OPTIONAL PURE / VECTORS & MATRICES

Cayley-Hamilton Theorem

Mark scheme 10 questions · 72 marks

EXAM BOARDS Edexcel FP2 9FM0/4A · OCR MEI Extra Pure Y435 · CIE 9231

Questions

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	Total	**72**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last accuracy mark of the working that reaches a printed result: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	$(4-\lambda)(1-\lambda)-(-1)\times 2=\ldots$	**M1**	Forms $\det(\mathbf{A}-\lambda\mathbf{I})$, or $\det(\lambda\mathbf{I}-\mathbf{A})$, and expands to a quadratic in λ. Condone one sign slip.
	$\lambda^2-5\lambda+6=0$	**A1**	Correct simplified equation oe; condone a missing $=0$.
		[2]	
1(b)	$\mathbf{A}^2-5\mathbf{A}+6\mathbf{I}=\mathbf{0}$ $\Rightarrow \mathbf{A}-5\mathbf{I}+6\mathbf{A}^{-1}=\mathbf{0}$	**M1**	Uses their quadratic characteristic equation with $\mathbf{A}$ for λ and $\mathbf{I}$ on the constant, multiplied by $\mathbf{A}^{-1}$, to express $\mathbf{A}^{-1}$ in $\mathbf{A}$ and $\mathbf{I}$. Condone sign slips.
	$\mathbf{A}^{-1}=\frac{1}{6}(5\mathbf{I}-\mathbf{A})$	**A1**	oe
	$\mathbf{I}-5\mathbf{A}^{-1}+6\mathbf{A}^{-2}=\mathbf{0}$ $\Rightarrow \mathbf{A}^{-2}=\frac{1}{6}\left(5\times\text{“}\frac{1}{6}(5\mathbf{I}-\mathbf{A})\text{”}-\mathbf{I}\right)$	**dM1**	Multiplies by $\mathbf{A}^{-1}$ again and substitutes their $\mathbf{A}^{-1}$, or squares their $\mathbf{A}^{-1}$ and replaces $\mathbf{A}^2$, reaching $\mathbf{A}^{-2}$ in $\mathbf{A}$ and $\mathbf{I}$ only.
	$\mathbf{A}^{-2}=-\frac{5}{36}\mathbf{A}+\frac{19}{36}\mathbf{I}$	**A1**	Correct expression, or $p=-\frac{5}{36}$ and $q=\frac{19}{36}$ stated, as exact fractions.
		[4]	
	Total for Question 1	**6**	

Question	Scheme	Marks	Notes
2(a)	$\det(\mathbf{A}-\lambda\mathbf{I}) = (1-\lambda)\left[-\lambda(2-\lambda)-2\right]-2$	**M1**	Expands $\det(\mathbf{A}-\lambda\mathbf{I})$ or $\det(\lambda\mathbf{I}-\mathbf{A})$ by any row, column or rule to a cubic in λ. Condone sign slips.
	$\lambda^3-3\lambda^2+4=0$	**A1**	Or $-\lambda^3+3\lambda^2-4=0$; condone a missing $=0$.
		[2]	
2(b)	$\mathbf{A}^3-3\mathbf{A}^2+4\mathbf{I}=\mathbf{0}$	**M1**	Replaces λ by $\mathbf{A}$ in their cubic, with $\mathbf{I}$ on the constant. Condone 4 for $4\mathbf{I}$.
	$\mathbf{A}^2-3\mathbf{A}+4\mathbf{A}^{-1}=\mathbf{0}$ $\Rightarrow \mathbf{A}-3\mathbf{I}+4\mathbf{A}^{-2}=\mathbf{0} \Rightarrow \mathbf{A}^{-2}=\ldots$	**dM1**	Multiplies by $\mathbf{A}^{-1}$ twice, or by $\mathbf{A}^{-2}$ once, reaching an equation in $\mathbf{A}^{-2}$, $\mathbf{A}$ and $\mathbf{I}$ only, and makes $\mathbf{A}^{-2}$ the subject.
	$\mathbf{A}^{-2}=\frac{3}{4}\mathbf{I}-\frac{1}{4}\mathbf{A}$, so $p=\frac{3}{4}$, $q=-\frac{1}{4}$	**A1**	Either p and q stated or the expression for $\mathbf{A}^{-2}$. cso.
		[3]	
	Total for Question 2	**5**	

Question	Scheme	Marks	Notes
3	$\det(\mathbf{A}-\lambda\mathbf{I})$ $=(4-\lambda)(-1-\lambda)(1-\lambda)-2(1+\lambda)$	**M1**	Expands $\det(\mathbf{A}-\lambda\mathbf{I})$ along any row or column, attempting every nonzero term. Condone sign slips; need not be simplified.
	$\lambda^3-4\lambda^2+\lambda+6=0$	**A1**	oe, such as $-\lambda^3+4\lambda^2-\lambda-6=0$.
	$\mathbf{A}^3=4\mathbf{A}^2-\mathbf{A}-6\mathbf{I}$ $\mathbf{A}^4=4\mathbf{A}^3-\mathbf{A}^2-6\mathbf{A}$ $=15\mathbf{A}^2-10\mathbf{A}-24\mathbf{I}$	**M1**	Uses their cubic with $\mathbf{A}$ for λ to reduce $\mathbf{A}^4$, or another higher power, to $\mathbf{A}^2$, $\mathbf{A}$ and $\mathbf{I}$. Condone slips.
	$\mathbf{A}^5=15\mathbf{A}^3-10\mathbf{A}^2-24\mathbf{A}$ $=15(4\mathbf{A}^2-\mathbf{A}-6\mathbf{I})-\ldots$	**dM1**	Multiplies their $\mathbf{A}^4$ by $\mathbf{A}$ and substitutes their $\mathbf{A}^3$, or completes any valid reduction, giving $\mathbf{A}^5$ in $\mathbf{A}^2$, $\mathbf{A}$ and $\mathbf{I}$.
	$\mathbf{A}^5=50\mathbf{A}^2-39\mathbf{A}-90\mathbf{I}$	**A1**	cao, or $a=50$, $b=-39$, $c=-90$ stated.
	Total for Question 3	**5**	

Question	Scheme	Marks	Notes
4	$\det(\mathbf{A}-\lambda\mathbf{I}) = (2-\lambda)\left[(-2-\lambda)^2-1\right]$	**M1**	Expands $\det(\mathbf{A}-\lambda\mathbf{I})$ by any row or column, or uses the trace, minors and determinant, reaching a cubic in λ. Condone sign slips.
	$\lambda^3+2\lambda^2-5\lambda-6=0$	**A1**	Any equivalent equation, including factorised or with leading term $-\lambda^3$.
	$(\mathbf{A}-2\mathbf{I})^3 = \mathbf{A}^3+3\mathbf{A}^2(-2\mathbf{I})$ $+3\mathbf{A}(-2\mathbf{I})^2+(-2\mathbf{I})^3$	**M1**	Expands by the binomial theorem or repeated multiplication, with terms in $\mathbf{A}^3$, $\mathbf{A}^2$, $\mathbf{A}$ and $\mathbf{I}$. Condone coefficient slips.
	$= \mathbf{A}^3-6\mathbf{A}^2+12\mathbf{A}-8\mathbf{I}$	**A1**	
	$\mathbf{A}^3 =$ "$-2\mathbf{A}^2+5\mathbf{A}+6\mathbf{I}$" $(\mathbf{A}-2\mathbf{I})^3 = -2\mathbf{A}^2+5\mathbf{A}+6\mathbf{I}$ $-6\mathbf{A}^2+12\mathbf{A}-8\mathbf{I}$	**M1**	Replaces $\mathbf{A}^3$ in their expansion using their characteristic equation in $\mathbf{A}$, and collects terms. Condone a missing $\mathbf{I}$. Not if any power of $\mathbf{A}$ is calculated.
	$= -8\mathbf{A}^2+17\mathbf{A}-2\mathbf{I}$ $p=-8,\ q=17,\ r=-2$	**A1**	Correct expression, or p, q and r stated.
	Total for Question 4	**6**	

Question	Scheme	Marks	Notes
5(a)	$\mathbf{M}^2 = \begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix}$ $\mathbf{M}^2 - (a+d)\mathbf{M} + (ad-bc)\mathbf{I}$ $= \begin{pmatrix} a^2+bc-a^2-ad+ad-bc & \dots \\ \dots & \dots \end{pmatrix}$	**M1**	Squares $\mathbf{M}$, at least two entries correct, and combines it entry by entry with $(a+d)\mathbf{M}$ and $(ad-bc)\mathbf{I}$. Condone sign slips.
	$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \mathbf{0}$	**A1 AG**	Correct $\mathbf{M}^2$ and combination leading without error to all four entries zero.
		[2]	
5(b)	$a+d=0,\ d=-a \Rightarrow \mathbf{M}^2 + (-a^2-bc)\mathbf{I} = \mathbf{0}$	**M1**	Uses $a+d=0$ in the result of (a) and replaces d by $-a$ in $ad-bc$. Condone sign slips.
	$\mathbf{M}^2 = (a^2+bc)\mathbf{I}$	**A1**	Printed result from (a)'s equation with $d=-a$, reached without error; not from the printed answer worked backwards.
	$\mathbf{N}^2 = \left(5^2 + (-8)\times 2\right)\mathbf{I} = 9\mathbf{I}$	**B1**	Accept by direct multiplication.
	$(\mathbf{N}+\mathbf{I})^3 = \mathbf{N}^3 + 3\mathbf{N}^2 + 3\mathbf{N} + \mathbf{I}$ $= 9\mathbf{N} + 27\mathbf{I} + 3\mathbf{N} + \mathbf{I}$	**M1**	Expands $(\mathbf{N}+\mathbf{I})^3$, in one step or by squaring first, replacing each $\mathbf{N}^2$ by their $k\mathbf{I}$. Condone coefficient slips. Not for cubing the numerical matrix.
	$= 12\mathbf{N} + 28\mathbf{I}$	**A1 AG**	Correct expansion with correct substitutions leading without error to the printed result.
		[5]	
	Total for Question 5	**7**	

Question	Scheme	Marks	Notes
6(a)	$(a-\lambda)(5-\lambda)+12=0$	**M1**	Forms $\det(\mathbf{A}-\lambda\mathbf{I})$ as $(a-\lambda)(5-\lambda)\pm 12$, or uses $\lambda^2-(\text{trace})\lambda+\det\mathbf{A}$, and expands. Condone sign slips.
	$\lambda^2-(a+5)\lambda+5a+12=0$	**A1**	Correct expanded equation, oe, terms in any order; condone a missing $=0$.
		[2]	
6(b)	$\mathbf{A}^2=(a+5)\mathbf{A}-(5a+12)\mathbf{I}$ $\mathbf{A}^3=(a+5)\mathbf{A}^2-(5a+12)\mathbf{A}$ $=(a+5)\left[(a+5)\mathbf{A}-(5a+12)\mathbf{I}\right]$ $-(5a+12)\mathbf{A}$	**M1**	Replaces λ by $\mathbf{A}$ in their equation, multiplies by $\mathbf{A}$ and substitutes for $\mathbf{A}^2$, reaching the form $p\mathbf{A}+q\mathbf{I}$. Condone sign slips.
	$\mathbf{A}^3=(a^2+5a+13)\mathbf{A}-(a+5)(5a+12)\mathbf{I}$	**A1**	oe. Coefficients need not be expanded.
	Equates to $7\mathbf{A}+b\mathbf{I}$ and compares $a^2+5a+13=7$, $b=-(a+5)(5a+12)$	**M1**	Equates their $\mathbf{A}^3$ to $7\mathbf{A}+b\mathbf{I}$, comparing coefficients or matrix entries, for an equation in a and an expression for b.
	$a^2+5a+6=0\Rightarrow a=\ldots,\ b=\ldots$	**dM1**	Solves their quadratic in a by any means and substitutes each root into their expression for b.
	$a=-2$, $b=-6$ and $a=-3$, $b=6$	**A1**	Both pairs, correctly matched, and no others.
		[5]	
	Total for Question 6	**7**	

Question	Scheme	Marks	Notes
7(a)	$\det(\mathbf{M}-\lambda\mathbf{I}) = (3-\lambda)\big[(-1-\lambda)(2-\lambda)-2k\big]$ $-2\big[-3(2-\lambda)-k\big]$	**M1**	Expands $\det(\mathbf{M}-\lambda\mathbf{I})$ or $\det(\lambda\mathbf{I}-\mathbf{M})$ by any row or column, attempting every nonzero term. Condone sign slips.
	$= -\lambda^3+4\lambda^2+(2k-7)\lambda+6-4k$ $\Rightarrow \lambda^3-4\lambda^2+(7-2k)\lambda+4k-6=0$	**A1 AG**	Correct expansion leading without error to the printed equation, set equal to zero; combined steps accepted.
		[2]	
7(b)(i)	$27-36+3(7-2k)+4k-6=0 \Rightarrow k=\ldots$	**M1**	Substitutes $\lambda=3$ into the characteristic equation, or uses $\det(\mathbf{M}-3\mathbf{I})=0$, and solves for k.
	$k=3$	**A1**	
	$[\lambda^3-4\lambda^2+\lambda+6=(\lambda-3)(\lambda-2)(\lambda+1)]$ $\lambda=2,\ \lambda=-1$	**B1**	Both values.
		[3]	
7(b)(ii)	$\mathbf{M}^3-4\mathbf{M}^2+\mathbf{M}+6\mathbf{I}=\mathbf{0}$ $\Rightarrow \mathbf{M}^2-4\mathbf{M}+\mathbf{I}+6\mathbf{M}^{-1}=\mathbf{0}$	**M1**	Replaces λ by $\mathbf{M}$ in their cubic with their k, and multiplies by $\mathbf{M}^{-1}$. Condone a missing $\mathbf{I}$.
	$\mathbf{M}^{-1}=\frac{1}{6}\left(4\mathbf{M}-\mathbf{M}^2-\mathbf{I}\right)$	**A1**	oe, with $\mathbf{I}$ present.
	$\mathbf{M}^2=\begin{pmatrix}3&-3&-1\\4&1&4\\6&3&10\end{pmatrix}$	**B1ft**	Correct $\mathbf{M}^2$, ft their k; calculator allowed, soi.
	$4\mathbf{M}-\mathbf{M}^2-\mathbf{I}=\begin{pmatrix}8&-9&5\\4&-6&4\\-6&9&-3\end{pmatrix}$	**dM1**	Combines their $\mathbf{M}^2$, $\mathbf{M}$ and $\mathbf{I}$ entry by entry as in their expression for $\mathbf{M}^{-1}$.
	$\mathbf{M}^{-1}=\frac{1}{6}\begin{pmatrix}8&-9&5\\4&-6&4\\-6&9&-3\end{pmatrix}$	**A1**	oe, such as the factor taken inside.
		[5]	
	Total for Question 7	**10**	

Question	Scheme	Marks	Notes
8(a)	$(a-\lambda)(a-3-\lambda)-4=0$ $\Rightarrow \mathbf{P}^2-(2a-3)\mathbf{P}+\ldots\mathbf{I}=\mathbf{0}$	**M1**	Forms $\det(\mathbf{P}-\lambda\mathbf{I})=0$ as a quadratic in λ, then replaces λ by $\mathbf{P}$ and the constant by a multiple of $\mathbf{I}$. Condone sign slips.
	$\mathbf{P}^2-(2a-3)\mathbf{P}+\left(a^2-3a-4\right)\mathbf{I}=\mathbf{0}$	**A1**	oe, need not be expanded.
	Compares with $\mathbf{P}^2+r\mathbf{P}-6\mathbf{I}=\mathbf{0}$: $(r+2a-3)\mathbf{P}=\left(a^2-3a+2\right)\mathbf{I}$ $\Rightarrow r=3-2a,\ a^2-3a+2=0\Rightarrow a=\ldots$	**M1**	Uses both equations, by subtracting and equating entries or by equating coefficients of $\mathbf{P}$ and $\mathbf{I}$, to get equations in a and r, and solves for a. Condone sign slips.
	$a=1,\ r=1$ or $a=2,\ r=-1$	**A1**	Both pairs, correctly matched.
		[4]	
8(b)	$\mathbf{P}^2=$ "$\mathbf{P}+6\mathbf{I}$" $\Rightarrow \mathbf{P}^3=\mathbf{P}^2+6\mathbf{P}=7\mathbf{P}+6\mathbf{I}$ $\Rightarrow \mathbf{P}^4=7\mathbf{P}^2+6\mathbf{P}=\ldots$	**M1**	Uses their $\mathbf{P}^2=p\mathbf{P}+q\mathbf{I}$ for $a>1$, multiplying by $\mathbf{P}$ or squaring, and substitutes for $\mathbf{P}^2$ to reach $\mathbf{P}^4$ in $\mathbf{P}$ and $\mathbf{I}$. Not for calculating $\mathbf{P}^4$.
	$\mathbf{P}^4=13\mathbf{P}+42\mathbf{I}$ $s=13,\ t=42$	**A1**	
	$\mathbf{P}^2-\mathbf{P}=6\mathbf{I}\Rightarrow\mathbf{P}-\mathbf{I}=6\mathbf{P}^{-1}$	**M1**	Multiplies their equation in $\mathbf{P}^2$, $\mathbf{P}$ and $\mathbf{I}$ by $\mathbf{P}^{-1}$, or finds $\mathbf{P}^{-1}$ and compares entries with $\alpha\mathbf{P}+\beta\mathbf{I}$. Condone sign slips.
	$\mathbf{P}^{-1}=\frac{1}{6}\mathbf{P}-\frac{1}{6}\mathbf{I}$	**A1**	oe, or $\alpha=\frac{1}{6}$, $\beta=-\frac{1}{6}$ stated.
		[4]	
Total for Question 8		**8**	

Question	Scheme	Marks	Notes
9(a)	$\det(\mathbf{A}-\lambda\mathbf{I})$ $=(k-\lambda)\left[(1-\lambda)(-1-\lambda)-8\right]$	**M1**	Expands $\det(\mathbf{A}-\lambda\mathbf{I})$ along any row or column, with λ subtracted from each diagonal entry. Condone sign slips.
	$(k-\lambda)\left(\lambda^2-9\right)=0$ $\Rightarrow \lambda^3-k\lambda^2-9\lambda+9k=0$	**A1 AG**	Correct expansion leading without error to the printed equation; collected steps accepted.
		[2]	
9(b)	$\mathbf{A}^3-k\mathbf{A}^2-9\mathbf{A}+9k\mathbf{I}=\mathbf{0}$ $\Rightarrow \mathbf{A}^2-k\mathbf{A}-9\mathbf{I}+9k\mathbf{A}^{-1}=\mathbf{0}$	**M1**	Replaces λ by $\mathbf{A}$ in the characteristic equation and multiplies through by $\mathbf{A}^{-1}$. Condone $\mathbf{I}$ omitted.
	$\mathbf{A}^{-1}=\frac{1}{9k}\left(9\mathbf{I}+k\mathbf{A}-\mathbf{A}^2\right)$	**A1**	oe, with $\mathbf{I}$ present.
	$\mathbf{A}^2=\begin{pmatrix}9 & k^2+6k+1 & 0\\ 0 & k^2 & 0\\ 0 & 2k^2+2k+4 & 9\end{pmatrix}$	**M1**	Multiplies $\mathbf{A}$ by $\mathbf{A}$, rows by columns, attempting all nine entries. Condone slips; need not be simplified.
	$\mathbf{A}^{-1}=\frac{1}{9k}\left(9\mathbf{I}+k\mathbf{A}-\text{“}\mathbf{A}^2\text{”}\right)$ $=\ldots$	**dM1**	Substitutes $\mathbf{A}$ and their $\mathbf{A}^2$ into their expression in $\mathbf{I}$, $\mathbf{A}$ and $\mathbf{A}^2$, and combines entry by entry.
	$\mathbf{A}^{-1}=\frac{1}{9k}\begin{pmatrix}k & -5k-1 & 2k\\ 0 & 9 & 0\\ 4k & -2k-4 & -k\end{pmatrix}$	**A1**	oe, such as $\frac{1}{9k}$ taken into the matrix.
		[5]	
	Total for Question 9	**7**	

Question	Scheme	Marks	Notes
10(a)	$\det(\mathbf{M}-\lambda\mathbf{I}) = (b-\lambda)\big(-\lambda(3-\lambda)-2a\big)$ $-2\left(0-a^2\right)-(0+a\lambda)$	**M1**	Expands $\det(\mathbf{M}-\lambda\mathbf{I})$ or $\det(\lambda\mathbf{I}-\mathbf{M})$ by any row or column, with three terms. Condone sign slips.
	$\lambda^3-(b+3)\lambda^2+(3b-a)\lambda+2ab-2a^2=0$	**A1**	Correct cubic, oe, with either overall sign.
	$-(b+3)=1 \Rightarrow b=\ldots$	**M1**	Equates the coefficient of λ^2 or the constant term of their cubic with the given equation's, or uses the trace or determinant of $\mathbf{M}$. Condone sign slips.
	$b=-4$	**A1**	
	$2a\text{“}(-4)\text{”}-2a^2=-24 \Rightarrow a^2+4a-12=0$ $\Rightarrow a=\ldots$	**dM1**	Forms a quadratic in a using their b, solves it by any means and selects a positive root.
	$a=2, \qquad c=3b-a=-14$	**A1**	Both values. Withhold if $a=-6$ is also given as an answer.
		[6]	
10(b)	$\mathbf{M}^3+\mathbf{M}^2-14\mathbf{M}-24\mathbf{I}=\mathbf{0}$ $\Rightarrow (\mathbf{M}-2\mathbf{I})\left(\mathbf{M}^2+3\mathbf{M}-8\mathbf{I}\right)=40\mathbf{I}$	**M1**	Writes their characteristic equation in $\mathbf{M}$, with $\mathbf{I}$ on the constant, and factorises out $\mathbf{M}-2\mathbf{I}$ leaving a multiple of $\mathbf{I}$. Condone slips in the division.
	$(\mathbf{M}-2\mathbf{I})^{-1}=\frac{1}{40}\left(\mathbf{M}^2+3\mathbf{M}-8\mathbf{I}\right)$	**A1**	oe
	$\mathbf{M}^2=\begin{pmatrix}14&-10&5\\4&4&6\\-2&10&11\end{pmatrix}$	**B1ft**	Correct $\mathbf{M}^2$, ft their a and b, or $(\mathbf{M}-2\mathbf{I})^2$ where $\mathbf{M}-2\mathbf{I}$ was substituted; calculator allowed, soi.
	$\frac{1}{40}\left(\mathbf{M}^2+3\mathbf{M}-8\mathbf{I}\right)=\frac{1}{40}\begin{pmatrix}-6&-4&2\\4&-4&12\\4&16&12\end{pmatrix}$	**dM1**	Combines their square, matrix and $\mathbf{I}$ as in their expression for $(\mathbf{M}-2\mathbf{I})^{-1}$, including its scalar factor.
	$(\mathbf{M}-2\mathbf{I})^{-1}=\frac{1}{20}\begin{pmatrix}-3&-2&1\\2&-2&6\\2&8&6\end{pmatrix}$	**A1**	Any equivalent form, including exact decimal entries.
		[5]	
	Total for Question 10	**11**	