OPTIONAL PURE / VECTORS & MATRICES

Shortest Distances

Mark scheme 10 questions · 112 marks

EXAM BOARDS Edexcel FP1 9FM0/3A · Edexcel 9FM0 · AQA 7367 · OCR A H245 · OCR MEI H645 CIE 9231

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	Total	**112**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last accuracy mark of the working that reaches a printed result: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	$\overrightarrow{AP} = \begin{pmatrix} 2\lambda - 3 \\ \lambda - 3 \\ -2\lambda \end{pmatrix}$ $\begin{pmatrix} 2\lambda - 3 \\ \lambda - 3 \\ -2\lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = 0$ $\Rightarrow 9\lambda - 9 = 0 \Rightarrow \lambda = \ldots$	**M1**	Uses a general point on ℓ with a perpendicularity condition (scalar product zero, minimum of $\lvert\overrightarrow{AP}\rvert^2$, or plane through A normal to ℓ) and solves for λ.
	$\lambda = 1$	**A1**	
	$\overrightarrow{ON} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + \text{“1”} \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = \ldots$	**dM1**	Substitutes their λ into the equation of ℓ.
	$N(3, -1, 1)$	**A1**	Accept the position vector of N as a column.
		[4]	
1(b)	$\left\lvert\overrightarrow{AN}\right\rvert = \sqrt{(-1)^2 + (-2)^2 + (-2)^2} = 3$	**B1ft**	ft their N from (a). Correct magnitude of their $\overrightarrow{AN}$.
		[1]	
Total for Question 1		**5**	

Question	Scheme	Marks	Notes
2(a)	$\begin{pmatrix}4\\-2\\4\end{pmatrix} = 2\begin{pmatrix}2\\-1\\2\end{pmatrix} \Rightarrow$ normals parallel, so planes parallel	**B1**	Normal of Π_2 shown or stated to be a multiple of the normal of Π_1, with a conclusion.
	$(0,-1,0)$ on Π_1 dist $= \frac{\lvert 4(0)-2(-1)+4(0)-11\rvert}{\sqrt{4^2+(-2)^2+4^2}}$	**M1**	Uses a point on one plane in the point-to-plane distance formula with the other plane, or divides the difference of constants, normals made equal, by the normal's modulus.
	dist $= \frac{3}{2}$	**A1**	oe
		[3]	
2(b)	$\begin{pmatrix}-2\\-4\\4\end{pmatrix} = -2\begin{pmatrix}1\\2\\-2\end{pmatrix} \Rightarrow$ lines parallel	**B1**	Direction of ℓ_2 shown or stated to be a multiple of the direction of ℓ_1, with a conclusion.
	$\overrightarrow{BP} = \begin{pmatrix}\lambda-3\\2\lambda-3\\-2\lambda\end{pmatrix}$ $(\lambda-3)+2(2\lambda-3)+4\lambda=0 \Rightarrow \lambda = \ldots$	**M1**	Forms a joining vector to a general point of one line and sets its scalar product with the direction to zero (or minimises its modulus) and solves; or forms a joining vector $\times$ the direction.
	$\lambda = 1$	**A1**	The correct parameter for their choice of point and line, or the correct vector product.
	$\lambda =$ "1" $\Rightarrow \overrightarrow{BP} = \begin{pmatrix}-2\\-1\\-2\end{pmatrix}$ $\lvert\overrightarrow{BP}\rvert = \sqrt{(-2)^2+(-1)^2+(-2)^2}$	**dM1**	Substitutes their parameter into their joining vector and finds its magnitude, or divides the vector product's modulus by the direction's modulus.
	dist $= 3$	**A1**	
		[5]	
	Total for Question 2	**8**	

Question	Scheme	Marks	Notes
3(a)	$\frac{\lvert 3(5)+1(-5)+2(3)+4\rvert}{\sqrt{3^2+1^2+2^2}}$	**M1**	Substitutes P and the plane into the distance formula, or finds the foot of the perpendicular. Condone one sign slip.
	$= \frac{20}{\sqrt{14}}$ $\left[= \frac{10\sqrt{14}}{7}\right]$	**A1**	$\frac{20}{\sqrt{14}}$ oe, or awrt 5.35.
		[2]	
3(b)	$2+\lambda = 5 \Rightarrow \lambda = 3$	**M1**	Equates one component of the general point of L to that of P and solves for λ.
	$y = 1-2(3) = -5$, $z = -3+2(3) = 3$ so P lies on L	**A1 AG**	Both other components shown to match P at $\lambda = 3$, or $\lambda = 3$ from all three, with a conclusion.
		[2]	
3(c)	$3(2+\lambda)+(1-2\lambda)+2(-3+2\lambda) = -4$ $\Rightarrow \lambda = \ldots$	**M1**	Substitutes all three components of L into the plane equation and solves for λ. Condone sign slips.
	$\lambda =$ "-1" $\Rightarrow (2-1,\ 1+2,\ -3-2)$	**dM1**	Substitutes their λ into the equation of L.
	$(1, 3, -5)$	**A1**	
		[3]	
3(d)	$\mathbf{u}\cdot\mathbf{n} = 3-2+4 = 5$, $\lvert\mathbf{u}\rvert = 3$, $\lvert\mathbf{n}\rvert = \sqrt{14}$	**B1**	All three values, seen or used.
	$\sin\theta = \frac{\text{"5"}}{\text{"3"}\ \text{"}\sqrt{14}\text{"}} \Rightarrow \theta = \ldots$	**M1**	Uses $\sin\theta = \frac{\lvert\mathbf{u}\cdot\mathbf{n}\rvert}{\lvert\mathbf{u}\rvert\lvert\mathbf{n}\rvert}$ with their values, or finds the angle with the normal by cosine and subtracts it from 90°.
	$\theta = 26.5°$	**A1**	awrt 26.5°, or awrt 0.462 radians.
		[3]	
3(e)	$PC = (3-(-1))\times 3 = \ldots$ $[= 12]$	**M1**	Finds PC as the change in λ times their $\lvert\mathbf{u}\rvert$, or as the magnitude of $\overrightarrow{PC}$ using their point from (c).
	"12" $\times$ "$\frac{5}{3\sqrt{14}}$"	**dM1**	Multiplies their PC by their $\sin\theta$ from (d), or by the cosine of their angle with the normal.

Question 3 continued

Question	Scheme	Marks	Notes
	$= \frac{20}{\sqrt{14}}$, as in (a)	**A1**	$\frac{20}{\sqrt{14}}$ oe, or awrt 5.35, with a statement that it agrees with (a).
		[3]	
	Total for Question 3	**13**	

Question	Scheme	Marks	Notes
4(a)	$\overrightarrow{CD} = \begin{pmatrix} 3 \\ 3 \\ -3 \end{pmatrix}$	**B1**	Or any nonzero multiple, such as $\mathbf{i}+\mathbf{j}-\mathbf{k}$.
	$\sin\alpha = \frac{\lvert(1)(1)+(1)(1)+(-1)(1)\rvert}{\sqrt{3}\sqrt{3}}$	**M1**	Uses the scalar product of their direction of CD and $\mathbf{i}+\mathbf{j}+\mathbf{k}$ over the product of magnitudes, as $\sin\alpha$, or as a cosine then subtracts from $90°$.
	$\sin\alpha = \frac{1}{3}$	**A1**	Or the cosine of the angle with the normal $= \frac{1}{3}$.
	$\alpha = 19.5°$	**A1**	
		[4]	
4(b)	$\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$	**M1**	Uses C or D and their direction of CD from (a) in a vector or parametric equation.
	$(x, y, z) = (t, t, 2-t)$	**A1**	oe for their point and parameter.
	P_1: $t+t+2-t=3 \Rightarrow t = \ldots$ P_2: $2t-t=1 \Rightarrow t = \ldots$	**M1**	Substitutes their general point into both plane equations and solves each, or into one and tests the resulting point in the other.
	$t=1$ from both, so the point is common to P_1 and P_2	**A1**	The same parameter from both, or the point from one shown to satisfy the other, with a conclusion that it is common. A0 if the conclusion is missing.
	$(1, 1, 1)$	**A1**	
		[5]	
4(c)(i)	$(\mathbf{i}+\mathbf{j}+\mathbf{k}) \times (2\mathbf{i}-\mathbf{j})$ $= \mathbf{i}(0+1) - \mathbf{j}(0-2) + \mathbf{k}(-1-2)$	**M1**	Expands a determinant or uses the component formula for all three components. Condone one sign slip.
	$= \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$	**A1**	
		[2]	
4(c)(ii)	$\lvert\mathbf{i}+2\mathbf{j}-3\mathbf{k}\rvert = \sqrt{14}$ $\lvert\mathbf{i}+\mathbf{j}+\mathbf{k}\rvert = \sqrt{3},\ \lvert2\mathbf{i}-\mathbf{j}\rvert = \sqrt{5}$	**B1**	All three needed.
	$\sin\theta = \frac{\text{“}\sqrt{14}\text{”}}{\sqrt{3}\sqrt{5}} \Rightarrow \theta = \ldots$	**M1**	Divides the magnitude of their vector from (c)(i) by the product of the normals' magnitudes as $\sin\theta$, and finds θ. Not for a scalar-product route that doesn't use (c)(i).

Question 4 continued

Question	Scheme	Marks	Notes
	$\theta = 75.0°$	**A1**	
		[3]	
4(c)(iii)	$\begin{pmatrix}0\\0\\2\end{pmatrix} - \begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}-1\\-1\\1\end{pmatrix}$ $\begin{pmatrix}-1\\-1\\1\end{pmatrix} \times \begin{pmatrix}1\\2\\-3\end{pmatrix} = \ldots$	**M1**	Forms the vector from a point on both planes, such as their point from (b), to C, and crosses it with their direction from (c)(i). Condone sign slips.
	$\begin{pmatrix}1\\-2\\-1\end{pmatrix}$	**A1**	Or its negative.
	dist $= \frac{\text{“}\sqrt{6}\text{”}}{\sqrt{14}}$	**dM1**	Divides the magnitude of their cross product by the magnitude of their direction.
	$= \sqrt{\frac{3}{7}} \approx 0.655$	**A1**	
		[4]	
Total for Question 4		**18**	

Question	Scheme	Marks	Notes
5(a)	$(1)(1)+(-2)(2)+(3)(1)=\ldots$ $(1)(4)+(-2)(-1)+(3)(-2)=\ldots$	**M1**	Forms the scalar product of the vector with both direction vectors, or finds the vector product of the direction vectors.
	$1-4+3=0$ and $4+2-6=0$ Perpendicular to both direction vectors $\Rightarrow$ perpendicular to Π	**A1 AG**	Both products shown evaluated to zero, or the vector product shown to be a multiple of the vector, with a conclusion.
		[2]	
5(b)	$x-2y+3z=(1)(2)+(-2)(-1)+(3)(3)=\ldots$	**M1**	Substitutes $(2,-1,3)$, or another point of Π, into $x-2y+3z=d$, or expands $(\mathbf{i}-2\mathbf{j}+3\mathbf{k})\cdot(\mathbf{r}-\mathbf{a})=0$. Condone sign slips.
	$x-2y+3z=13$	**A1**	Cartesian form, oe. $\mathbf{r}\cdot(\mathbf{i}-2\mathbf{j}+3\mathbf{k})=13$ alone scores A0.
		[2]	
5(c)	$A=(-2+t,\ 1+t,\ 4+t)$ $\frac{\lvert(-2+t)-2(1+t)+3(4+t)-\text{“13”}\rvert}{\sqrt{1+4+9}}$ $=\frac{9}{\sqrt{14}}$	**M1**	Uses a general point of ℓ in a point-to-plane distance with their plane from (b), set equal to $\frac{9}{\sqrt{14}}$, or an equivalent method such as parallel planes. Condone sign slips.
	$\lvert 2t-5\rvert=9$	**A1**	oe, such as $(2t-5)^2=81$ or $2t-5=\pm 9$.
	$2t-5=9$ or $2t-5=-9\Rightarrow t=\ldots$ $[=7,-2]$	**dM1**	Solves both cases, or their quadratic, for two values of t.
	$A=(5,8,11)$ or $(-4,-1,2)$	**A1**	Both points, as coordinates or position vectors, and no others.
		[4]	
	Total for Question 5	**8**	

Question	Scheme	Marks	Notes
6(a)	$\overrightarrow{AB} = \begin{pmatrix}1\\2\\0\end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix}1\\0\\-1\end{pmatrix}$ $\overrightarrow{AB} \times \overrightarrow{AC} = \ldots$	**M1**	Finds two non-parallel vectors in the plane and attempts their cross product, or solves two scalar products with them equal to zero. Condone sign slips.
	$\mathbf{n} = \begin{pmatrix}-2\\1\\-2\end{pmatrix}$	**A1**	Any nonzero multiple.
	$2x - y + 2z = 2(1) - (-2) + 2(3)$	**M1**	Substitutes a point of Π into $ax + by + cz = d$ or $\mathbf{r} \cdot \mathbf{n} = d$, with their normal, to find d.
	$2x - y + 2z = 10$	**A1**	oe, including $\mathbf{r} \cdot \begin{pmatrix}2\\-1\\2\end{pmatrix} = 10$ or any multiple.
	$\text{dist} = \frac{\lvert 2(4)-1+2(0)-10\rvert}{\sqrt{2^2+(-1)^2+2^2}} = \ldots$	**M1**	Uses the point-to-plane formula with D and their plane $ax + by + cz = d$, or projects $\overrightarrow{AD}$ onto their normal. Condone sign slips.
	$= 1$	**A1**	
		[6]	
6(b)	$\begin{pmatrix}2\\1\\2\end{pmatrix} \times \begin{pmatrix}1\\-2\\1\end{pmatrix} = \ldots$	**M1**	Attempts the cross product of the two direction vectors, or solves two scalar products with them equal to zero. Condone sign slips.
	$= \begin{pmatrix}5\\0\\-5\end{pmatrix}$	**A1**	Any nonzero multiple.
	$\begin{pmatrix}2\\-3\\5\end{pmatrix} \cdot \text{“}\begin{pmatrix}5\\0\\-5\end{pmatrix}\text{”}$ $[= -15]$ $\text{dist} = \frac{15}{\sqrt{5^2+0^2+(-5)^2}} = \ldots$	**dM1**	Divides the scalar product of a vector joining the lines with their normal by the normal's magnitude. Condone a missing modulus.
	$= \frac{3\sqrt{2}}{2}$	**A1**	Any exact form, such as $\frac{3}{\sqrt{2}}$.
		[4]	
	Total for Question 6	**10**	

Question	Scheme	Marks	Notes
7(i)	Sets $z = t$: $x - y + t = 6,\ 2x + y - t = 3$ $\Rightarrow x = \ldots,\ y = \ldots$	**M1**	Solves the plane equations with one variable as parameter, or crosses the normals and finds a common point. Condone slips.
	$\mathbf{r} = \begin{pmatrix} 3 \\ -3 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$	**A1**	Any point on ℓ_2 and any multiple of the direction.
	$\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \times \text{“}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\text{”} = \ldots$	**M1**	Forms the vector product of the two directions, or finds a vector perpendicular to both. At least two components correct for their vectors.
	$= \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$	**A1**	Or any nonzero multiple.
	$\dfrac{\left\lvert \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} \cdot \text{“}\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}\text{”} \right\rvert}{\sqrt{11}} = \ldots$	**dM1**	Divides the scalar product of a vector joining the two lines with their normal by the normal's magnitude. Condone a missing modulus.
	$\dfrac{2}{\sqrt{11}}$	**A1**	oe, or awrt 0.603.
		[6]	
7(ii)	$\text{“}\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}\text{”} \cdot \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = \ldots$	**M1**	Forms $ax + by + cz = d$ with their normal from (i) and finds d from a point on ℓ_1.
	$3x - y + z = 10$	**A1**	Any Cartesian equivalent. Not for a vector equation alone.
		[2]	
	Total for Question 7	**8**	

Question	Scheme	Marks	Notes
8(a)	$\det\begin{pmatrix}1 & \lambda & 1\\ 2 & -1 & 2\\ 2 & 1 & -1\end{pmatrix}$ $= 1(1-2) - \lambda(-2-4) + 1(2+2)$	**M1**	Expands the determinant of the coefficient matrix by any row or column, giving an expression in λ. Condone sign slips.
	$= 6\lambda + 3$	**A1**	oe, such as $3(2\lambda+1)$.
	$6\lambda + 3 = 0 \Rightarrow \lambda = -\frac{1}{2}$ Unique point for all $\lambda \neq -\frac{1}{2}$	**A1**	Needs the conclusion that the point is unique for every other λ. The case $\lambda = -\frac{1}{2}$ need not be examined further.
		[3]	
8(b)	$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1 & 2 & 1\\ 2 & -1 & 2\\ 2 & 1 & -1\end{pmatrix}^{-1}\begin{pmatrix}4\\3\\5\end{pmatrix}$	**M1**	Writes the system as $\mathbf{Ax} = \mathbf{b}$ with $\lambda = 2$ and uses $\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$. Not for solving the equations without matrices.
	$\mathbf{A}^{-1} = \frac{1}{15}\begin{pmatrix}-1 & 3 & 5\\ 6 & -3 & 0\\ 4 & 3 & -5\end{pmatrix}$	**B1**	Any equivalent form; an inverse from a calculator is accepted.
	$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}2\\1\\0\end{pmatrix}$, $P = (2,1,0)$	**A1**	
		[3]	
8(c)	$\frac{x+1}{1} = \frac{y-1}{-1} = \frac{z}{1} = t$ $\Rightarrow x = t-1,\ y = 1-t,\ z = t$	**M1**	Sets the ratios equal to a parameter and writes x, y, z in terms of it, or reads a point and a direction from the Cartesian form. Condone sign slips.
	$\mathbf{r} = \begin{pmatrix}-1\\1\\0\end{pmatrix} + t\begin{pmatrix}1\\-1\\1\end{pmatrix}$	**A1**	Any point of ℓ and any multiple of its direction, with $\mathbf{r} =$ or equivalent.
		[2]	
8(d)	"$\begin{pmatrix}2\\1\\0\end{pmatrix}$" $- \begin{pmatrix}-1\\1\\0\end{pmatrix} = \begin{pmatrix}3\\0\\0\end{pmatrix}$ $\begin{pmatrix}3\\0\\0\end{pmatrix} \times \begin{pmatrix}1\\-1\\1\end{pmatrix} = \ldots$	**M1**	Forms the vector from any point of ℓ to their P and its vector product with the direction of ℓ. Condone sign slips.
	$= \begin{pmatrix}0\\-3\\-3\end{pmatrix}$	**A1**	oe, such as its negative, or the multiple that their multiple of the direction gives.

Question 8 continued

Question	Scheme	Marks	Notes
	$\frac{\sqrt{0^2+(-3)^2+(-3)^2}}{\sqrt{1^2+(-1)^2+1^2}} = \ldots$	**dM1**	Divides the magnitude of their vector product by the magnitude of their direction of ℓ.
	$= \sqrt{6}$	**A1**	$\sqrt{6}$ or awrt 2.45.
		[4]	
8(e)(i)	$\begin{pmatrix}1\\-1\\1\end{pmatrix} \cdot \begin{pmatrix}2\\1\\-1\end{pmatrix} = 2-1-1$	**M1**	Forms the scalar product of the direction of ℓ with the normal of the plane.
	$= 0$	**A1**	
	$2(-1)+1-0 = -1 \neq 5$ so ℓ is not in the plane, and is parallel to it	**A1 AG**	Needs the zero scalar product and a point of ℓ shown not to satisfy the plane's equation, then a conclusion of parallel in words.
		[3]	
8(e)(ii)	$\frac{\lvert 2(-1)+1(1)-1(0)-5\rvert}{\sqrt{2^2+1^2+(-1)^2}} = \ldots$	**M1**	Substitutes any point of ℓ into the point-to-plane distance formula, or uses the parallel plane through ℓ. Unquoted, implied only by a correct substitution.
	$= \frac{6}{\sqrt{6}} = \sqrt{6}$	**A1**	$\sqrt{6}$ or awrt 2.45.
		[2]	
	Total for Question 8	**17**	

Question	Scheme	Marks	Notes
9(a)(i)	$\mathbf{u}\cdot\mathbf{v}=\lambda-4$	**B1**	
		[1]	
9(a)(ii)	$\mathbf{u}\times\mathbf{v}=(2-2)\mathbf{i}-(-\lambda-2)\mathbf{j}$ $+(\lambda+2)\mathbf{k}$	**M1**	Attempts all three components by a determinant or the component formula. Condone sign slips.
	$=(\lambda+2)(\mathbf{j}+\mathbf{k})$	**A1**	oe, such as a column vector; need not be factorised.
		[2]	
9(b)(i)	$\lvert 2\mathbf{i}-2\mathbf{j}+2\mathbf{k}\rvert=2\sqrt{3}$, $\lvert\mathbf{i}+\mathbf{j}-\mathbf{k}\rvert=\sqrt{3}$	**B1**	
	$\cos\theta=\dfrac{\lvert\text{“}2-4\text{”}\rvert}{2\sqrt{3}\times\sqrt{3}}=\ldots$	**M1**	Divides their (a)(i) at $\lambda=2$ by the product of their magnitudes in the cosine formula. Condone a missing modulus.
	$\theta=70.5^\circ$	**A1**	awrt 70.5° or 1.23 rad, or $\arccos\frac{1}{3}$.
		[3]	
9(b)(ii)	$(\mathbf{i}-2\mathbf{j}+2\mathbf{k})\times(\mathbf{i}+\mathbf{j}-\mathbf{k})=3(\mathbf{j}+\mathbf{k})$	**B1ft**	ft their (a)(ii) at $\lambda=1$; any nonzero multiple accepted.
	$\dfrac{\lvert(\mathbf{i}+3\mathbf{j}+4\mathbf{k})\cdot\text{“}(3\mathbf{j}+3\mathbf{k})\text{”}\rvert}{\lvert\text{“}3\mathbf{j}+3\mathbf{k}\text{”}\rvert}$ $=\ldots$	**M1**	Divides the scalar product of a vector joining the two lines with their common normal by its magnitude. Condone sign slips in the joining vector.
	$\dfrac{7\sqrt{2}}{2}$	**A1**	$\frac{7\sqrt{2}}{2}$ or $\frac{7}{2}\sqrt{2}$. Not for $\frac{7}{\sqrt{2}}$ or a decimal alone.
		[3]	
	Total for Question 9	**9**	

Question	Scheme	Marks	Notes
10(a)	$\overrightarrow{AD} = \begin{pmatrix}1\\1\\m\end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix}-1\\1\\2\end{pmatrix}$, $\overrightarrow{AB} = \begin{pmatrix}1\\0\\0\end{pmatrix}$	**B1**	All three, oe; any vector joining a point of one line to a point of the other in place of $\overrightarrow{AB}$.
	$\begin{pmatrix}1\\1\\m\end{pmatrix} \times \begin{pmatrix}-1\\1\\2\end{pmatrix} = \begin{pmatrix}2-m\\-2-m\\2\end{pmatrix}$	**M1 A1**	**M1** Attempts the vector product of their two direction vectors. Condone sign slips. **A1** oe, such as its negative.
	$\dfrac{\left\lvert\overrightarrow{AB} \cdot \text{“}\mathbf{n}\text{”}\right\rvert}{\lvert\text{“}\mathbf{n}\text{”}\rvert} = \dfrac{1}{\sqrt{14}}$, $\mathbf{n}$ their vector product	**M1**	Uses the scalar product of their joining vector with their vector product, divided by its magnitude, and sets it equal to $\frac{1}{\sqrt{14}}$.
	$\dfrac{\lvert 2-m\rvert}{\sqrt{2m^2+12}} = \dfrac{1}{\sqrt{14}}$	**A1**	Correct equation oe, need not be simplified. Condone a missing modulus.
	$14(2-m)^2 = 2m^2+12$ $\Rightarrow 3m^2 - 14m + 11 = 0 \Rightarrow m = \ldots$	**dM1**	Squares and solves their three-term quadratic in m by any means, a calculator included.
	$m = 1$ or $m = \frac{11}{3}$; m is an integer, so $m = 1$	**A1 AG**	cso. Both roots from a correct quadratic, with $\frac{11}{3}$ rejected as not an integer. A0 if $m = 1$ is only verified.
		[7]	
10(b)	$\overrightarrow{AC} \times \overrightarrow{AD} = \begin{pmatrix}0\\1\\2\end{pmatrix} \times \begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}-1\\2\\-1\end{pmatrix}$	**M1 A1**	**M1** Attempts the vector product of a vector from a point on the line to C with the line's direction, using $m = 1$. Condone sign slips. **A1** Or its negative.
	$d = \dfrac{\lvert\text{“}-\mathbf{i}+2\mathbf{j}-\mathbf{k}\text{”}\rvert}{\left\lvert\overrightarrow{AD}\right\rvert} = \dfrac{\sqrt{6}}{\sqrt{3}}$	**dM1**	Divides the magnitude of their vector product by the magnitude of their direction vector.
	$d = \sqrt{2}$	**A1**	Or awrt 1.41.
		[4]	

Question 10 continued

Question	Scheme	Marks	Notes
10(c)	$\overrightarrow{AB} \times \overrightarrow{AD} = \begin{pmatrix}1\\0\\0\end{pmatrix} \times \begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}0\\-1\\1\end{pmatrix}$	**M1 A1**	**M1** Attempts the vector product of two non-parallel vectors in plane ABD, or finds a normal from two scalar products set to zero. Condone sign slips. **A1** Any nonzero multiple.
	$\mathbf{n}_{ACD} = \begin{pmatrix}-1\\2\\-1\end{pmatrix}$	**B1**	Any nonzero multiple; may be taken from (b).
	$\cos\theta = \dfrac{\lvert\text{“}(0,-1,1)\text{”} \cdot \text{“}(-1,2,-1)\text{”}\rvert}{\sqrt{2}\sqrt{6}}$	**M1**	Uses the scalar product of their two normals divided by the product of their magnitudes.
	$\cos\theta = \dfrac{3}{\sqrt{12}} = \dfrac{\sqrt{3}}{2}$ so the acute angle is $\arccos\left(\frac{\sqrt{3}}{2}\right)$	**A1 AG**	cso, from correct normals. A negative cosine must be made positive, or the obtuse angle replaced by its supplement. Needs a concluding statement.
		[5]	
	Total for Question 10	**16**	