OPTIONAL PURE / VECTORS & MATRICES

# Scalar Triple Product

Mark scheme 10 questions · 105 marks

EXAM BOARDS Edexcel FP1 9FM0/3A · OCR A Y545

## Questions

| Question | Marks by part | Marks | Page |
|---|---|---|---|
| Question 1 | (a) 2 (b) 2 (c) 2 (d) 2 | 8 | 4 |
| Question 2 | (a) 4 (b) 4 (c) 4 | 12 | 5 |
| Question 3 | (a) 4 (b)(i) 3 (b)(ii) 2 | 9 | 7 |
| Question 4 | (a) 2 (b) 5 (c) 2 (d) 4 | 13 | 8 |
| Question 5 | (a) 4 (b) 2 (c) 5 | 11 | 10 |
| Question 6 | (a) 6 (b) 6 (c) 3 | 15 | 12 |
| Question 7 | (a) 4 (b) 3 | 7 | 14 |
| Question 8 | (a) 3 (b) 4 (c) 6 | 13 | 15 |
| Question 9 | (a) 7 (b) 1 (c) 2 (d) 3 | 13 | 17 |
| Question 10 | | 4 | 19 |
| | **Total** | **105** | |

# Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

### Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

| Code | Meaning |
|---|---|
| **M1** | Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted. |
| **dM1** | Dependent method: as M1, but only if the method mark before it was earned, unless its note names another. |
| **A1** | Accuracy: a correct answer or step. It needs the method mark it follows. |
| **A1ft** | As A1, following through from an earlier answer that the note names. |
| **A1 AG** | The last accuracy mark of the working that reaches a printed result: complete working, with no errors. |
| **B1** | A correct value, fact or statement. It needs no method mark. |
| **B1ft** | As B1, following through from an earlier answer that the note names. |
| **E1** | A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark. |

### Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

| | | | |
|---|---|---|---|
| **ft** | follow through<br>Correct work from the student's own earlier wrong value earns the mark. | **AG** | answer given<br>The answer is printed in the question, so every step to it must be shown. |
| **cao** | correct answer only<br>No follow-through from an earlier error. | **cso** | correct solution only<br>No error anywhere in the part, not just in its answer. |
| **oe** | or equivalent | **awrt** | answers which round to |
| **soi** | seen or implied<br>Earned if written, or if later working shows it was used. | **isw** | ignore subsequent working<br>A correct answer, once reached, isn't spoiled by later working. |
| **SC** | special case<br>Credit, as the note says, for work outside the main method. | **their** | the student's own earlier value |
| **condone** | a slip that doesn't cost the mark | [ ] | working the mark doesn't need |
| **s.f.** | significant figures | **d.p.** | decimal places |

### Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

### Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

### Value of $g$

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state $g$, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **1(a)** | $\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\2&-1&1\\1&3&-2\end{vmatrix}$<br>$=(2-3)\mathbf{i}-(-4-1)\mathbf{j}+(6+1)\mathbf{k}$ | **M1** | Expands the determinant, or uses the component formula, all three components attempted. Condone sign slips; a product written without working needs two components correct. |
| | $=\begin{pmatrix}-1\\5\\7\end{pmatrix}$ | **A1** | Or $-\mathbf{i}+5\mathbf{j}+7\mathbf{k}$. |
| | | **[2]** | |
| **1(b)** | Area $=\frac{1}{2}\sqrt{\text{“}(-1)^2+5^2+7^2\text{”}}$ | **M1** | Finds half the magnitude of their three-component $\mathbf{a}\times\mathbf{b}$. |
| | $=\frac{5\sqrt{3}}{2}$ | **A1** | Any exact equivalent, such as $\frac{\sqrt{75}}{2}$. |
| | | **[2]** | |
| **1(c)** | $\begin{pmatrix}\lambda\\-1\\2\end{pmatrix}\cdot\text{“}\begin{pmatrix}-1\\5\\7\end{pmatrix}\text{”}=-\lambda-5+14$ | **M1** | Sums the three products of components of $\mathbf{c}$ and their three-component $\mathbf{a}\times\mathbf{b}$, or evaluates the determinant with rows $\mathbf{c}$, $\mathbf{a}$, $\mathbf{b}$. Condone sign slips. |
| | $\mathbf{c}\cdot(\mathbf{a}\times\mathbf{b})=9-\lambda$ | **A1** | |
| | | **[2]** | |
| **1(d)** | $\frac{1}{6}\lvert\text{“}9-\lambda\text{”}\rvert=1\Rightarrow\lambda=\ldots$ | **M1** | Sets $\frac{1}{6}$ of their expression in $\lambda$ from (c) equal to 1 or $-1$, and solves for $\lambda$. Condone a missing modulus. |
| | $\lambda=3$ or $\lambda=15$ | **A1** | Both values and no others. |
| | | **[2]** | |
| | **Total for Question 1** | **8** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **2(a)** | $\mathbf{r} = \begin{pmatrix} 3+\lambda \\ 2-2\lambda \\ 5+2\lambda \end{pmatrix}$ | **B1** | Any correct general point of $\ell$, in any parameter, oe. |
| | $\text{“}\begin{pmatrix} 3+\lambda \\ 2-2\lambda \\ 5+2\lambda \end{pmatrix}\text{”} \cdot \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = 0$<br>$\Rightarrow 9 + 9\lambda = 0 \Rightarrow \lambda = \ldots$ | **M1** | Sets the scalar product of their general point with the direction of $\ell$ equal to zero, or uses an equivalent condition, and solves for the parameter. Condone sign slips. |
| | $\lambda = -1$ | **A1** | Correct value for their parameter. |
| | $\overrightarrow{OC} = \begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix}$ | **A1** | Or $2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, in any vector form. |
| | | [4] | |
| **2(b)** | $\overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & -1 \\ 1 & 2 & 0 \end{vmatrix}$<br>$= \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix}$ | **M1 A1** | **M1** Attempts the vector product of two of $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$, all three components, or their $3 \times 3$ determinant. Condone sign slips; a product written without working needs two components correct.<br>**A1** Correct for the pair used, in either order, or the determinant's three $2 \times 2$ minors, or its value $\pm 18$. |
| | $V = \frac{1}{6}\left\lvert \text{“}\begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix}\text{”} \cdot \text{“}\begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix}\text{”} \right\rvert = \ldots$ | **dM1** | Forms the scalar product of their vector product with the third position vector, or combines the minors with its entries or uses their determinant, and takes one sixth of it. |
| | $V = 3$ | **A1** | |
| | | [4] | |
| **2(c)** | $\det(\mathbf{M}) = k(k-2) - 1(3-0) + 0 = \ldots$ | **M1** | Expands the determinant of $\mathbf{M}$ along any row or column, or by any valid rule. Condone sign slips. |
| | $\det(\mathbf{M}) = k^2 - 2k - 3$ | **A1** | |
| | $\text{“3”} \left\lvert \text{“}k^2 - 2k - 3\text{”} \right\rvert = 36$<br>$\Rightarrow k^2 - 2k - 15 = 0 \Rightarrow k = \ldots$ | **M1** | Sets their volume from (b) times their determinant, or its modulus, equal to 36 or $\pm 36$, and solves the resulting quadratic by any means. |

*Question 2 continued*

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| | $k = 5$ | **A1** | $k = 5$ only, with $k = -3$ rejected or not given. The case $\det(\mathbf{M}) = -12$ need not be considered. |
| | | **[4]** | |
| | **Total for Question 2** | **12** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **3(a)** | $\mathbf{a}\times\mathbf{b} = (4+2q)\mathbf{i} + (q-2p)\mathbf{j}$ $-(2p+2)\mathbf{k}$ | **M1 A1** | **M1** Expands a determinant or uses the component formula for all three components in $p$ and $q$, or forms $\mathbf{a}\cdot(-2\mathbf{i}+\mathbf{j}+2\mathbf{k}) = 0$. Condone sign slips.<br>**A1** Correct vector product, in any form, or $-2p+2+2q=0$ oe. |
| | "$q-2p$" $=\frac{1}{2}$"$(-2p-2)$" $\Rightarrow q = p-1$<br>$p-4+2q=-15 \Rightarrow p=\ldots,\ q=\ldots$ | **M1** | Uses two components of their $\mathbf{a}\times\mathbf{b}$ in the ratio $-2:1:2$, or $\mathbf{a}\cdot(-2\mathbf{i}+\mathbf{j}+2\mathbf{k}) = 0$, with the scalar product equation, and solves for $p$ and $q$. Condone sign slips. |
| | $p=-3, \quad q=-4$ | **A1** | |
| | | **[4]** | |
| **3(b)(i)** | $\mathbf{c}\cdot(\mathbf{a}\times\mathbf{b}) = \begin{pmatrix} d \\ e \\ f \end{pmatrix} \cdot$ "$\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$" $= -4d+2e+4f$ | **M1** | Forms the scalar triple product of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ in any order, or as a determinant, with their $p$ and $q$. Condone sign slips. |
| | $\frac{1}{6}\lvert$"$-4d+2e+4f$"$\rvert = 4$ | **dM1** | Sets $\frac{1}{6}$ of their scalar triple product, or of its modulus, equal to 4 or $\pm 4$. |
| | $-2d+e+2f=12$ or $-2d+e+2f=-12$ | **A1** | Both equations, oe such as $-4d+2e+4f=\pm 24$. |
| | | **[3]** | |
| **3(b)(ii)** | Two planes parallel to plane $OAB$,<br>one on each side of it, with $C$ in one | **E1** | States two planes parallel to plane $OAB$, one on each side of it or equidistant from it, with $C$ lying in one of them. |
| | dist $= \frac{12}{\sqrt{4+1+4}} = 4$ | **B1** | |
| | | **[2]** | |
| | **Total for Question 3** | **9** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **4(a)** | $\overrightarrow{AB} = \begin{pmatrix}3\\p\\0\end{pmatrix} - \begin{pmatrix}2\\-1\\1\end{pmatrix} = \ldots$ | **M1** | Subtracts the position vector of $A$ from those of at least two of $B$, $C$ and $D$. Condone sign slips. |
| | $\overrightarrow{AB} = \begin{pmatrix}1\\p+1\\-1\end{pmatrix}$<br>$\overrightarrow{AC} = \begin{pmatrix}p-2\\2\\2\end{pmatrix}$<br>$\overrightarrow{AD} = \begin{pmatrix}2\\1\\1\end{pmatrix}$ | **A1** | All three correct, as column vectors or in **i**, **j**, **k** form. |
| | | **[2]** | |
| **4(b)** | $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\1 & p+1 & -1\\p-2 & 2 & 2\end{vmatrix}$<br>$= \big(2(p+1)+2\big)\mathbf{i} - \big(2+(p-2)\big)\mathbf{j}$<br>$+ \big(2-(p+1)(p-2)\big)\mathbf{k}$ | **M1** | Expands the vector product of their $\overrightarrow{AB}$ and $\overrightarrow{AC}$ by determinant or component formula, all three components attempted, or the $3\times 3$ determinant of $\overrightarrow{AD}$, $\overrightarrow{AB}$, $\overrightarrow{AC}$. Condone sign slips. |
| | $= \begin{pmatrix}2p+4\\-p\\-p^2+p+4\end{pmatrix}$ | **A1** | Correct vector product, or its three $2\times 2$ minors, oe unsimplified. |
| | $\begin{pmatrix}2\\1\\1\end{pmatrix} \cdot$ "$\begin{pmatrix}2p+4\\-p\\-p^2+p+4\end{pmatrix}$"<br>$= 2(2p+4) - p + \big(-p^2+p+4\big)$ | **dM1** | Forms the scalar product of their vector product with their $\overrightarrow{AD}$, or combines the minors with the entries of $\overrightarrow{AD}$, summing three products. Condone sign slips. |
| | $= -p^2 + 4p + 12$ | **A1** | |
| | $= (6-p)(p+2)$ | **A1 AG** | Correct vector product and scalar product leading without error to $(6-p)(p+2)$; expanding the printed form to match is accepted. |
| | | **[5]** | |
| **4(c)** | Sets the triple product to zero:<br>$(6-p)(p+2) = 0 \Rightarrow p = \ldots$ | **M1** | Sets the scalar triple product, or the volume, equal to zero and solves. |

*Question 4 continued*

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| | $p = 6$ or $p = -2$ | **A1** | Both values, with the reason that a zero triple product, or zero volume, means the points are coplanar. |
| | | **[2]** | |
| **4(d)** | $\frac{1}{6}\|(6-p)(p+2)\| = 8$ | **M1** | Sets $\frac{1}{6}$ of the triple product, or of its modulus, equal to 8. Implied by $(6-p)(p+2) = 48$, $-48$ or $\pm 48$. |
| | $p^2 - 4p + 36 = 0$, $p^2 - 4p - 60 = 0$ | **A1** | Both equations oe, or $-p^2 + 4p + 12 = \pm 48$; or $= -48$ alone with $= 48$ shown impossible. |
| | $p^2 - 4p + 36 = 0$: $b^2 - 4ac = -128 < 0$<br>$\Rightarrow$ no real roots<br>$(p-10)(p+6) = 0 \Rightarrow p = \ldots$ | **dM1** | Solves at least one of their quadratics by any means, including a calculator. |
| | $p = 10$ or $p = -6$ | **A1** | Both values and no others. |
| | | **[4]** | |
| | **Total for Question 4** | **13** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **5(a)** | $\overrightarrow{OC} = \overrightarrow{OD} + \overrightarrow{AB} \Rightarrow C(2, -7, 1)$ | **B1** | Accept as a position vector. |
| | $\overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & -1 \\ -3 & -4 & 1 \end{vmatrix}$ $= (-1-4)\mathbf{i} - (2-3)\mathbf{j} + (-8-3)\mathbf{k}$ | **M1** | Forms the vector product of two non-parallel sides of $P$ by determinant or component formula, all three components attempted. Condone sign slips; a product written without working needs two components correct. |
| | $= \begin{pmatrix} -5 \\ 1 \\ -11 \end{pmatrix}$ | **A1** | Correct for the two sides of $P$ used, so its negative also scores. |
| | Area $= \sqrt{(-5)^2 + 1^2 + (-11)^2} = 7\sqrt{3}$ | **A1** | $7\sqrt{3}$ only; $\sqrt{147}$ or a decimal alone does not score. |
| | | **[4]** | |
| **5(b)** | $\left(\mathbf{r} - \text{“}\begin{pmatrix} 2 \\ -7 \\ 1 \end{pmatrix}\text{”}\right) \times \text{“}\begin{pmatrix} -5 \\ 1 \\ -11 \end{pmatrix}\text{”} = \mathbf{0}$ | **M1 A1** | **M1** Uses their $C$, or another point of $\ell$, as **a** and their vector product from (a), or a multiple, as **b** in the required form. **A1** Correct equation in the required form, **b** any non-zero multiple of $5\mathbf{i} - \mathbf{j} + 11\mathbf{k}$, **a** any point on $\ell$. |
| | | **[2]** | |
| **5(c)** | $\overrightarrow{AQ} = \begin{pmatrix} k-3 \\ k+2 \\ 3 \end{pmatrix}$ | **B1** | Or the vector between $Q$ and any vertex of $P$, in either direction, or the equation of the plane of $P$, oe $5x - y + 11z = 28$. |
| | $\overrightarrow{AQ} \cdot \left(\overrightarrow{AB} \times \overrightarrow{AD}\right) = \begin{pmatrix} k-3 \\ k+2 \\ 3 \end{pmatrix} \cdot \text{“}\begin{pmatrix} -5 \\ 1 \\ -11 \end{pmatrix}\text{”}$ $= -5(k-3) + (k+2) - 33$ | **M1** | Forms the scalar product of their vector product from (a) with their vector to $Q$, or area times the distance from $Q$ to the plane of $P$. Condone sign slips. |
| | $= -4k - 16$ | **A1** | oe, including $4k + 16$. |
| | $\lvert -4k - 16 \rvert = 36 \Rightarrow 4k + 16 = 36$ or $4k + 16 = -36$ $\Rightarrow k = \ldots$ | **dM1** | Sets their linear expression, or its modulus, equal to 36 or $\pm 36$, or squares both sides, and solves for $k$. |

*Question 5 continued*

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| | $k = 5$ or $k = -13$ | **A1** | Both values and no others. |
| | | **[5]** | |
| | **Total for Question 5** | **11** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **6(a)** | $2(1+3t)-1+2(4-6t)=7 \Rightarrow t=\ldots$ | **M1** | Substitutes a parametric form of line $AB$ or $AC$ into the equation of $\Pi$ and solves for the parameter. Condone sign slips. |
| | $P(2,1,2)$, $Q(1,-1,2)$ | **A1** | Both points, as coordinates or position vectors, or $\overrightarrow{AP}=\frac{1}{3}\overrightarrow{AB}$ and $\overrightarrow{AQ}=\frac{1}{2}\overrightarrow{AC}$. |
| | $\overrightarrow{AB}\times\overrightarrow{AC}=\begin{pmatrix}-24\\12\\-12\end{pmatrix}$<br>Area of $ABC=\frac{1}{2}\sqrt{(-24)^2+12^2+(-12)^2}$ | **M1** | Finds a vector product of two edge vectors of a triangle, all three components attempted, and takes half its modulus. Condone sign slips in the product; a product written without working needs two components correct. |
| | $=6\sqrt{6}$ | **A1** | Any form, such as $\frac{1}{2}\sqrt{864}$, or awrt 14.7; or the correct area of a triangle in another valid split of $PBCQ$. |
| | Area of $APQ=\frac{1}{2}\left\lvert\overrightarrow{AP}\times\overrightarrow{AQ}\right\rvert=\sqrt{6}$<br>Area of $PBCQ=$ "$6\sqrt{6}$" $-\sqrt{6}$ | **dM1** | Subtracts an area of $APQ$, by vector product or as $\frac{AP}{AB}\times\frac{AQ}{AC}$ of area $ABC$, from their area of $ABC$; or adds the areas of triangles making up $PBCQ$. |
| | $=5\sqrt{6}$ | **A1** | Any exact equivalent, such as $\sqrt{150}$. |
| | | **[6]** | |
| **6(b)** | $2(1-3u)-(1+3u)+2(4+3u)=7 \Rightarrow u=\ldots$ | **M1** | Substitutes a parametric form of line $AD$ into the equation of $\Pi$ and solves for the parameter. Condone sign slips. |
| | $R(-1,3,6)$ | **A1** | As coordinates or position vector, or $\overrightarrow{AR}=\frac{2}{3}\overrightarrow{AD}$ oe. |
| | $\overrightarrow{AR}\cdot\left(\overrightarrow{AP}\times\overrightarrow{AQ}\right)$<br>$=$ "$\begin{pmatrix}-2\\2\\2\end{pmatrix}$" $\cdot$ "$\begin{pmatrix}-4\\2\\-2\end{pmatrix}$" $=\ldots$ | **M1** | Forms a scalar triple product of three edge vectors of $APQR$ from one vertex, by a scalar product with a vector product or a determinant. Condone sign slips. |
| | $=8$ | **A1** | Or $-8$ from another arrangement. |
| | $V=\frac{1}{6}\lvert$"8"$\rvert=\ldots$ | **dM1** | Takes $\frac{1}{6}$ of the modulus of their triple product. Condone the modulus omitted. |

*Question 6 continued*

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| | $= \frac{4}{3}$ | **A1** | Or awrt 1.33. |
| | | **[6]** | |
| **6(c)** | Volume of $ABCD = \frac{1}{6}\left\lvert\overrightarrow{AD} \cdot \left(\overrightarrow{AB} \times \overrightarrow{AC}\right)\right\rvert$<br>$= \frac{1}{6}\left\lvert\begin{pmatrix}-3\\3\\3\end{pmatrix} \cdot \text{“}\begin{pmatrix}-24\\12\\-12\end{pmatrix}\text{”}\right\rvert = \ldots$ | **M1** | Takes $\frac{1}{6}$ of the modulus of a scalar triple product of three edges of $ABCD$ from one vertex, or multiplies their volume of $APQR$ by $\frac{AB}{AP} \times \frac{AC}{AQ} \times \frac{AD}{AR}$. Condone sign slips. |
| | $= 12$ | **A1** | |
| | $\frac{4}{3} : \left(12 - \frac{4}{3}\right) = 1 : 8$ | **A1** | $1 : 8$ or any equivalent ratio, such as $\frac{4}{3} : \frac{32}{3}$, with the smaller piece first. |
| | | **[3]** | |
| | **Total for Question 6** | **15** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **7(a)** | $(\mathbf{r}-\mathbf{c})\times\mathbf{d}=\mathbf{0}\Rightarrow\mathbf{r}-\mathbf{c}$ parallel to $\mathbf{d}$<br>$\Rightarrow\mathbf{r}=\mathbf{c}+\mu\mathbf{d}$ | **B1** | Needs the reason that a zero vector product with $\mathbf{d}$ makes $\mathbf{r}-\mathbf{c}$ a multiple of $\mathbf{d}$. |
| | $\mathbf{a}+\lambda^*\mathbf{b}=\mathbf{c}+\mu^*\mathbf{d}$<br>$\Rightarrow\mathbf{a}\cdot(\mathbf{b}\times\mathbf{d})+\lambda^*\,\mathbf{b}\cdot(\mathbf{b}\times\mathbf{d})$<br>$\quad=\mathbf{c}\cdot(\mathbf{b}\times\mathbf{d})+\mu^*\,\mathbf{d}\cdot(\mathbf{b}\times\mathbf{d})$ | **M1** | Equates points of the two lines at the intersection, or writes $\mathbf{a}-\mathbf{c}$ as a combination of $\mathbf{b}$ and $\mathbf{d}$, and takes the scalar product with $\mathbf{b}\times\mathbf{d}$. |
| | $\mathbf{b}\times\mathbf{d}$ perpendicular to $\mathbf{b}$ and $\mathbf{d}$<br>$\Rightarrow\mathbf{b}\cdot(\mathbf{b}\times\mathbf{d})=\mathbf{d}\cdot(\mathbf{b}\times\mathbf{d})=0$ | **A1** | Both zero, with the perpendicularity reason. |
| | $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{d})=\mathbf{c}\cdot(\mathbf{b}\times\mathbf{d})$ | **A1 AG** | Printed result reached without error, with reasons for the common point and for distributing the scalar product. |
| | | **[4]** | |
| **7(b)** | $\mathbf{n}=\mathbf{b}\times\mathbf{d}$ | **B1** | Or any non-zero multiple, such as $\mathbf{d}\times\mathbf{b}$. |
| | $\mathbf{a}$ on $\ell_1$, $\mathbf{c}$ on $\ell_2$, so both in $\Pi$<br>$\Rightarrow\mathbf{a}\cdot\mathbf{n}=p,\ \mathbf{c}\cdot\mathbf{n}=p$ | **M1** | Substitutes the position vector of a known point of $\ell_1$ or $\ell_2$, such as $\mathbf{a}$ or $\mathbf{c}$, into $\mathbf{r}\cdot\mathbf{n}=p$ with their $\mathbf{n}$. |
| | $p=\mathbf{a}\cdot(\mathbf{b}\times\mathbf{d})$, $p=\mathbf{c}\cdot(\mathbf{b}\times\mathbf{d})$ | **A1** | Both, with $\mathbf{a}$ on $\ell_1$ and $\mathbf{c}$ on $\ell_2$ as the reason. Accept equivalent arrangements, or the multiple matching their $\mathbf{n}$. |
| | | **[3]** | |
| | **Total for Question 7** | **7** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **8(a)** | $\overrightarrow{OR}\cdot\left(\overrightarrow{OP}\times\overrightarrow{OQ}\right)=\begin{pmatrix}0\\0\\r\end{pmatrix}\cdot\begin{pmatrix}0\\0\\pq\end{pmatrix}$ | **M1** | Forms the scalar triple product of $\overrightarrow{OP}$, $\overrightarrow{OQ}$ and $\overrightarrow{OR}$ in any order, or their $3\times3$ determinant. Condone sign slips. |
| | $=pqr$ | **A1** | $pqr$, or $-pqr$ from another order. |
| | $V=\frac{1}{6}\lvert\text{“}pqr\text{”}\rvert=\frac{pqr}{6}$ | **A1** | $\frac{pqr}{6}$ oe. Must be positive. |
| | | **[3]** | |
| **8(b)** | $\overrightarrow{RP}=\begin{pmatrix}p\\0\\-r\end{pmatrix},\quad\overrightarrow{RQ}=\begin{pmatrix}0\\q\\-r\end{pmatrix}$ | **B1** | Both correct. |
| | $\overrightarrow{RP}\times\overrightarrow{RQ}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\p&0&-r\\0&q&-r\end{vmatrix}$<br>$=(0+qr)\mathbf{i}-(-pr-0)\mathbf{j}+(pq-0)\mathbf{k}$ | **M1** | Expands the vector product of their $\overrightarrow{RP}$ and $\overrightarrow{RQ}$, each a difference of two position vectors, by determinant or component formula, all three components attempted. Condone sign slips. |
| | $=\begin{pmatrix}qr\\rp\\pq\end{pmatrix}$ | **A1** | oe in **i**, **j**, **k** form. |
| | Area $=\frac{1}{2}\sqrt{(qr)^2+(rp)^2+(pq)^2}$<br>$=\frac{1}{2}\sqrt{q^2r^2+r^2p^2+p^2q^2}$ | **A1 AG** | Correct substituted $\frac{1}{2}\left\lvert\overrightarrow{RP}\times\overrightarrow{RQ}\right\rvert$ leading without error to the printed area. |
| | | **[4]** | |
| **8(c)** | $V=\frac{1}{3}\times\frac{1}{2}\sqrt{q^2r^2+r^2p^2+p^2q^2}\times h$ | **M1 A1** | **M1** Uses one third of the area of triangle $PQR$ times $h$. Condone a slip in copying the area.<br>**A1** Correct volume in terms of $h$, oe. |
| | $\frac{h}{6}\sqrt{q^2r^2+r^2p^2+p^2q^2}=\text{“}\frac{pqr}{6}\text{”}$<br>$\Rightarrow\frac{1}{h^2}=\ldots$ | **dM1** | Equates their volume in terms of $h$ with their volume from (a) and makes $h^2$ or $\frac{1}{h^2}$ the subject. Condone slips in rearranging. |
| | $\frac{1}{h^2}=\frac{q^2r^2+r^2p^2+p^2q^2}{p^2q^2r^2}=\frac{1}{p^2}+\frac{1}{q^2}+\frac{1}{r^2}$ | **A1** | Correct equation of volumes leading without error to the printed result; combined steps accepted. |
| | $\frac{1}{h^2}=\frac{1}{4}+\frac{1}{9}+\frac{1}{36}\ \left[=\frac{7}{18}\right]\Rightarrow h=\ldots$ | **M1** | Substitutes the given values into the printed result, or their expression for $h$, and attempts $h$. Condone arithmetic slips. |

*Question 8 continued*

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| | $h = \frac{3\sqrt{14}}{7}$ | **A1** | Or any exact equivalent, such as $\sqrt{\frac{18}{7}}$. Positive value only. |
| | | **[6]** | |
| | **Total for Question 8** | **13** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **9(a)** | $\mathbf{q} \times \mathbf{r} = \begin{pmatrix} 3 \\ -9 \\ -12 \end{pmatrix}$ | **B1** | |
| | $\begin{pmatrix} 2 \\ -1 \\ k \end{pmatrix} \cdot \text{“}\begin{pmatrix} 3 \\ -9 \\ -12 \end{pmatrix}\text{”} = 15 - 12k = -21$<br>$\Rightarrow k = \ldots$ | **M1** | Forms the scalar product of $\mathbf{p}$ with their three-component $\mathbf{q} \times \mathbf{r}$, or a determinant, linear in $k$, set equal to $-21$ and solved. Condone sign slips. |
| | $k = 3$ | **A1** | |
| | $\begin{pmatrix} 2 \\ -1 \\ \text{“3”} \end{pmatrix} \times \text{“}\begin{pmatrix} 3 \\ -9 \\ -12 \end{pmatrix}\text{”} = \ldots$ | **M1** | Forms $\mathbf{p} \times (\mathbf{q} \times \mathbf{r})$ with their $k$ and their $\mathbf{q} \times \mathbf{r}$, by determinant or component formula, all three components attempted. Condone sign slips; a product written without working needs two components correct. |
| | $\mathbf{p} \times (\mathbf{q} \times \mathbf{r}) = \begin{pmatrix} 39 \\ 33 \\ -15 \end{pmatrix}$ | **A1** | Any vector form, such as $39\mathbf{i} + 33\mathbf{j} - 15\mathbf{k}$. |
| | $\mathbf{p} \times \mathbf{q} = \begin{pmatrix} -7 \\ 7 \\ 7 \end{pmatrix}$<br>$\text{“}\begin{pmatrix} -7 \\ 7 \\ 7 \end{pmatrix}\text{”} \times \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = \ldots$ | **M1** | Forms $\mathbf{p} \times \mathbf{q}$ with their $k$, then their $(\mathbf{p} \times \mathbf{q}) \times \mathbf{r}$, each by determinant or component formula, all three components attempted. Condone sign slips; each product written without working needs two components correct. |
| | $(\mathbf{p} \times \mathbf{q}) \times \mathbf{r} = \begin{pmatrix} 7 \\ 35 \\ -28 \end{pmatrix}$ | **A1** | Any vector form. |
| | | **[7]** | |
| **9(b)** | $\begin{pmatrix} 39 \\ 33 \\ -15 \end{pmatrix} \neq \begin{pmatrix} 7 \\ 35 \\ -28 \end{pmatrix}$, so not associative | **E1** | States not associative, because the two products found in (a) are unequal. Not for the conclusion alone. |
| | | **[1]** | |
| **9(c)** | $\mathbf{a} \times \mathbf{b}$ is perpendicular to $\mathbf{a}$ and $\mathbf{b}$,<br>so normal to the plane containing them | **E1** | States that $\mathbf{a} \times \mathbf{b}$ is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$, or normal to the plane containing them. |

*Question 9 continued*

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| | $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}$ is perpendicular to $\mathbf{a}\times\mathbf{b}$,<br>so lies in the plane of $\mathbf{a}$ and $\mathbf{b}$ | **E1** | States that $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}$ is perpendicular to $\mathbf{a}\times\mathbf{b}$, so lies in the plane of $\mathbf{a}$ and $\mathbf{b}$. Condone omitting the case of $\mathbf{a}$ parallel to $\mathbf{b}$. |
| | | **[2]** | |
| **9(d)** | $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})=(\mathbf{a}\cdot\mathbf{c})\mathbf{b}-(\mathbf{a}\cdot\mathbf{b})\mathbf{c}$<br>$\mathbf{b}\times(\mathbf{c}\times\mathbf{a})=(\mathbf{b}\cdot\mathbf{a})\mathbf{c}-(\mathbf{b}\cdot\mathbf{c})\mathbf{a}$<br>$\mathbf{c}\times(\mathbf{a}\times\mathbf{b})=(\mathbf{c}\cdot\mathbf{b})\mathbf{a}-(\mathbf{c}\cdot\mathbf{a})\mathbf{b}$ | **M1 A1** | **M1** Applies $(*)$ to the second or third term, with the letters replaced consistently. Condone sign slips.<br>**A1** All three correct, with scalar products in either order. |
| | LHS $=(\mathbf{b}\cdot\mathbf{c}-\mathbf{b}\cdot\mathbf{c})\mathbf{a}$<br>$+(\mathbf{a}\cdot\mathbf{c}-\mathbf{a}\cdot\mathbf{c})\mathbf{b}$<br>$+(\mathbf{a}\cdot\mathbf{b}-\mathbf{a}\cdot\mathbf{b})\mathbf{c}=\mathbf{0}$ | **A1 AG** | Adds the three correct expansions, using commutativity of the scalar product, so every term cancels, reaching $\mathbf{0}$ without error. Cancelling in pairs is accepted. |
| | | **[3]** | |
| | **Total for Question 9** | **13** | |

| Question | Scheme | Marks | Notes |
|---|---|---|---|
| **10** | $\mathbf{c}\cdot(\mathbf{a}\times\mathbf{b})=0$ for every $\mathbf{c}$<br>$\Rightarrow \mathbf{a}\times\mathbf{b}=\mathbf{0}$ | **M1** | Uses the zero triple product for every $\mathbf{c}$, by a choice of $\mathbf{c}$ or a coplanarity argument, to show $\mathbf{a}\times\mathbf{b}=\mathbf{0}$ or that $\mathbf{a}$ and $\mathbf{b}$ are parallel. |
| | $\mathbf{a}\neq k\mathbf{b}\Rightarrow \mathbf{b}=\mathbf{0}$ | **A1** | $\mathbf{b}=\mathbf{0}$, with the reason that $\mathbf{a}$ is not a multiple of $\mathbf{b}$. |
| | $p=2q,\ r=q^2\Rightarrow q^3-4q=0$<br>$\Rightarrow q=\ldots$ | **M1** | Sets all three components of $\mathbf{b}$ to zero, eliminates two variables and solves the resulting equation. Condone sign slips. |
| | $q=2$: $p=4,\ r=4\Rightarrow \mathbf{a}=\mathbf{0}$, rejected<br>$p=q=r=0$ or $p=-4,\ q=-2,\ r=4$ | **A1** | Both sets and no others, with $q=2$ rejected because $\mathbf{a}=\mathbf{0}$ there. |
| | **Total for Question 10** | **4** | |