OPTIONAL PURE / VECTORS & MATRICES

Plane Intersections

Questions by topic 10 questions · 83 marks

EXAM BOARDS Edexcel FP1 9FM0/3A · Edexcel 9FM0 · AQA 7367 · OCR A H245 · OCR MEI H645 CIE 9231

Questions

Question 1	*5 marks*	2
Question 2	*7 marks*	3
Question 3	*5 marks*	4
Question 4	*5 marks*	5
Question 5	*8 marks*	6
Question 6	*8 marks*	7
Question 7	*11 marks*	8
Question 8	*10 marks*	9
Question 9	*10 marks*	10
Question 10	*14 marks*	11

1. Three planes have equations

$$x + y + z = 4$$
$$2x - y + z = 8$$
$$x + 3y - 2z = -7$$

(a) Show that the three planes meet at a single point. **[2]**

(b) Find the coordinates of this point. **[3]**

2. The planes Π_1 and Π_2 have equations $x+2y+z=4$ and $2x-y+3z=3$ respectively, and meet in the line L.

(a) Find a vector equation of L. **[4]**

The plane Π_3 has equation $3x-4y+5z=2$.

(b) Show that L lies in Π_3. **[2]**

(c) Hence describe the geometrical arrangement of the planes Π_1, Π_2 and Π_3. **[1]**

3. Three planes have equations

$$\begin{aligned} \Pi_1 : &\quad 2x - y + 3z = 4 \\ \Pi_2 : &\quad 4x - 2y + 6z = 5 \\ \Pi_3 : &\quad x + y - z = 2 \end{aligned}$$

(a) Show that Π_1 and Π_2 are parallel and distinct. **[1]**

(b) Hence explain why the three equations have no common solution, and describe the geometrical arrangement of the three planes. **[2]**

The equation of Π_2 is now changed to $4x - 2y + 6z = k$, where k is a constant.

(c) Find the value of k for which the three equations have infinitely many solutions, and describe the geometrical arrangement of the planes in this case. **[2]**

4. **(a)** Consider the linear system $\mathbf{Ax} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 3 & -2 & 1 \\ 2 & 1 & -4 \\ 7 & 0 & -7 \end{pmatrix}, \qquad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 5 \\ 1 \\ 7 \end{pmatrix}$$

Show that $\mathbf{A}$ is singular, and deduce that the system does not have a unique solution. **[2]**

(b) Show that the system is consistent. Interpret the solution set geometrically. **[3]**

5. **(a)** Show that the system of equations

$$\begin{aligned} x + 2y - z &= 3 \\ 2x - y + 4z &= 1 \\ 3x + y + 3z &= a \end{aligned}$$

where a is a constant, does not have a unique solution. **[2]**

(b) Given that $a = 4$, show that the system of equations in part (a) is consistent. Interpret this situation geometrically. **[3]**

(c) Given instead that $a \neq 4$, show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically. **[3]**

6. Three planes have equations

$$x - y + 2z = 3$$
$$2x + y + z = 4$$
$$3x + (m+1)z = 7$$

where m is a constant.

(a) Investigate the arrangement of the planes:
when $m = 2$;
when $m \neq 2$. **[6]**

(b) A student claims that the position vectors $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $2\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $3\mathbf{i} + 4\mathbf{k}$ all lie in a plane through the origin. Determine whether or not the student is correct. **[2]**

7.

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 0 \\ 3 & k & 1 \\ 2 & -1 & 1 \end{pmatrix}$$

where k is a constant.

(a) Find the values of k for which the matrix $\mathbf{M}$ has an inverse. **[2]**

(b) Find, in terms of p, the coordinates of the point where the following planes intersect:

$$\begin{aligned} x + y &= 1 \\ 3x + 2y + z &= p \\ 2x - y + z &= 4 \end{aligned}$$ **[5]**

(c) (i) Find the value of q for which the set of simultaneous equations

$$\begin{aligned} x + y &= 1 \\ 3x + z &= q \\ 2x - y + z &= 4 \end{aligned}$$ **[2]**

has at least one solution.

(ii) For this value of q, interpret the solution set geometrically. **[2]**

8. The equations

$$\begin{aligned} x - y + 2z &= 4 \\ (k+1)x + ky + (2-k)z &= 4 - k \\ (5-2k)x + (2-2k)y + (k+2)z &= 8 - k \end{aligned}$$

represent three planes, where k is a real constant.
The planes do not intersect in exactly one point.

(a) Find the possible values of k. **[4]**

(b) For each value of k found in part (a), describe the configuration of the planes.
Fully justify your answer, stating in each case whether the corresponding system is consistent or inconsistent. **[6]**

9. Three planes have equations

$$\begin{aligned} x + ay &= -1 \\ 2x + y + z &= 3 \\ 3x + by + 2z &= c \end{aligned}$$

where a, b and c are constants.

(a) In the case where the planes **do not** intersect at a unique point,

(i) find b in terms of a. **[2]**

(ii) find the value of c for which the planes form a sheaf. **[3]**

(b) In the case where $b = 1 - a$ and $c = 5$, find the coordinates of the point of intersection of the planes in terms of a. **[5]**

10. You are given that a is a parameter which can take only real values.
The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 3 & 2 & a \\ 1 & 3 & 0 \\ 2 & -1 & 1 \end{pmatrix}$$

(a) Find an expression for the determinant of $\mathbf{A}$ in terms of a. **[2]**

You are given the following system of equations in x, y and z.

$$\begin{aligned} 3x + 2y + az &= 8 \\ x + 3y &= 5 \\ 2x - y + z &= 3 \end{aligned}$$

The system can be written in the form

$$\mathbf{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 5 \\ 3 \end{pmatrix}$$

(b) (i) In the case where $\mathbf{A}$ is not singular, solve the given system by using $\mathbf{A}^{-1}$. **[6]**

(ii) In the case where $\mathbf{A}$ is singular, describe the configuration of the planes whose equations are the three equations of the system. **[3]**

The transformation represented by $\mathbf{A}$ is denoted by T.
A 3-D solid of volume $a^2 + 3$ is transformed by T to an image.

(c) (i) Determine the range of values of a for which the orientation of the image is the reverse of the orientation of the object. **[1]**

(ii) Determine the range of values of a for which the volume of the image is less than the volume of the object. **[2]**