

5. The plane Π has equation

$$\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -1 \\ -2 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Show that vector $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ is perpendicular to Π. **[2]**

(b) Hence find a Cartesian equation of Π. **[2]**

The line ℓ has equation

$$\mathbf{r} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

where t is a scalar parameter.

The point A lies on ℓ. Given that the shortest distance between A and Π is $\frac{9}{\sqrt{14}}$

(c) determine the possible coordinates of A. **[4]**

6. **(a)** The plane Π passes through the points $A(1, -2, 3)$, $B(2, 0, 3)$ and $C(2, -2, 2)$.

The point D has coordinates $(4, 1, 0)$. Find the shortest distance between D and Π. **[6]**

(b) Lines ℓ_1 and ℓ_2 have the following equations.

$$\ell_1 : \ \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

$$\ell_2 : \ \mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

Find, in exact form, the distance between ℓ_1 and ℓ_2. **[4]**

7. The line ℓ_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

The line ℓ_2 is the line of intersection of the planes

$$x - y + z = 6$$

and

$$2x + y - z = 3$$

(i) Find the shortest distance between ℓ_1 and ℓ_2. **[6]**

(ii) Find a Cartesian equation of the plane which contains ℓ_1 and is parallel to ℓ_2. **[2]**

8. **(a)** Show that the three planes with equations

$$\begin{aligned} x + \lambda y + z &= 4 \\ 2x - y + 2z &= 3 \\ 2x + y - z &= 5 \end{aligned} \quad \textbf{[3]}$$

where λ is a constant, meet at a unique point except for one value of λ, which is to be determined.

(b) In the case $\lambda = 2$, use matrices to find the point of intersection P of the planes, showing your method clearly. **[3]**

The line ℓ has equation

$$\frac{x+1}{1} = \frac{y-1}{-1} = \frac{z}{1}$$

(c) Find a vector equation of ℓ. **[2]**

(d) Find the shortest distance between the point P and ℓ. **[4]**

(e) (i) Show that ℓ is parallel to the plane $2x + y - z = 5$. **[3]**

(ii) Find the distance between ℓ and the plane $2x + y - z = 5$. **[2]**

9. **(a)** Given that $\mathbf{u} = \lambda\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{v} = \mathbf{i} + \mathbf{j} - \mathbf{k}$, find the following, giving your answers in terms of λ.

(i) $\mathbf{u} \cdot \mathbf{v}$ **[1]**

(ii) $\mathbf{u} \times \mathbf{v}$ **[2]**

(b) Hence determine

(i) the acute angle between the planes $2x - 2y + 2z = 11$ and $x + y - z = 7$ **[3]**

(ii) the shortest distance between the lines

$$\mathbf{r} = \mathbf{i} - \mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$$

and

$$\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k} + t(\mathbf{i} + \mathbf{j} - \mathbf{k})$$

giving your answer as a multiple of $\sqrt{2}$. **[3]**

10. The position vectors of the points A, B, C, D are

$$\mathbf{i}+2\mathbf{j}, \quad 2\mathbf{i}+2\mathbf{j}, \quad \mathbf{i}+3\mathbf{j}+2\mathbf{k}, \quad 2\mathbf{i}+3\mathbf{j}+m\mathbf{k}$$

respectively, where m is an integer.

It is given that the shortest distance between the line through A and D and the line through B and C is $\frac{1}{\sqrt{14}}$.

(a) Show that the only possible value of m is 1. **[7]**

(b) Find the shortest distance of C from the line through A and D. **[4]**

(c) Show that the acute angle between the planes ABD and ACD is $\arccos\left(\frac{\sqrt{3}}{2}\right)$. **[5]**