OPTIONAL PURE / VECTORS & MATRICES

Vector Product

Mark scheme 13 questions · 111 marks

EXAM BOARDS Edexcel FP1 9FM0/3A · OCR A Y545 · AQA 7367 · OCR A H245 · OCR MEI H645 CIE 9231

Questions

Question	Marks by part	Marks	Page
Question 1	(a) 3 (b) 3	6	4
Question 2	(a) 4 (b) 3	7	5
Question 3	(a) 2 (b)(i) 4 (b)(ii) 2	8	6
Question 4	(a) 3 (b) 3 (c) 3	9	7
Question 5	(a) 6 (b) 3	9	8
Question 6	(a) 5 (b) 2 (c) 2	9	9
Question 7	(a) 2 (b) 2 (c) 3	7	10
Question 8	(a) 3 (b) 4	7	11
Question 9		5	12
Question 10	(a)(i) 1 (a)(ii) 2 (a)(iii) 2 (b)(i) 1 (b)(ii) 1	7	13
Question 11	(a) 4 (b) 2 (c) 4	10	14
Question 12		9	15
Question 13	(a) 6 (b) 4 (c) 4 (d) 4	18	16
	Total	**111**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last accuracy mark of the working that reaches a printed result: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [***n***] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	$\mathbf{q} \times \mathbf{p} = (2m+3)\mathbf{i} - 5\mathbf{j} + (6-m)\mathbf{k}$	**B1**	Any form; the components need not be simplified.
	$-5 = -\lambda$, $2m+3 = \lambda$ or $6-m = \lambda$ $\Rightarrow \lambda = \ldots,\ m = \ldots$	**M1**	Equates at least two components of their $\mathbf{q} \times \mathbf{p}$ to those of $\lambda\mathbf{n}$ and solves for λ and m. Condone sign slips.
	$\lambda = 5,\ m = 1$	**A1**	
		[3]	
1(b)	$\lvert\mathbf{q} \times \mathbf{p}\rvert = \sqrt{75}$ $[= 5\sqrt{3}]$ $\lvert\mathbf{p}\rvert = \sqrt{14},\ \lvert\mathbf{q}\rvert = \sqrt{6}$	**B1**	All three, in any form.
	$\sin\theta = \frac{\text{“}5\sqrt{3}\text{”}}{\text{“}\sqrt{6}\text{”}\ \text{“}\sqrt{14}\text{”}}$ $\left[= \frac{5}{2\sqrt{7}}\right]$	**M1**	Uses $\lvert\mathbf{q} \times \mathbf{p}\rvert = \lvert\mathbf{q}\rvert\lvert\mathbf{p}\rvert \sin\theta$ with their magnitudes to find $\sin\theta$. Condone the ratio inverted.
	$\theta = 70.9°$	**A1**	awrt $70.9°$ or awrt 1.24 rad. A0 for the obtuse angle.
		[3]	
	Total for Question 1	**6**	

Question	Scheme	Marks	Notes
2(a)	$\overrightarrow{AB} = \begin{pmatrix}-4\\-1\\4\end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix}-1\\-4\\1\end{pmatrix}$ $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-4 & -1 & 4\\-1 & -4 & 1\end{vmatrix} = \ldots$	**M1**	Finds two sides as differences of position vectors and expands their vector product by determinant or component formula, all three components attempted. Condone sign slips; a product written without working needs two components correct.
	$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix}15\\0\\15\end{pmatrix}$	**A1**	$\pm(15\mathbf{i} + 15\mathbf{k})$, from any two sides.
	Area $= \frac{1}{2}\sqrt{\text{“15”}^2 + \text{“0”}^2 + \text{“15”}^2} = \ldots$	**dM1**	Finds half the magnitude of their vector product, using all three components.
	Area $= \frac{15\sqrt{2}}{2}$	**A1**	Accept $\frac{15}{2}\sqrt{2}$ oe; not $\frac{\sqrt{450}}{2}$, $\frac{15}{\sqrt{2}}$ or a decimal.
		[4]	
2(b)	$\mathbf{a} \times \mathbf{b} = \begin{pmatrix}8\\-12\\5\end{pmatrix}$ $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \text{“}\begin{pmatrix}8\\-12\\5\end{pmatrix}\text{”} \times \begin{pmatrix}2\\-2\\1\end{pmatrix} = \ldots$	**M1**	Expands $\mathbf{a} \times \mathbf{b}$, then the vector product of their result with $\mathbf{c}$, all three components of each attempted. Condone sign slips; each product written without working needs two components correct.
	$\mathbf{b} \times \mathbf{c} = \begin{pmatrix}9\\9\\0\end{pmatrix}$ $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \begin{pmatrix}3\\2\\0\end{pmatrix} \times \text{“}\begin{pmatrix}9\\9\\0\end{pmatrix}\text{”} = \ldots$	**M1**	Expands $\mathbf{b} \times \mathbf{c}$, then the vector product of $\mathbf{a}$ with their result, all three components of each attempted. Condone sign slips; each product written without working needs two components correct.
	$\begin{pmatrix}-2\\2\\8\end{pmatrix} \neq \begin{pmatrix}0\\0\\9\end{pmatrix}$ $\Rightarrow (\mathbf{a} \times \mathbf{b}) \times \mathbf{c} \neq \mathbf{a} \times (\mathbf{b} \times \mathbf{c})$	**A1 AG**	Both sides correct with no errors, and a conclusion that they differ; the two vectors joined by $\neq$ is enough.
		[3]	
Total for Question 2		**7**	

Question	Scheme	Marks	Notes
3(a)	$\overrightarrow{AB} \times \overrightarrow{AC} = \mathbf{0}$ only if $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel, or one is $\mathbf{0}$	**B1**	Accept oe such as $\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert \sin\theta = 0$ only when $\theta = 0$ or π, or a zero vector. The zero-vector case may be omitted.
	$\overrightarrow{AB} = \begin{pmatrix} t-1 \\ 2 \\ 4-t \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \neq \mathbf{0}$ y-components 2 and 0, so $\overrightarrow{AB} \neq k\overrightarrow{AC}$ for any t $\Rightarrow$ statement false for all t	**E1**	Shows $\overrightarrow{AB}$ is never a multiple of $\overrightarrow{AC}$, by components, and concludes false for all t. Not for computing the vector product.
		[2]	
3(b)(i)	$\begin{pmatrix} t-1 \\ 2 \\ 4-t \end{pmatrix} \times \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2\times 1 - (4-t)\times 0 \\ (4-t)\times 3 - (t-1)\times 1 \\ (t-1)\times 0 - 2\times 3 \end{pmatrix}$	**M1**	Forms two sides of the triangle as vectors in t and attempts their vector product, all three components. Condone sign slips.
	$= \begin{pmatrix} 2 \\ 13-4t \\ -6 \end{pmatrix}$	**A1**	Correct vector, or its negative.
	Area $= \frac{1}{2}\sqrt{2^2 + \text{“}(13-4t)\text{”}^2 + (-6)^2}$	**dM1**	Takes half the magnitude of their vector product, squaring all three components.
	$= \frac{1}{2}\sqrt{(13-4t)^2 + 40}$	**A1**	Correct expression oe, such as $\frac{1}{2}\sqrt{16t^2 - 104t + 209}$.
		[4]	
3(b)(ii)	$(13-4t)^2 + 40$ least when $13 - 4t = 0 \Rightarrow t = \ldots$	**M1**	Finds the minimum of the area or the expression under their root, by a squared term set to zero, completing the square, $t = -\frac{b}{2a}$ or a derivative set to zero.
	$t = \frac{13}{4}$	**A1**	oe, such as 3.25.
		[2]	
	Total for Question 3	**8**	

Question	Scheme	Marks	Notes
4(a)	$\overrightarrow{AB} = (p-1)\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ $\overrightarrow{AB} \cdot \mathbf{j} = \lvert\overrightarrow{AB}\rvert \cos 60°$ $\Rightarrow 2 = \frac{1}{2}\sqrt{(p-1)^2 + 2^2 + 2^2}$	**M1 A1**	**M1** Equates the scalar product of their $\overrightarrow{AB}$, a difference of position vectors, with $\mathbf{j}$ to their $\lvert\overrightarrow{AB}\rvert \cos 60°$. Condone sign slips. **A1** Any correct equation in p alone.
	$(p-1)^2 = 8 \Rightarrow p = 1 \pm 2\sqrt{2}$	**A1**	Both values, oe, or awrt 3.83 and -1.83.
		[3]	
4(b)	$\overrightarrow{OA} \times \overrightarrow{AB} = 2\mathbf{i} - 2\mathbf{j} + (3-p)\mathbf{k}$	**B1**	
	"$2\mathbf{i} - 2\mathbf{j} + (3-p)\mathbf{k}$" $= \lambda(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ $\Rightarrow \lambda = 1,\ p = \ldots$	**M1**	Equates their vector product, with a component in p, to a multiple of the given vector, or equates ratios of components, and solves for p.
	$p = 2$	**A1**	cso.
		[3]	
4(c)	$\overrightarrow{OA} \times \overrightarrow{OB} = 2\mathbf{i} - 2\mathbf{j} + (3-p)\mathbf{k}$ $\frac{1}{2}\sqrt{2^2 + (-2)^2 + (3-p)^2} = \frac{3}{2}$	**M1 A1**	**M1** Sets half the magnitude of a vector product of two sides, in terms of p, equal to $\frac{3}{2}$. Condone slips in the vector product; needs the $\frac{1}{2}$. **A1** Any correct equation in p alone.
	$(3-p)^2 = 1 \Rightarrow p = 2$ or $p = 4$	**A1**	
		[3]	
Total for Question 4		**9**	

Question	Scheme	Marks	Notes
5(a)	$\mathbf{p} \times \mathbf{q} = \mathbf{i}(1-0) - \mathbf{j}(1-2x) + \mathbf{k}(0-x)$	**M1**	Attempts $\mathbf{p} \times \mathbf{q}$ by determinant or component formula, giving three components in terms of x. Condone sign slips.
	$= \mathbf{i} + (2x-1)\mathbf{j} - x\mathbf{k}$	**A1**	oe, as a column vector.
	$1^2 + (2x-1)^2 + (-x)^2 = 2$ $\Rightarrow 5x^2 - 4x = 0 \Rightarrow x = \ldots$	**M1**	Equates the squared magnitude of their $\mathbf{p} \times \mathbf{q}$ to 2, or its magnitude to $\sqrt{2}$, and solves for x by any means.
	$x = 0$ or $x = \frac{4}{5}$	**A1**	Both values; 0.8 accepted.
	$1 + 3(2x-1) - mx = 0$ $[\Rightarrow x(6-m) = 2]$	**M1**	Sets the scalar product of their $\mathbf{p} \times \mathbf{q}$ with $\mathbf{i} + 3\mathbf{j} + m\mathbf{k}$ equal to zero. Need not be simplified.
	$x \neq 0$, so $x = \frac{4}{5}$, $m = \frac{7}{2}$	**A1**	Both values, oe such as 0.8 and 3.5. A0 if $x = 0$ is retained as a solution.
		[6]	
5(b)	$\lvert\mathbf{p}\rvert = \sqrt{6}$, $\lvert\mathbf{q}\rvert = \frac{\sqrt{41}}{5}$	**B1**	Both, with $x = \frac{4}{5}$; oe such as $\sqrt{1.64}$.
	$\sqrt{2} = \sqrt{6} \times \frac{\sqrt{41}}{5} \sin\theta \Rightarrow \theta = \ldots$	**M1**	Uses $\lvert\mathbf{p} \times \mathbf{q}\rvert = \lvert\mathbf{p}\rvert\lvert\mathbf{q}\rvert \sin\theta$ with $\sqrt{2}$ and their magnitudes to find $\sin\theta$. Condone the ratio inverted. A scalar-product method scores M0 A0.
	$\theta = 26.8°$	**A1**	awrt 26.8° or 0.468 radians, or $\arcsin\frac{5}{\sqrt{123}}$ oe.
		[3]	
Total for Question 5		**9**	

Question	Scheme	Marks	Notes
6(a)	$\overrightarrow{OR} = 2\overrightarrow{OM} - \overrightarrow{OP} \Rightarrow R(0,1,3)$ $\overrightarrow{PR} = -2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$	**B1**	Correct R or $\overrightarrow{PR}$, soi.
	$\overrightarrow{PQ} \times \overrightarrow{PR} = (\mathbf{i} + 2\mathbf{j}) \times (-2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$	**M1**	Expands the vector product of two sides of the triangle, using their R, by determinant or component formula, all three components attempted. Condone sign slips; a product written without working needs two components correct.
	$= 4\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$	**A1**	Or its negative.
	Area $= \frac{1}{2}\sqrt{4^2 + (-2)^2 + 6^2}$	**dM1**	Takes half the magnitude of their vector product.
	$= \frac{1}{2}\sqrt{56} = \sqrt{14}$	**A1 AG**	Correct vector product and magnitude leading without error to $\sqrt{14}$.
		[5]	
6(b)	$\mathbf{n} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ $k = (2\mathbf{i} - \mathbf{j} + \mathbf{k}) \cdot (2\mathbf{i} - \mathbf{j} + 3\mathbf{k}) = \ldots$	**M1**	Uses their vector product from (a), or a multiple, as $\mathbf{n}$, and finds its scalar product with the position vector of P, Q, R or M.
	$\mathbf{r} \cdot (2\mathbf{i} - \mathbf{j} + 3\mathbf{k}) = 8$	**A1**	Or any multiple, such as $\mathbf{r} \cdot (4\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}) = 16$. A Cartesian equation alone scores A0.
		[2]	
6(c)	$\text{dist} = \dfrac{8}{\sqrt{2^2 + (-1)^2 + 3^2}}$	**M1**	Divides $\lvert k \rvert$ by $\lvert \mathbf{n} \rvert$, both from their plane equation in (b).
	$= \frac{8}{\sqrt{14}} = \frac{4\sqrt{14}}{7}$	**A1**	Either form, or any exact equivalent.
		[2]	
	Total for Question 6	**9**	

Question	Scheme	Marks	Notes
7(a)	$\begin{pmatrix}2\\1\\20\end{pmatrix}\cdot\begin{pmatrix}10\\0\\-1\end{pmatrix}=20+0-20$ $\begin{pmatrix}2\\1\\20\end{pmatrix}\cdot\begin{pmatrix}0\\20\\-1\end{pmatrix}=0+20-20$	**M1**	Forms the scalar product of $\mathbf{n}$ with both direction vectors, or finds the vector product of the direction vectors. Condone one slip.
	Both $=0$, so $\mathbf{n}$ is normal to the field	**A1 AG**	Both products correctly evaluated as zero, or the vector product shown to be $10\mathbf{n}$, with a conclusion in words.
		[2]	
7(b)	$\mathbf{n}\times\mathbf{v_A}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\2&1&20\\4&-1&-2\end{vmatrix}$ $=(-2+20)\mathbf{i}-(-4-80)\mathbf{j}+(-2-4)\mathbf{k}$	**M1**	Expands $\mathbf{n}\times\mathbf{v_A}$ by determinant or component formula, all three components attempted. Condone sign slips; a product written without working needs two components correct.
	$=\begin{pmatrix}18\\84\\-6\end{pmatrix}=6\begin{pmatrix}3\\14\\-1\end{pmatrix}$, $\lambda=6$	**A1 AG**	Correct expansion leading without error to $6(3\mathbf{i}+14\mathbf{j}-\mathbf{k})$ oe, with $\lambda=6$ stated.
		[2]	
7(c)	Uses $\mathbf{v_L}$ perpendicular to $\mathbf{n}\times\mathbf{v_A}$ and to $\mathbf{n}$: $\mathbf{v_L}\cdot\begin{pmatrix}3\\14\\-1\end{pmatrix}=0,\quad\mathbf{v_L}\cdot\begin{pmatrix}2\\1\\20\end{pmatrix}=0$	**M1**	States or uses both perpendicularities, or writes $\mathbf{v_L}=\mathbf{v_A}+\mu\mathbf{n}$ with $\mathbf{v_L}\cdot\mathbf{n}=0$.
	$\begin{pmatrix}3\\14\\-1\end{pmatrix}\times\begin{pmatrix}2\\1\\20\end{pmatrix}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\3&14&-1\\2&1&20\end{vmatrix}=\ldots$	**dM1**	Finds a vector perpendicular to both, by a vector product or by solving the two scalar equations, or solves for μ. Condone sign slips.
	$\begin{pmatrix}281\\-62\\-25\end{pmatrix}$	**A1**	Or any positive multiple. A0 for a negative multiple. No justification of the sign is needed.
		[3]	
	Total for Question 7	**7**	

Question	Scheme	Marks	Notes
8(a)	$\mathbf{p} \times \mathbf{q} = 6\mathbf{i} - 6\mathbf{j} + 6\mathbf{k}$	**B1**	Or any non-zero multiple of $\mathbf{i} - \mathbf{j} + \mathbf{k}$; or, by scalar products, $\mathbf{r} \cdot \mathbf{p} = 2t - 2$ and $\mathbf{r} \cdot \mathbf{q} = 0$.
	$t\mathbf{i} - \mathbf{j} + \mathbf{k} = \lambda\,(\text{“}\mathbf{i} - \mathbf{j} + \mathbf{k}\text{”})$ $\Rightarrow \lambda = 1,\ t = \ldots$	**M1**	Sets $\mathbf{r}$ as a multiple of their normal vector and compares components, or sets $\mathbf{r}\times$ their normal $= \mathbf{0}$, or sets $\mathbf{r} \cdot \mathbf{p}$ and $\mathbf{r} \cdot \mathbf{q}$ equal to zero, to find t.
	$t = 1$	**A1**	
		[3]	
8(b)	$\mathbf{r} \cdot (\mathbf{p} \times \mathbf{q})$ $= (t\mathbf{i} - \mathbf{j} + \mathbf{k}) \cdot \text{“}(6\mathbf{i} - 6\mathbf{j} + 6\mathbf{k})\text{”}$	**M1**	Forms the scalar product of their vector $\mathbf{p} \times \mathbf{q}$ with $\mathbf{r}$, or the determinant with rows $\mathbf{p}$, $\mathbf{q}$, $\mathbf{r}$ in any order.
	$= 6(t + 2)$	**A1**	$\pm 6(t + 2)$ oe, the sign depending on the order.
	$\frac{1}{6}\,\lvert\text{“}6(t + 2)\text{”}\rvert = 9 \Rightarrow t + 2 = \pm 9$ $\Rightarrow t = \ldots$	**M1**	Sets $\frac{1}{6}$ of their triple product, or its modulus, equal to 9 or ± 9 and solves for t. Needs the $\frac{1}{6}$.
	$t = 7$ or $t = -11$	**A1**	Both values and no others.
		[4]	
Total for Question 8		**7**	

Question	Scheme	Marks	Notes
9	$3\mathbf{a}\times\mathbf{a}-\mathbf{a}\times\mathbf{b}+\mathbf{a}\times\mathbf{c}$ $+6\mathbf{b}\times\mathbf{a}-2\mathbf{b}\times\mathbf{b}+2\mathbf{b}\times\mathbf{c}$ $-3\mathbf{c}\times\mathbf{a}+\mathbf{c}\times\mathbf{b}-\mathbf{c}\times\mathbf{c}$	**M1**	Expands both brackets distributively into at least six products of pairs of $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$. Condone sign and coefficient slips.
	$-\mathbf{a}\times\mathbf{b}-\mathbf{c}\times\mathbf{a}-6\mathbf{a}\times\mathbf{b}$ $+2\mathbf{b}\times\mathbf{c}-3\mathbf{c}\times\mathbf{a}-\mathbf{b}\times\mathbf{c}$	**M1**	Uses $\mathbf{u}\times\mathbf{u}=\mathbf{0}$ and $\mathbf{v}\times\mathbf{u}=-\mathbf{u}\times\mathbf{v}$ to reduce to multiples of the three given products. Condone one sign slip.
	$-7\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}-4\mathbf{c}\times\mathbf{a}$	**A1**	oe, such as with $+4\mathbf{a}\times\mathbf{c}$.
	$-7\begin{pmatrix}1\\2\\-1\end{pmatrix}+\begin{pmatrix}0\\3\\1\end{pmatrix}-4\begin{pmatrix}2\\-1\\4\end{pmatrix}$	**dM1**	Substitutes the given vectors into their combination of $\mathbf{a}\times\mathbf{b}$, $\mathbf{b}\times\mathbf{c}$, $\mathbf{c}\times\mathbf{a}$ and combines componentwise.
	$\begin{pmatrix}-15\\-7\\-8\end{pmatrix}$	**A1**	cao, in any vector form.
	Total for Question 9	**5**	

Question	Scheme	Marks	Notes
10(a)(i)	$(\mathbf{r}-\mathbf{q})\times(\mathbf{q}-\mathbf{p})=-2\mathbf{p}$	**B1**	
		[1]	
10(a)(ii)	$\mathbf{p}\times 2\mathbf{p}=2(\mathbf{p}\times\mathbf{p})=\ldots$	**M1**	Replaces the vector product by $2\mathbf{p}$ and uses $\mathbf{p}\times\mathbf{p}=\mathbf{0}$, or that parallel vectors have zero vector product.
	$\mathbf{0}$	**A1**	Identified as the zero vector, not the scalar 0.
		[2]	
10(a)(iii)	$\mathbf{p}\cdot 2\mathbf{p}=2\lvert\mathbf{p}\rvert^2=2(5^2)$	**M1**	Replaces the vector product by $2\mathbf{p}$ and uses $\mathbf{p}\cdot\mathbf{p}=\lvert\mathbf{p}\rvert^2$ with $\lvert\mathbf{p}\rvert=5$.
	50	**A1**	
		[2]	
10(b)(i)	OP is perpendicular to the plane PQR	**E1**	States OP is normal to plane PQR, oe such as perpendicular to both PQ and QR. Not for perpendicular to one line alone.
		[1]	
10(b)(ii)	$50\neq 0$, so O, P, Q and R are not coplanar	**E1**	States the four points are not coplanar, oe such as O not in plane PQR. Need not mention $50\neq 0$.
		[1]	
	Total for Question 10	**7**	

Question	Scheme	Marks	Notes
11(a)	$\mathbf{c} = 3\mathbf{g} - \mathbf{a} - \mathbf{b} = \ldots$	**M1**	Rearranges $\mathbf{g} = \frac{1}{3}(\mathbf{a} + \mathbf{b} + \mathbf{c})$ for $\mathbf{c}$ and substitutes the given vectors.
	$\mathbf{c} = 5\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$	**A1**	
	$\overrightarrow{AB} = 3\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$, $\overrightarrow{AC} =$ "$3\mathbf{i} + 4\mathbf{j}$" $\overrightarrow{AB} \times \overrightarrow{AC}$ $= (0 + 12)\mathbf{i} - (0 + 9)\mathbf{j} + (12 - 6)\mathbf{k}$	**M1**	Finds $\overrightarrow{AB}$ and $\overrightarrow{AC}$, with their $\mathbf{c}$, and expands their vector product by determinant or component formula, all three components attempted. Condone sign slips; a product written without working needs two components correct.
	$= 12\mathbf{i} - 9\mathbf{j} + 6\mathbf{k}$	**A1**	Any vector form.
		[4]	
11(b)	$\mathbf{n} =$ "$4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$" $p = \mathbf{a} \cdot \mathbf{n} = 2(4) + (-1)(-3) + 3(2) = \ldots$	**M1**	Takes their (a) result, or a multiple of it, as $\mathbf{n}$ and finds its scalar product with the position vector of A, B, C or G.
	$\mathbf{r} \cdot (4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}) = 17$	**A1**	Or any multiple, such as $\mathbf{r} \cdot (12\mathbf{i} - 9\mathbf{j} + 6\mathbf{k}) = 51$. A0 for $4x - 3y + 2z = 17$ alone.
		[2]	
11(c)	$\overrightarrow{AD} = \mathbf{d} - \mathbf{a} = -\mathbf{i} + 6\mathbf{j} + \mathbf{k}$	**B1**	Or any vector from a point of Π to D, either direction, such as $\overrightarrow{BD} = -4\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$; or, by area $\times$ distance, the distance $\frac{20}{\sqrt{29}}$ from D to Π, oe.
	"$(-\mathbf{i} + 6\mathbf{j} + \mathbf{k})$" $\cdot$ "$(12\mathbf{i} - 9\mathbf{j} + 6\mathbf{k})$" $= -12 - 54 + 6$ $[= -60]$	**M1**	Forms the scalar product of their (a) result with their vector to D, or a three-edge determinant; or finds the area of ABC as $\frac{1}{2}$|their (a) result|.
	$V = \frac{1}{6}\,\lvert$"-60"$\rvert = \ldots$	**M1**	Takes one sixth of the modulus of their scalar triple product, or their $\frac{1}{3}\times$ area $\times$ distance.
	$V = 10$	**A1**	Must be positive.
		[4]	
	Total for Question 11	**10**	

Question	Scheme	Marks	Notes
12	$L_3: \ (x, y, z) = (3 + t, \ -6 - 5t, \ 1 - 2t)$ $8 + 3\lambda = 9 - 4\mu$ $3 + 2\lambda = -2 + 3\mu$ $1 - \lambda = -1 + 3\mu$	**M1**	Writes two of the lines in parametric form, with different parameters, and equates all three coordinates. Condone sign slips.
	$\Rightarrow \lambda = \ldots, \ \mu = \ldots$	**dM1**	Solves two of their equations simultaneously for both parameters, by any method.
	$\lambda = -1, \ \mu = 1 \Rightarrow L_1 \cap L_2 = (5, \ 1, \ 2)$	**A1**	One correct vertex. The vertices may be found in any order.
	$L_1 \cap L_3 = (2, \ -1, \ 3)$	**A1**	
	$L_2 \cap L_3 = (1, \ 4, \ 5)$	**A1**	
	"$(-3\mathbf{i} - 2\mathbf{j} + \mathbf{k})$" $\times$ "$(-4\mathbf{i} + 3\mathbf{j} + 3\mathbf{k})$" $= \ldots$	**M1**	Finds two sides as differences of their vertices and expands their vector product by determinant or component formula, all three components attempted. Condone sign slips; a product written without working needs two components correct.
	$= -9\mathbf{i} + 5\mathbf{j} - 17\mathbf{k}$	**A1**	Or its negative, depending on the sides chosen.
	Area $= \frac{1}{2}\sqrt{(-9)^2 + 5^2 + (-17)^2}$	**dM1**	Takes half the magnitude of their vector product, using all three components.
	$= \frac{1}{2}\sqrt{395}$	**A1**	Or awrt 9.94.
	Total for Question 12	**9**	

Question	Scheme	Marks	Notes
13(a)	$z = t \Rightarrow x = t - 2,\ y = \ldots$	**M1**	Uses both plane equations to express two coordinates in terms of a parameter, or finds a point on ℓ and the vector product of the normals.
	$\left(t-2,\ \frac{7-t}{2},\ t\right)$	**A1**	Any correct parametrisation.
	$(t-11)^2 + \left(\frac{t+13}{2}\right)^2 + (t+1)^2 = 225$	**M1**	Sets the squared distance from A to their general point equal to 225, with all three components. Condone sign slips.
	$t^2 - 6t - 27 = 0$	**A1**	Correct three-term quadratic, oe.
	$t = \ldots \Rightarrow (\ldots,\ \ldots,\ \ldots)$	**dM1**	Solves their three-term quadratic (factorising with the constants' product equal to the constant term, the correct formula with their values, or completing the square; calculator allowed) and substitutes a root into their general point.
	$B(-5, 5, -3)$, $C(7, -1, 9)$	**A1**	Both points, with B the one whose z coordinate is negative.
		[6]	
13(b)	$\overrightarrow{AB} = -14\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$ $\overrightarrow{AC} = -2\mathbf{i} - 11\mathbf{j} + 10\mathbf{k}$ $\overrightarrow{AB} \times \overrightarrow{AC} = \ldots$	**M1**	Forms two sides of triangle ABC from their points, then their vector product by determinant or component formula, all three components attempted. Condone sign slips; a product written without working needs two components correct.
	$= -72\mathbf{i} + 144\mathbf{j} + 144\mathbf{k}$	**A1**	Any non-zero multiple of $-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$.
	$-x + 2y + 2z = -9 + 20 - 2$	**M1**	Forms $\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$ with their normal and a point of the plane, or substitutes such a point to find the constant.
	$x - 2y - 2z + 9 = 0$	**A1**	Any Cartesian form, such as $-x + 2y + 2z = 9$; a vector equation scores A0.
		[4]	

Question 13 continued

Question	Scheme	Marks	Notes
13(c)	$\overrightarrow{AD} = -\mathbf{i} - 6\mathbf{j} + (\alpha+1)\mathbf{k}$	**B1**	Or any vector from a point of plane ABC to D, either direction; or, by area × distance, the distance $\frac{\lvert 2\alpha-9\rvert}{3}$ from D to plane ABC, oe.
	$\overrightarrow{AD} \cdot$ "$-72\mathbf{i} + 144\mathbf{j} + 144\mathbf{k}$" $= 72 - 864 + 144(\alpha+1)$ $[= 144\alpha - 648]$	**M1**	Forms the scalar product of their $\overrightarrow{AB} \times \overrightarrow{AC}$ (factor restored if rescaled) with their vector to D, or a determinant, linear in α; or finds the area of ABC as $\frac{1}{2}$|their $\overrightarrow{AB} \times \overrightarrow{AC}$|.
	$\frac{1}{6}\lvert$"$144\alpha - 648$"$\rvert = 156 \Rightarrow \alpha = \ldots$	**M1**	Sets $\frac{1}{6}$ of their triple product, or its modulus, or their $\frac{1}{3}$× area × distance, equal to 156 or ± 156 and solves for α. One value is enough.
	$\alpha = 11$ or $\alpha = -2$	**A1**	Both values and no others.
		[4]	
13(d)	Direction of ℓ: $2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ $\overrightarrow{AD} = -\mathbf{i} - 6\mathbf{j} + 12\mathbf{k}$ $(2\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \times \overrightarrow{AD} = \ldots$	**M1**	Forms both lines' direction vectors, with their positive α, then their vector product by determinant or component formula, all three components attempted. Condone sign slips; a product written without working needs two components correct.
	$= -26\mathbf{j} - 13\mathbf{k}$	**A1**	Any non-zero multiple of $2\mathbf{j} + \mathbf{k}$.
	$\overrightarrow{BA} = 14\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ $\frac{\lvert 14(0) + 5(-26) + 2(-13)\rvert}{\sqrt{26^2 + 13^2}} = \ldots$	**dM1**	Divides the scalar product of a vector joining a point of each line with their common normal by the normal's magnitude.
	$= \frac{12}{\sqrt{5}} = 5.4$	**A1**	5.4, to 2 s.f. as required; an exact or 3 s.f. answer alone scores A0.
		[4]	
	Total for Question 13	**18**	