OPTIONAL PURE / VECTORS & MATRICES

Reflections in Planes

Mark scheme 10 questions · 100 marks

EXAM BOARDS OCR MEI Extra Pure Y435 · Edexcel 9FM0 · AQA 7367 · OCR A H245

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	Total	**100**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last accuracy mark of the working that reaches a printed result: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1	$\mathbf{p}\cdot\mathbf{n} = 5+6+4 = 15, \quad \lvert\mathbf{n}\rvert^2 = 9$	**M1**	Finds $\mathbf{p}\cdot\mathbf{n}$ and $\lvert\mathbf{n}\rvert^2$ with $\mathbf{n}$ from Π's coefficients. Condone sign slips.
	$\frac{\text{“15”}-6}{\text{“9”}} = 1$	**A1**	$\frac{\mathbf{p}\cdot\mathbf{n}-d}{\lvert\mathbf{n}\rvert^2} = 1$.
	$\mathbf{p}' = \begin{pmatrix}5\\3\\-2\end{pmatrix} - 2\times\text{“1”}\begin{pmatrix}1\\2\\-2\end{pmatrix} = \ldots$	**dM1**	Subtracts twice their multiple of $\mathbf{n}$ from $\mathbf{p}$.
	$(3,-1,2)$	**A1**	Accept as a position vector.
	Alternative: Perpendicular line through A		
	$(5+t)+2(3+2t)-2(-2-2t) = 6 \Rightarrow t = \ldots$	**M1**	Substitutes the general point of the line through A in direction $\mathbf{n}$ into Π and solves for the parameter. Condone sign slips.
	$t=-1, \quad M(4,1,0)$	**A1**	Correct parameter, or the point $M(4,1,0)$, oe.
	$\overrightarrow{OA'} = \begin{pmatrix}5\\3\\-2\end{pmatrix} + 2\begin{pmatrix}\text{“4”}-5\\\text{“1”}-3\\\text{“0”}+2\end{pmatrix} = \ldots$	**dM1**	Uses the point where their line meets Π as the midpoint of A and its image, or puts twice their parameter into their line.
	$(3,-1,2)$	**A1**	Accept as a position vector.
	Total for Question 1	**4**	

Question	Scheme	Marks	Notes
2(a)	$\mathbf{p}\cdot\mathbf{n} = 10+4+6 = 20$ $\lvert\mathbf{n}\rvert^2 = 4+1+4 = 9$	**M1**	Finds $\mathbf{p}\cdot\mathbf{n}$ and $\lvert\mathbf{n}\rvert^2$ with $\mathbf{n}$ from the equation of Π. Condone sign slips.
	$\frac{\mathbf{p}\cdot\mathbf{n}-d}{\lvert\mathbf{n}\rvert^2} = \frac{\text{“20”}-2}{\text{“9”}} = 2$	**A1**	$\frac{\mathbf{p}\cdot\mathbf{n}-d}{\lvert\mathbf{n}\rvert^2} = 2$.
	$\mathbf{p}' = \begin{pmatrix}5\\4\\-3\end{pmatrix} - 2\times\text{“2”}\begin{pmatrix}2\\1\\-2\end{pmatrix}$	**dM1**	Subtracts twice their multiple of $\mathbf{n}$ from $\mathbf{p}$.
	$\mathbf{p}' = \begin{pmatrix}-3\\0\\5\end{pmatrix}$	**A1**	Accept as coordinates.
	Alternative: Perpendicular line through P		
	$\mathbf{r} = \begin{pmatrix}5\\4\\-3\end{pmatrix} + t\begin{pmatrix}2\\1\\-2\end{pmatrix}$ $2(5+2t)+(4+t)-2(-3-2t) = 2$ $\Rightarrow t = \ldots$	**M1**	Substitutes the general point of the line through P in direction $\mathbf{n}$ into Π and solves for the parameter. Condone sign slips.
	$t = -2$	**A1**	Correct parameter, or the point $(1,2,1)$, oe.
	$\overrightarrow{OP'} = \begin{pmatrix}5\\4\\-3\end{pmatrix} + 2\text{“}\begin{pmatrix}-4\\-2\\4\end{pmatrix}\text{”}$	**dM1**	Uses twice the displacement from P to the point where their line meets Π, puts twice their parameter into their line, or uses that point as the midpoint of PP'.
	$\overrightarrow{OP'} = \begin{pmatrix}-3\\0\\5\end{pmatrix}$	**A1**	Accept as coordinates.
		[4]	
2(b)	$2(5+\lambda)+(4+2\lambda)-2(-3-\lambda) = 2$ $\Rightarrow \lambda = \ldots$ [$= -3$]	**M1**	Substitutes the general point of ℓ into the equation of Π and solves the linear equation for λ. Condone sign slips.
	$\mathbf{r} = \begin{pmatrix}5\\4\\-3\end{pmatrix} + \text{“}(-3)\text{”}\begin{pmatrix}1\\2\\-1\end{pmatrix}$	**dM1**	Substitutes their λ into the equation of ℓ.
	$\begin{pmatrix}2\\-2\\0\end{pmatrix}$	**A1**	Accept as coordinates.
		[3]	

Question 2 continued

Question	Scheme	Marks	Notes
2(c)	$\mathbf{r} = \text{“}\begin{pmatrix}2\\-2\\0\end{pmatrix}\text{”} + \mu\left(\text{“}\begin{pmatrix}-3\\0\\5\end{pmatrix}\text{”} - \text{“}\begin{pmatrix}2\\-2\\0\end{pmatrix}\text{”}\right)$	**M1**	Forms a line through their point from (b) or their P', with direction their P' minus their point from (b), either order, or the reflected direction of ℓ.
	$\mathbf{r} = \begin{pmatrix}2\\-2\\0\end{pmatrix} + \mu\begin{pmatrix}-5\\2\\5\end{pmatrix}$	**A1**	oe, with any point of ℓ' and any nonzero multiple of the direction. Must be $\mathbf{r} = \ldots$ with a parameter; a Cartesian form alone scores A0.
		[2]	
	Total for Question 2	**9**	

Question	Scheme	Marks	Notes
3(a)	$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 4 \\ -2 \end{pmatrix} \Rightarrow \mathbf{n} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$	**B1**	Any nonzero multiple of $\overrightarrow{AB}$ as the normal, soi, including by their plane from equating squared distances.
	$M = (2, 3, 3)$ $2(2) + 2(3) - 3 = d \Rightarrow d = \ldots$	**M1**	Substitutes a midpoint attempted as $\frac{1}{2}\left(\overrightarrow{OA} + \overrightarrow{OB}\right)$ into $\mathbf{r} \cdot \mathbf{n} = d$ with their normal, or equates squared distances from A and B. Condone arithmetic slips.
	$2x + 2y - z = 7$	**A1**	Cartesian form only, any nonzero multiple; the $\mathbf{r} \cdot \mathbf{n} = 7$ form alone scores A0.
		[3]	
3(b)	$2(3 + \lambda) + 2(1) - (1 + 2\lambda) = \ldots$	**M1**	Substitutes the general point of ℓ into their plane equation, or checks a point of ℓ in their plane and forms $\mathbf{b} \cdot \mathbf{n}$.
	$6 + 2\lambda + 2 - 1 - 2\lambda = 7$ for all λ so ℓ lies in Π	**A1 AG**	Correct expansion giving 7 for all λ, or a point of ℓ in Π and $\mathbf{b} \cdot \mathbf{n} = 0$ both shown, with a conclusion that ℓ lies in Π.
	$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$	**B1**	Any correct equation of ℓ, such as the given one, as the reflection.
		[3]	
	Total for Question 3	**6**	

Question	Scheme	Marks	Notes
4	$\overrightarrow{CD} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\overrightarrow{CE} = \mathbf{i} + 3\mathbf{j}$ $\mathbf{n} = \overrightarrow{CD} \times \overrightarrow{CE} = \ldots$	**M1**	Finds two vectors in the plane from the given points and attempts their vector product, or solves $\mathbf{n} \cdot \overrightarrow{CD} = \mathbf{n} \cdot \overrightarrow{CE} = 0$. Condone sign slips.
	$\mathbf{n} = 3\mathbf{i} - \mathbf{j} + 5\mathbf{k}$	**A1**	Any nonzero multiple.
	$d = (\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \cdot$ "$(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$" $= \ldots$	**M1**	Forms the scalar product of their normal with the position vector of C, D or E.
	$3x - y + 5z = 14$	**A1**	oe, including $\mathbf{r} \cdot (3\mathbf{i} - \mathbf{j} + 5\mathbf{k}) = 14$ or any multiple.
	$\mathbf{p}' = \mathbf{p} - 2\dfrac{\mathbf{p} \cdot \mathbf{n} - d}{\lvert\mathbf{n}\rvert^2}\mathbf{n}$, $\mathbf{p} = 4\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$	**M1**	Uses the reflection formula with A and their plane, or forms the line through A in the direction of their normal.
	$\mathbf{p} \cdot \mathbf{n} - d = 12 + 2 + 35 - 14$ $[= 35]$ $\lvert\mathbf{n}\rvert^2 = 9 + 1 + 25$ $[= 35]$	**dM1**	Evaluates $\mathbf{p} \cdot \mathbf{n} - d$ and $\lvert\mathbf{n}\rvert^2$ with their values, or substitutes their line into their plane and solves for the parameter.
	$\mathbf{p}' = (4\mathbf{i} - 2\mathbf{j} + 7\mathbf{k})$ $\quad - 2\left(\frac{\text{"35"}}{\text{"35"}}\right)(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$	**dM1**	Subtracts twice their projection term times $\mathbf{n}$ from $\mathbf{p}$, or uses the foot F with $\overrightarrow{OB} = \overrightarrow{OA} + 2\overrightarrow{AF}$ oe.
	$B = (-2, 0, -3)$	**A1**	Accept a position vector.
	Total for Question 4	**8**	

Question	Scheme	Marks	Notes
5(i)	$\mathbf{n} = \begin{pmatrix}2\\1\\0\end{pmatrix} \times \begin{pmatrix}2\\-1\\1\end{pmatrix} = \ldots$	**M1**	Forms the vector product of the two direction vectors, or finds a vector perpendicular to both by any method. Condone sign slips.
	$\mathbf{n} = \begin{pmatrix}1\\-2\\-4\end{pmatrix}$	**A1**	Or any nonzero multiple.
	$p = \begin{pmatrix}1\\-2\\-4\end{pmatrix} \cdot \begin{pmatrix}1\\1\\2\end{pmatrix} = \ldots$	**M1**	Forms the scalar product of their normal with a point on m.
	$\mathbf{r} \cdot \begin{pmatrix}1\\-2\\-4\end{pmatrix} = -9$	**A1**	oe, the same multiple of normal and p. Must be in the form $\mathbf{r} \cdot \mathbf{n} = p$; a Cartesian equation alone scores A0.
		[4]	
5(ii)	$\dfrac{\left\lvert \begin{pmatrix}4\\2\\-3\end{pmatrix} \cdot \begin{pmatrix}1\\-2\\-4\end{pmatrix} - (-9) \right\rvert}{\sqrt{1^2 + (-2)^2 + (-4)^2}}$	**M1**	Uses the point-to-plane distance with a point on ℓ and their plane from (i), or any valid method such as the foot of the perpendicular. Condone sign slips.
	$= \sqrt{21}$	**A1**	oe, or awrt 4.58.
		[2]	
5(iii)	$\begin{pmatrix}2\\-1\\1\end{pmatrix} \cdot \begin{pmatrix}1\\-2\\-4\end{pmatrix} = 0 \Rightarrow$ direction $\begin{pmatrix}2\\-1\\1\end{pmatrix}$ unchanged	**B1**	Direction of ℓ or a multiple, justified by $\mathbf{b} \cdot \mathbf{n} = 0$ or ℓ parallel to Π, or found from two reflected points.
	$\begin{pmatrix}4\\2\\-3\end{pmatrix} - 2\dfrac{\begin{pmatrix}4\\2\\-3\end{pmatrix} \cdot \mathbf{n} - (-9)}{\lvert\mathbf{n}\rvert^2}\mathbf{n}$	**M1**	Applies the reflection formula to a point of ℓ with their normal, or forms the line through that point along their normal.
	$\begin{pmatrix}4\\2\\-3\end{pmatrix} \cdot \mathbf{n} - (-9) = 21, \quad \lvert\mathbf{n}\rvert^2 = 21$	**dM1**	Evaluates both quantities, or substitutes that line into their plane and solves for its parameter. Condone sign slips.

Question 5 continued

Question	Scheme	Marks	Notes
	$\begin{pmatrix}4\\2\\-3\end{pmatrix} - 2\begin{pmatrix}1\\-2\\-4\end{pmatrix} = \ldots \ [= (2,6,5)]$	**dM1**	Subtracts twice their component along the normal, or adds to the point twice its displacement to their foot of the perpendicular.
	$\mathbf{r} = \begin{pmatrix}2\\6\\5\end{pmatrix} + t\begin{pmatrix}2\\-1\\1\end{pmatrix}$	**A1**	oe, any point of the line with any multiple of the direction. Must be an equation $\mathbf{r} = \ldots$; the point alone scores A0.
		[5]	
Total for Question 5		**11**	

Question	Scheme	Marks	Notes
6(a)	$3x - y + 2z = 6$ $3(1+2\lambda) - (4+\lambda) + 2\lambda = 6$ $\Rightarrow \lambda = \ldots$ $[= 1]$	**M1**	Forms a Cartesian or scalar-product equation of Π_1, substitutes the general point of ℓ_1 and solves for λ. Condone sign or arithmetic slips.
	$(1 + 2 \times$ "1", $4 +$ "1", "1"$)$	**dM1**	Substitutes their value of λ into the equation of ℓ_1.
	$(3, 5, 1)$	**A1**	Accept the position vector.
		[3]	
6(b)	Reflects $(1, 4, 0)$ along the normal: $\mathbf{p}' = \mathbf{p} - 2\,\frac{\mathbf{p}\cdot\mathbf{n}-6}{\lvert\mathbf{n}\rvert^2}\,\mathbf{n}$	**M1**	Uses the reflection formula with a point of ℓ_1 not on Π_1, or forms the line through it along $\mathbf{n}$.
	$\mathbf{p}\cdot\mathbf{n} - 6 = -7$, $\lvert\mathbf{n}\rvert^2 = 14$	**dM1**	Evaluates $\mathbf{p}\cdot\mathbf{n} - d$ and $\lvert\mathbf{n}\rvert^2$, or solves for the parameter where the line along $\mathbf{n}$ meets Π_1. Condone arithmetic slips.
	$\mathbf{p}' = \begin{pmatrix}1\\4\\0\end{pmatrix} - 2\left(\frac{\text{"}-7\text{"}}{\text{"}14\text{"}}\right)\begin{pmatrix}3\\-1\\2\end{pmatrix}$	**dM1**	Subtracts twice their projection term times $\mathbf{n}$, or adds twice the displacement from the point to their foot of the perpendicular.
	$= \begin{pmatrix}4\\3\\2\end{pmatrix}$	**A1**	Correct image, with no incorrect values seen.
	$\mathbf{b}' = \begin{pmatrix}2\\1\\1\end{pmatrix} - 2\left(\frac{7}{14}\right)\begin{pmatrix}3\\-1\\2\end{pmatrix}$	**M1**	Finds the reflected direction of ℓ_1 by the formula with no d term, or as the vector joining their $(3, 5, 1)$ from (a) to their image.
	$= \begin{pmatrix}-1\\2\\-1\end{pmatrix}$ $\mathbf{r} = \begin{pmatrix}4\\3\\2\end{pmatrix} + \mu\begin{pmatrix}-1\\2\\-1\end{pmatrix}$	**A1 AG**	cso. Correct image and a direction parallel to $(-1, 2, -1)$ reached without error, then the printed equation, showing their image lies on the printed line, or $(4, 3, 2)$ on theirs, if another point was reflected.
	Alternative: Fixed point from (a)		
	$P(3, 5, 1)$ from (a) lies in Π_1, so is on ℓ_2	**B1**	Stated, or used as a point of ℓ_2.
	$\mathbf{b}' = \mathbf{b} - 2\,\frac{\mathbf{b}\cdot\mathbf{n}}{\lvert\mathbf{n}\rvert^2}\,\mathbf{n}$	**M1**	Finds the reflected direction of ℓ_1 by the formula with no d term, or reflects a second point of ℓ_1 in Π_1.

Question 6 continued

Question	Scheme	Marks	Notes
	$= \begin{pmatrix}2\\1\\1\end{pmatrix} - 2\left(\frac{7}{14}\right)\begin{pmatrix}3\\-1\\2\end{pmatrix}$	**dM1**	Evaluates $\mathbf{b}\cdot\mathbf{n}$ and $\lvert\mathbf{n}\rvert^2$ and subtracts twice the projection, or joins their image point to their point from (a). Condone arithmetic slips.
	$= \begin{pmatrix}-1\\2\\-1\end{pmatrix}$	**A1**	Or any nonzero multiple, with no incorrect values seen.
	$\mu = 1$: $\begin{pmatrix}4\\3\\2\end{pmatrix} + \begin{pmatrix}-1\\2\\-1\end{pmatrix} = \begin{pmatrix}3\\5\\1\end{pmatrix}$	**M1**	Substitutes their $(3,5,1)$ into the printed equation, finding μ and checking all three components; or checks $(4,3,2)$ on their line through it.
	Same point and same direction, so ℓ_2 is the printed line	**A1 AG**	cso. Direction parallel to $(-1,2,-1)$, and $(3,5,1)$ on the printed line or $(4,3,2)$ on theirs, without error, with a conclusion.
		[6]	
6(c)	$\mathbf{b}+\mathbf{b}' = \begin{pmatrix}2\\1\\1\end{pmatrix} + \begin{pmatrix}-1\\2\\-1\end{pmatrix} = \ldots$ through $(3,5,1)$	**M1**	Finds a direction in both planes, such as $\mathbf{b}+\mathbf{b}'$ or the cross product of the planes' normals, and uses it with a point on both, such as their $(3,5,1)$.
	$\mathbf{r} = \begin{pmatrix}3\\5\\1\end{pmatrix} + t\begin{pmatrix}1\\3\\0\end{pmatrix}$	**A1**	oe, with any point on the line and any nonzero multiple of $(1,3,0)$.
		[2]	
	Total for Question 6	**11**	

Question	Scheme	Marks	Notes
7(a)	$\overrightarrow{BA} = -\mathbf{i} - 2\mathbf{j} - \mathbf{k},\ \overrightarrow{BC} = -\mathbf{i} - \mathbf{j} + (c-1)\mathbf{k}$ $\overrightarrow{BA} \times \overrightarrow{BC} = \ldots$	**M1**	Forms both vectors and expands their vector product by determinant or components. Condone sign slips in the vectors.
	$(1-2c)\mathbf{i} + c\mathbf{j} - \mathbf{k}$	**A1**	oe, column vector accepted.
		[2]	
7(b)	$(1-2c)\mathbf{i} + c\mathbf{j} - \mathbf{k} = d\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ $\Rightarrow c = \ldots,\ d = 1 - 2c$	**M1**	Equates corresponding components of their vector from (a), in terms of c, and the given vector to find c.
	$c = 2,\ d = 1 - 2(2) = -3$	**A1 AG**	$c = 2$ and $d = -3$ from a correct (a), with $c = 2$ substituted into $d = 1 - 2c$.
		[2]	
7(c)	$\mathbf{n} = -3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ $p = \mathbf{i} \cdot (-3\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = \ldots$	**M1**	Takes a normal parallel to $-3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and forms its scalar product with the position vector of A, B or C, using $c = 2$ for C.
	$\mathbf{r} \cdot (-3\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = -3$	**A1**	Or any multiple, such as $\mathbf{r} \cdot (3\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = 3$. A Cartesian equation alone scores A0.
		[2]	
7(d)	$\mathbf{s}' = \mathbf{s} - 2\,\frac{\mathbf{s}\cdot\mathbf{n} - p}{\lvert\mathbf{n}\rvert^2}\,\mathbf{n}$	**M1**	Uses a correct reflection formula with $\mathbf{s} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and their $\mathbf{n}$ and p from (c). If not quoted, implied by correct working.
	$\mathbf{s} \cdot \mathbf{n} - p = -9 - (-3) = -6$ $\lvert\mathbf{n}\rvert^2 = 14$	**dM1**	Evaluates both $\mathbf{s} \cdot \mathbf{n} - p$ and $\lvert\mathbf{n}\rvert^2$ for their $\mathbf{n}$ and p. Condone arithmetic slips.
	$\mathbf{s}' = \mathbf{s} - 2\left(\text{“}-\frac{6}{14}\text{”}\right)\mathbf{n}$ $= (3\mathbf{i} + \mathbf{j} + 2\mathbf{k}) + \frac{6}{7}(-3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$	**dM1**	Subtracts from $\mathbf{s}$ twice their scalar $\frac{\mathbf{s}\cdot\mathbf{n} - p}{\lvert\mathbf{n}\rvert^2}$ times their $\mathbf{n}$. Need not be simplified.
	$\frac{3}{7}\mathbf{i} + \frac{19}{7}\mathbf{j} + \frac{8}{7}\mathbf{k}$	**A1**	oe, or awrt $0.429\mathbf{i} + 2.71\mathbf{j} + 1.14\mathbf{k}$. Column vector accepted.
	Alternative: Line through S normal to Π		
	$\mathbf{r} = (3\mathbf{i} + \mathbf{j} + 2\mathbf{k}) + \lambda(-3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$	**M1**	Forms the line through S with direction their normal from (c).

Question 7 continued

Question	Scheme	Marks	Notes
	$-3(3-3\lambda)+2(1+2\lambda)-(2-\lambda)=-3$ $\Rightarrow \lambda = \ldots$ $[=\frac{3}{7}]$	**dM1**	Substitutes the general point of their line into their plane equation and solves for λ. Condone sign slips.
	$\mathbf{s}' = \mathbf{s} + 2 \times$ "$\frac{3}{7}$" $\mathbf{n}$ $= (3\mathbf{i}+\mathbf{j}+2\mathbf{k}) + \frac{6}{7}(-3\mathbf{i}+2\mathbf{j}-\mathbf{k})$	**dM1**	Uses the foot of the perpendicular as the midpoint of SS', by doubling their λ or by $\mathbf{s}' = 2\mathbf{m} - \mathbf{s}$ with $\mathbf{m}$ the foot.
	$\frac{3}{7}\mathbf{i} + \frac{19}{7}\mathbf{j} + \frac{8}{7}\mathbf{k}$	**A1**	oe, or awrt $0.429\mathbf{i} + 2.71\mathbf{j} + 1.14\mathbf{k}$. Column vector accepted.
		[4]	
Total for Question 7		**10**	

Question	Scheme	Marks	Notes
8(a)	$2(1+\lambda)-2\lambda+(1+3\lambda)=9$ $\Rightarrow \lambda = \ldots$ $[=2]$	**M1**	Substitutes the general point of ℓ into the equation of Π, in either form, and solves for λ. Condone sign slips.
	$\mathbf{r}=(\mathbf{i}+\mathbf{k})+$ "2"$(\mathbf{i}+2\mathbf{j}+3\mathbf{k})$	**dM1**	Substitutes their λ into the equation of ℓ.
	$(3,4,7)$	**A1**	Accept a position vector.
		[3]	
8(b)	$\mathbf{r}=4\mathbf{j}+\mathbf{k}+t(2\mathbf{i}-\mathbf{j}+\mathbf{k})$	**M1**	Forms the line through A along $2\mathbf{i}-\mathbf{j}+\mathbf{k}$, in any form, or finds $\overrightarrow{AB}$ and compares it with the normal.
	$2(2t)-(4-t)+(1+t)=9 \Rightarrow t=\ldots$	**dM1**	Substitutes their general point into the equation of Π and solves for the parameter, or checks $(4,2,3)$ in $2x-y+z=9$.
	$t=2 \Rightarrow B=(2\times 2,\ 4-2,\ 1+2)$ $=(4,2,3)$	**A1 AG**	Correct substituted equation leading without error to $t=2$ and $(4,2,3)$; or both checks correct, with $\overrightarrow{AB}=2(2\mathbf{i}-\mathbf{j}+\mathbf{k})$ and a conclusion.
		[3]	
8(c)	$\mathbf{p}'=(4\mathbf{j}+\mathbf{k})-2\frac{-3-9}{6}(2\mathbf{i}-\mathbf{j}+\mathbf{k})$	**M1**	Uses $\mathbf{p}-2\frac{\mathbf{p}\cdot\mathbf{n}-d}{\lvert\mathbf{n}\rvert^2}\mathbf{n}$ with $\mathbf{n}=2\mathbf{i}-\mathbf{j}+\mathbf{k}$ and $d=9$. Condone slips if quoted; if not, implied by correct values.
	$A'=(8,0,5)$	**A1**	Accept a position vector.
	Alternative: Using B as the midpoint		
	$\overrightarrow{OA'}=\overrightarrow{OA}+2\overrightarrow{AB}$ $=4\mathbf{j}+\mathbf{k}+2(4\mathbf{i}-2\mathbf{j}+2\mathbf{k})$	**M1**	Uses B as the midpoint of AA', by $\overrightarrow{OA}+2\overrightarrow{AB}$, $2\overrightarrow{OB}-\overrightarrow{OA}$ or $t=4$ in the line from (b).
	$A'=(8,0,5)$	**A1**	Accept a position vector.
		[2]	
8(d)	$(3,4,7)$ lies in Π, so is on the reflected line	**B1**	Stated, or used as a point of the reflected line.
	Reflects the direction of ℓ: $\mathbf{u}'=\mathbf{u}-2\frac{\mathbf{u}\cdot\mathbf{n}}{\lvert\mathbf{n}\rvert^2}\mathbf{n}$	**M1**	Finds the reflection of $\mathbf{u}=\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ using $\mathbf{n}$, by the formula with no constant term, or reflects a second point of ℓ in Π.

Question 8 continued

Question	Scheme	Marks	Notes
	$\mathbf{u}' = (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) - 2\left(\frac{3}{6}\right)(2\mathbf{i} - \mathbf{j} + \mathbf{k})$	**dM1**	Evaluates $\mathbf{u} \cdot \mathbf{n}$ and $\lvert\mathbf{n}\rvert^2$ and subtracts twice the projection, or joins the image point to their point from (a). Condone arithmetic slips.
	$\mathbf{u}' = -\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$	**A1**	Or any nonzero multiple.
	$\mathbf{r} = 3\mathbf{i} + 4\mathbf{j} + 7\mathbf{k} + \mu(-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$	**A1ft**	ft their point from (a) and their direction; soi by their $\mathbf{r} \times \mathbf{a} = \mathbf{b}$.
	$\mathbf{r} \times \mathbf{a} = \mathbf{c} \times \mathbf{a}$, $\mathbf{c} = 3\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}$, $\mathbf{a} =$ "$-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$"	**M1**	Takes $\mathbf{a}$ as their direction and $\mathbf{b}$ as the cross product of a point of their line with it. Condone the order of the product.
	$(3\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}) \times (-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$ $= (8 - 21)\mathbf{i} - (6 + 7)\mathbf{j} + (9 + 4)\mathbf{k}$	**dM1**	Attempts the cross product of their point and direction, by determinant or components. Condone sign slips.
	$\mathbf{r} \times (-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) = -13\mathbf{i} - 13\mathbf{j} + 13\mathbf{k}$	**A1**	Or with $\mathbf{a}$ and $\mathbf{b}$ both multiplied by the same nonzero k. Must be in the form $\mathbf{r} \times \mathbf{a} = \mathbf{b}$.
		[8]	
Total for Question 8		**16**	

Question	Scheme	Marks	Notes
9(a)	$\begin{pmatrix}2\\-1\\1\end{pmatrix}\times\begin{pmatrix}1\\1\\0\end{pmatrix}=\ldots$	**M1**	Attempts the vector product of the two direction vectors, or solves two scalar products equal to zero, for a normal.
	$\mathbf{n}=\begin{pmatrix}-1\\1\\3\end{pmatrix}$	**A1**	Or any nonzero multiple.
	$-x+y+3z=-2$	**B1**	oe, in vector or Cartesian form.
	$-(4-\lambda)+(1+\lambda)+3(2+3\lambda)=-2$ $\Rightarrow\lambda=\ldots$	**M1**	Substitutes the general point of a line through P along their normal into their plane equation and solves for the parameter.
	$\lambda=-\frac{5}{11}$	**A1**	Or the correct value for a multiple of $\mathbf{n}$.
	$\overrightarrow{OQ}=\begin{pmatrix}4\\1\\2\end{pmatrix}-\frac{5}{11}\begin{pmatrix}-1\\1\\3\end{pmatrix}$	**dM1**	Substitutes their λ into their line through P.
	$Q\left(\frac{49}{11},\frac{6}{11},\frac{7}{11}\right)$	**A1**	Or awrt $(4.45, 0.545, 0.636)$. A position vector is accepted.
		[7]	
9(b)	$\overrightarrow{PQ}=\frac{5}{11}\begin{pmatrix}1\\-1\\-3\end{pmatrix}$, $\overrightarrow{QR}=\frac{1}{11}\begin{pmatrix}-16\\5\\-7\end{pmatrix}$ $\begin{pmatrix}1\\-1\\-3\end{pmatrix}\times\begin{pmatrix}-16\\5\\-7\end{pmatrix}=\ldots$	**M1**	Forms two non-parallel vectors in the plane from P, R, their Q or $\mathbf{n}$, and attempts their vector product.
	$\begin{pmatrix}22\\55\\-11\end{pmatrix}=11\begin{pmatrix}2\\5\\-1\end{pmatrix}$	**A1**	Or any nonzero multiple.
	$2x+5y-z=2(4)+5(1)-1(2)$	**M1**	Uses their normal with P, R or their Q to find the constant, in vector or Cartesian form.
	$2x+5y-z=11$	**A1**	oe Cartesian equation. A vector form alone scores A0.
		[4]	
9(c)	R lies on Π, so is fixed: R is on ℓ_3	**B1**	soi by using R as a point of ℓ_3.
	Reflects the direction of ℓ_2 in Π: $\begin{pmatrix}1\\0\\2\end{pmatrix}-2\times\frac{5}{11}\begin{pmatrix}-1\\1\\3\end{pmatrix}=\ldots$	**M1**	Uses $\mathbf{b}-2\frac{\mathbf{b}\cdot\mathbf{n}}{\lvert\mathbf{n}\rvert^2}\mathbf{n}$ with their normal, or reflects P in Π, such as by $\overrightarrow{OP'}=2\overrightarrow{OQ}-\overrightarrow{OP}$ with their Q, and finds $\overrightarrow{RP'}$.

Question 9 continued

Question	Scheme	Marks	Notes
	$\frac{1}{11}\begin{pmatrix}21\\-10\\-8\end{pmatrix}$	**A1**	Or any nonzero multiple.
	$\mathbf{r}=\begin{pmatrix}3\\1\\0\end{pmatrix}+\mu\begin{pmatrix}21\\-10\\-8\end{pmatrix}$	**A1ft**	ft their direction, through R or their P'; any nonzero multiple. Needs $\mathbf{r}=\ldots$ with a parameter.
		[4]	
Total for Question 9		**15**	

Question	Scheme	Marks	Notes
10	$\begin{pmatrix}1\\1\\1\end{pmatrix} \times \begin{pmatrix}2\\-1\\1\end{pmatrix} = \begin{pmatrix}2\\1\\-3\end{pmatrix}$	**M1 A1**	**M1** Attempts the vector product of the two normals, or the difference of two points on ℓ_1. **A1** Or any nonzero multiple.
	$x+y+z=1,\ 2x-y+z=4$ $x+y+2z=3$	**M1**	Uses the equation of Π with both plane equations, or substitutes their parametric form of ℓ_1 into Π.
	$\Rightarrow (x,y,z) = \ldots$	**dM1**	Solves by any method, calculator allowed, to a point.
	$\left(\frac{1}{3}, -\frac{4}{3}, 2\right)$	**A1**	
	The point lies on Π, so is on ℓ_2	**B1**	Stated, or implied by its use as the point on ℓ_2.
	$\mathbf{u}' = \mathbf{u} - 2\,\frac{\mathbf{u}\cdot\mathbf{n}}{\lvert\mathbf{n}\rvert^2}\,\mathbf{n}$	**M1**	Finds the reflection of their direction in Π by the formula with no d term; or reflects a second point of ℓ_1 in Π.
	$= \text{“}\begin{pmatrix}2\\1\\-3\end{pmatrix}\text{”} - 2\left(\frac{-3}{6}\right)\begin{pmatrix}1\\1\\2\end{pmatrix}$	**dM1**	Finds $\mathbf{u}\cdot\mathbf{n}$ and $\lvert\mathbf{n}\rvert^2$ and subtracts twice the projection; or subtracts the fixed point from the image point. Condone arithmetic slips.
	$= \begin{pmatrix}3\\2\\-1\end{pmatrix}$	**A1**	Or any nonzero multiple.
	$\mathbf{r} = \text{“}\begin{pmatrix}\frac{1}{3}\\-\frac{4}{3}\\2\end{pmatrix}\text{”} + t\,\text{“}\begin{pmatrix}3\\2\\-1\end{pmatrix}\text{”}$	**A1ft**	ft their point and reflected direction; any point of ℓ_2, such as the image of their second point, and any nonzero multiple. Needs $\mathbf{r} = \ldots$ with a parameter.
	Total for Question 10	**10**	