OPTIONAL PURE / SURFACES & PARTIAL DIFFERENTIATION

Tangent Planes and Normal Lines

Questions by topic 10 questions · 103 marks

EXAM BOARDS OCR A Y545 · OCR MEI Extra Pure Y435

Instructions

OCR A (Y545): the specification names the tangent plane only for a surface given as $z = f(x, y)$. Questions 3, 4 and 8 to 10 use ∇g for a surface given as $g(x, y, z) = c$, which the specification does not name, and can be treated as extension.

Questions

Question 1	*5 marks*	2
Question 2	*7 marks*	3
Question 3	*7 marks*	4
Question 4	*9 marks*	5
Question 5	*8 marks*	6
Question 6	*11 marks*	7
Question 7	*12 marks*	8
Question 8	*7 marks*	9
Question 9	*18 marks*	10
Question 10	*19 marks*	11

1. A surface S has equation

$$z = 3y\sqrt{x} - \frac{16x}{y} + xy^2, \qquad x \geq 0,\ y > 0$$

The point P on S has x-coordinate 4 and y-coordinate 2.

Find the z-coordinate of P, and hence determine the equation of the tangent plane to S at P. Give your answer in the form $ax + by + cz = d$, where a, b, c and d are integers. **[5]**

2. The surface E has equation

$$z = \frac{48}{x^2 - xy + 2y^2}$$

The point P on E has x-coordinate 2 and y-coordinate 3.

(a) Find the z-coordinate of P, and determine the values of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at P. **[5]**

(b) Find the equation of the tangent plane to E at P, giving your answer in the form $ax + by + cz = d$, where a, b, c and d are integers. **[2]**

3. The function g is defined by

$$g(x, y, z) = (x^2 + yz)e^{xy+z}$$

The surface S has equation $g(x, y, z) = 2$, and the point $P(2, 1, -2)$ lies on S.

(a) Find $\frac{\partial g}{\partial x}$, $\frac{\partial g}{\partial y}$ and $\frac{\partial g}{\partial z}$. **[4]**

(b) Find a vector equation of the normal line to S at P, and hence find the coordinates of the two points on this normal line whose distance from P is 14. **[3]**

4. A surface S is defined by $g(x, y, z) = 2$, where

$$g(x, y, z) = x^2y - 2yz^2 + 3xz$$

The point $A(1, 1, a)$, where a is an integer, and the point $B(1, 2, 0)$ both lie on S.

(a) Find the value of a. **[2]**

(b) The normal line to S at A and the tangent plane to S at B intersect at the point P. Determine the position vector of P. **[7]**

5. A surface has equation

$$z = x^2y - 3x^2 + y^3 - 6y^2 + 24y + 5$$

Show that there are exactly four points on the surface at which the normal line is parallel to the vector $24\mathbf{j} - \mathbf{k}$, and find the coordinates of these points. Hence write down a vector equation of the normal line at the one of these points with the greatest z-coordinate. **[8]**

6. A surface has equation

$$z = 2x^3 - 6xy + 3y^2 + 5$$

The point Q with coordinates $(2, 1, 12)$ lies on the surface.

(a) Show that the equation of the tangent plane to the surface at Q is $18x - 6y - z = 18$. **[4]**

(b) There is exactly one other point R on the surface at which the tangent plane is parallel to the tangent plane at Q.
Find the coordinates of R and the equation of the tangent plane at R. **[7]**

7. A surface has equation

$$z = 2y(x-y)^2 - 2y^2 + 14y$$

(a) The point $P(3, 1, 20)$ lies on the surface. Find a vector equation of the normal line to the surface at P, and find the coordinates of the point where this normal line meets the plane $x = y$. **[5]**

(b) Show that there is exactly one point on the surface at which the tangent plane has an equation of the form $6y - z = d$, where d is a constant. Find the coordinates of this point and the value of d. **[7]**

8. A surface S has equation

$$x^2 + y^2 + 6z = 54$$

C is the set of points on S at which the tangent plane to S makes an angle of $\frac{1}{3}\pi$ radians with the x-y plane.

(a) Show that C lies in a plane, Π, and find the equation of Π. **[5]**

(b) Show that the tangent planes to S at all points of C pass through a common point, and find the coordinates of this point. **[2]**

9. A surface S has equation

$$x^2 + 2xz - y^2 + 4yz - z^2 + 10 = 0$$

(a) Find a normal vector to S at the general point (x, y, z) on S. **[3]**

(b) Find a Cartesian equation of the tangent plane to S at the point $Q(1, 6, 1)$. **[3]**

(c) Show that there is no point on S at which the normal line to S is parallel to the x-axis. **[4]**

(d) The plane with equation $x + 2y - 2z = k$, where k is a constant, is a tangent plane to S. Find the two possible values of k and, for each value, the coordinates of the point at which the plane touches S. **[8]**

10. A surface S has equation $g(x, y, z) = 0$, where

$$g(x, y, z) = x^2 + 3y^2 + 3z^2 - 2xy + 4yz - 2xz - 54$$

The point P has coordinates $(1, 2, 3)$.

(a) Verify that P lies on S, and find an equation of the tangent plane to S at P, giving your answer in the form $ax + by + cz = d$. **[5]**

(b) Find the coordinates of the two points on S at which the tangent plane is parallel to the plane $z = 0$, and write down the equations of the tangent planes at these points. **[6]**

(c) Show that the plane $4x + 4y - 3z = 54$ is a tangent plane to S, and find the coordinates of its point of contact with S. **[8]**