OPTIONAL PURE / SURFACES & PARTIAL DIFFERENTIATION

Tangent Planes and Normal Lines

Mark scheme 10 questions · 103 marks

EXAM BOARDS OCR A Y545 · OCR MEI Extra Pure Y435

Questions

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	Total	**103**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last accuracy mark of the working that reaches a printed result: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1	$z = 3(2)\sqrt{4} - \frac{16(4)}{2} + 4(2)^2 = -4$	**B1**	
	$g = 3y\sqrt{x} - \frac{16x}{y} + xy^2 - z$ $\frac{\partial g}{\partial x} = \frac{3y}{2\sqrt{x}} - \frac{16}{y} + y^2$ $\frac{\partial g}{\partial y} = 3\sqrt{x} + \frac{16x}{y^2} + 2xy$	**M1**	Differentiates partially with respect to x and y, at least one power decreased by 1 in each. Condone slips; $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ accepted.
	$\nabla g = \begin{pmatrix} -\frac{5}{2} \\ 38 \\ -1 \end{pmatrix}$ at P	**A1**	Both values $-\frac{5}{2}$ and 38, or a correct normal at P or any multiple of it.
	$5x - 76y + 2z = \begin{pmatrix} 4 \\ 2 \\ \text{“}-4\text{”} \end{pmatrix} \cdot \text{“}\begin{pmatrix} 5 \\ -76 \\ 2 \end{pmatrix}\text{”} = \ldots$	**M1**	Forms a plane through their P with their normal, by $\mathbf{r} \cdot \mathbf{n} = \overrightarrow{OP} \cdot \mathbf{n}$, a Cartesian constant found from P, or $z - z_0 = \ldots$. Condone sign slips.
	$5x - 76y + 2z = -140$	**A1**	Any integer multiple, such as $-5x + 76y - 2z = 140$.
	Total for Question 1	**5**	

Question	Scheme	Marks	Notes
2(a)	$z = \frac{48}{4-6+18} = 3$	**B1**	
	$\frac{\partial z}{\partial x} = -\frac{48(2x-y)}{(x^2-xy+2y^2)^2}$	**M1**	Differentiates holding y constant, by chain or quotient rule, to the form $k(2x-y)\left(x^2-xy+2y^2\right)^{-2}$. Condone sign and coefficient slips.
	$\frac{\partial z}{\partial x} = -\frac{48\times 1}{256} = -\frac{3}{16}$	**A1**	oe, such as -0.1875.
	$\frac{\partial z}{\partial y} = -\frac{48(4y-x)}{(x^2-xy+2y^2)^2}$	**M1**	Differentiates holding x constant, by chain or quotient rule, to the form $k(4y-x)\left(x^2-xy+2y^2\right)^{-2}$. Condone sign and coefficient slips.
	$\frac{\partial z}{\partial y} = -\frac{48\times 10}{256} = -\frac{15}{8}$	**A1**	oe, such as -1.875.
		[5]	
2(b)	$z-3 = \text{“}-\frac{3}{16}\text{”}(x-2)$ $+ \text{“}-\frac{15}{8}\text{”}(y-3)$	**M1**	Forms the tangent plane through their P with their partial derivatives, by the formula or a normal $\frac{\partial z}{\partial x}\mathbf{i} + \frac{\partial z}{\partial y}\mathbf{j} - \mathbf{k}$. Condone sign slips.
	$3x + 30y + 16z = 144$	**A1**	Or any integer multiple, in the form $ax + by + cz = d$.
		[2]	
	Total for Question 2	**7**	

Question	Scheme	Marks	Notes
3(a)	$\frac{\partial g}{\partial x} = 2xe^{xy+z} + (x^2+yz)ye^{xy+z}$	**M1 A1**	**M1** Applies the product rule in at least one derivative, differentiating e^{xy+z} to a multiple of itself. Condone a wrong chain-rule factor. **A1** oe; need not be simplified or factorised.
	$\frac{\partial g}{\partial y} = ze^{xy+z} + (x^2+yz)xe^{xy+z}$	**A1**	oe; need not be simplified or factorised.
	$\frac{\partial g}{\partial z} = ye^{xy+z} + (x^2+yz)e^{xy+z}$	**A1**	oe; need not be simplified or factorised.
		[4]	
3(b)	$\nabla g = \begin{pmatrix}6\\2\\3\end{pmatrix}$ at P $\mathbf{r} = \begin{pmatrix}2\\1\\-2\end{pmatrix} + \lambda\begin{pmatrix}6\\2\\3\end{pmatrix}$	**B1ft**	ft their partial derivatives from (a) evaluated at P. Line through P along that vector or any non-zero multiple of it; must be an equation in $\mathbf{r}$.
	$7\lvert\lambda\rvert = 14 \Rightarrow \lambda = \ldots$	**M1**	Sets their $\lvert\lambda\rvert$ times the direction's magnitude, or the distance of a general point from P, equal to 14 and solves for λ. One sign is enough.
	$\lambda = \pm 2 \Rightarrow (14, 5, 4)$ and $(-10, -3, -8)$	**A1**	Both points; position vectors accepted.
		[3]	
	Total for Question 3	**7**	

Question	Scheme	Marks	Notes
4(a)	$1-2a^2+3a=2 \Rightarrow 2a^2-3a+1=0$ $\Rightarrow a=\ldots$	**M1**	Substitutes $(1,1,a)$ into $g=2$ and solves the resulting quadratic in a by any means. Condone sign slips.
	$a=1$ $[a\neq\frac{1}{2}]$	**A1**	$a=1$ only; A0 if $\frac{1}{2}$ is also given as a value of a.
		[2]	
4(b)	$\frac{\partial g}{\partial x}=2xy+3z$ $\frac{\partial g}{\partial y}=x^2-2z^2$ $\frac{\partial g}{\partial z}=3x-4yz$	**M1 A1**	**M1** Differentiates g partially with respect to x, y and z, at least one power decreased by 1 in each. Condone sign slips. **A1** All three correct.
	$\mathbf{r}=\begin{pmatrix}1\\1\\1\end{pmatrix}+\lambda\begin{pmatrix}5\\-1\\-1\end{pmatrix}$	**A1**	Through $(1,1,1)$, direction any non-zero multiple of $5\mathbf{i}-\mathbf{j}-\mathbf{k}$; any equivalent line form.
	"4"x + "1"y + "3"z = "4"(1) + "1"(2) + "3"(0)	**M1**	Evaluates their partial derivatives at B as the plane's normal and finds the constant using B; vector form $\mathbf{r}\cdot\mathbf{n}=\overrightarrow{OB}\cdot\mathbf{n}$ accepted.
	$4x+y+3z=6$	**A1**	oe, including any multiple or $\mathbf{r}\cdot\begin{pmatrix}4\\1\\3\end{pmatrix}=6$.
	$4(1+5\lambda)+(1-\lambda)+3(1-\lambda)=6$ $\Rightarrow \lambda=\ldots$	**M1**	Substitutes the general point of their normal line at A into their tangent plane and solves for the parameter. Condone sign slips.
	$\overrightarrow{OP}=\begin{pmatrix}\frac{3}{8}\\\frac{9}{8}\\\frac{9}{8}\end{pmatrix}$	**A1**	cao; accept coordinates $\left(\frac{3}{8},\frac{9}{8},\frac{9}{8}\right)$, $\frac{3}{8}\mathbf{i}+\frac{9}{8}\mathbf{j}+\frac{9}{8}\mathbf{k}$ or exact decimals.
		[7]	
	Total for Question 4	**9**	

Question	Scheme	Marks	Notes
5	$\nabla g = \begin{pmatrix} 2xy - 6x \\ x^2 + 3y^2 - 12y + 24 \\ -1 \end{pmatrix}$	**M1 A1**	**M1** Finds both partial derivatives of the surface's expression, the power of at least one term decreased by 1 in each. Condone sign slips. **A1** Correct normal vector, or any non-zero multiple of it.
	Sets the normal proportional to $24\mathbf{j} - \mathbf{k}$: $2xy - 6x = 0, \quad x^2 + 3y^2 - 12y + 24 = 24$ $\Rightarrow x = 0$ or $y = 3$	**M1 A1**	**M1** Equates their normal to a multiple of $24\mathbf{j} - \mathbf{k}$, giving a zero first component and an equation in the other two. Condone sign slips. **A1** Both equations correct, or the second with the cases $x = 0$ or $y = 3$.
	$x = 0: \ 3y^2 - 12y = 0 \Rightarrow y = \ldots$ $y = 3: \ x^2 - 9 = 0 \Rightarrow x = \ldots$	**M1**	Substitutes at least one case into the other equation and solves the resulting equation by any method.
	$x = 0, \ y = 0$ or $4; \quad y = 3, \ x = \pm 3$	**A1**	All four pairs, and no others.
	$(0, 0, 5), \ (0, 4, 69), \ (3, 3, 50), \ (-3, 3, 50)$	**A1**	cso. All four points with correct z, and a conclusion that there are exactly four.
	$\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ 69 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 24 \\ -1 \end{pmatrix}$	**B1ft**	ft their point with the greatest z-coordinate. Direction any non-zero multiple of $24\mathbf{j} - \mathbf{k}$, any parameter letter; must be an equation in $\mathbf{r}$.
	Total for Question 5	**8**	

Question	Scheme	Marks	Notes
6(a)	$\frac{\partial z}{\partial x} = 6x^2 - 6y, \quad \frac{\partial z}{\partial y} = -6x + 6y$ $x = 2,\ y = 1 \Rightarrow \frac{\partial z}{\partial x} = \ldots,\ \frac{\partial z}{\partial y} = \ldots$	**M1**	Differentiates partially with respect to x and y, at least one power decreased by 1 in each, and substitutes $x = 2$, $y = 1$.
	$\frac{\partial z}{\partial x} = 18, \quad \frac{\partial z}{\partial y} = -6$	**A1**	Both values.
	$\mathbf{n} = \begin{pmatrix} \text{“18”} \\ \text{“}-6\text{”} \\ -1 \end{pmatrix}$ $18x - 6y - z = 18 \times 2 - 6 \times 1 - 12$	**M1**	Forms a plane through Q with normal from their partial derivatives and -1, or a multiple, or uses the tangent plane formula at Q with their partial derivatives. Condone sign slips.
	$18x - 6y - z = 18$	**A1 AG**	Correct substituted plane through Q leading without error to the printed equation.
		[4]	
6(b)	$6x^2 - 6y = 18, \quad -6x + 6y = -6$	**M1 A1**	**M1** Sets their partial derivatives equal to their values at Q, or the normal proportional to that at Q. **A1** Both equations, oe.
	$y = x - 1 \Rightarrow 6x^2 - 6(x - 1) = 18 \Rightarrow x = \ldots$	**M1**	Eliminates one variable to reach a three-term quadratic and solves it by any means.
	$x = -1 \quad (x = 2 \text{ gives } Q)$	**A1**	$x = -1$ or $y = -2$, with the root giving Q rejected or identified as Q.
	$R(-1, -2, 3)$	**A1**	
	$18x - 6y - z = 18 \times$ “(-1)” $- 6 \times$ “(-2)” $-$ “3”	**M1**	Forms a plane through their R with normal parallel to that at Q, or uses $z - z_0 = 18(x - x_0) - 6(y - y_0)$ at their R.
	$18x - 6y - z = -9$	**A1**	oe
		[7]	
	Total for Question 6	**11**	

Question	Scheme	Marks	Notes
7(a)	$\frac{\partial z}{\partial x} = 4y(x-y)$ $\frac{\partial z}{\partial y} = 2(x-y)^2 - 4y(x-y) - 4y + 14$ At P: $\frac{\partial z}{\partial x} = \ldots$, $\frac{\partial z}{\partial y} = \ldots$	**M1**	Differentiates partially with respect to x and y, at least one power decreased by 1 in each, and substitutes $x = 3$, $y = 1$. Condone sign and coefficient slips.
	$\frac{\partial z}{\partial x} = 8$, $\frac{\partial z}{\partial y} = 10$	**A1**	
	$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 20 \end{pmatrix} + \lambda \begin{pmatrix} 8 \\ 10 \\ -1 \end{pmatrix}$	**A1**	oe, with any non-zero multiple of the direction vector; must be an equation in $\mathbf{r}$.
	$3 + 8\lambda = 1 + 10\lambda \Rightarrow \lambda = \ldots$	**M1**	Equates the x and y components of their line's general point and solves for the parameter.
	$(11, 11, 19)$	**A1**	
		[5]	
7(b)	Normal parallel to $\begin{pmatrix} 0 \\ 6 \\ -1 \end{pmatrix}$: $\frac{\partial z}{\partial x} = 0$, $\frac{\partial z}{\partial y} = 6$	**M1**	Sets their $\frac{\partial z}{\partial x} = 0$ and $\frac{\partial z}{\partial y} = 6$, or equates their normal to a multiple of $6\mathbf{j} - \mathbf{k}$.
	$4y(x-y) = 0 \Rightarrow y = 0$ or $x = y$	**A1**	
	$y = 0$: $2x^2 + 14 = 6 \Rightarrow x^2 = \ldots$ $x = y$: $-4y + 14 = 6 \Rightarrow y = \ldots$	**M1**	Substitutes at least one of their cases into their $\frac{\partial z}{\partial y} = 6$ and attempts to solve.
	$x^2 = -4$, no real solutions; $y = 2$	**A1**	Both: $y = 0$ rejected as giving no real x, and $y = 2$.
	$(2, 2, 20)$ is the only point	**A1**	cso, with both cases resolved and a conclusion that there is exactly one point.
	$z - \text{“20”} = 0(x - \text{“2”}) + 6(y - \text{“2”})$	**M1**	Uses their point in the tangent plane formula with their partial derivatives, or substitutes their point into $6y - z = d$.
	$6y - z = -8$, $d = -8$	**A1**	
		[7]	
Total for Question 7		**12**	

Question	Scheme	Marks	Notes
8(a)	$\mathbf{n} = \nabla g = \begin{pmatrix} 2x \\ 2y \\ 6 \end{pmatrix}$	**B1**	Any non-zero multiple.
	$\cos\frac{1}{3}\pi = \dfrac{\lvert\mathbf{n}\cdot\mathbf{k}\rvert}{\lvert\mathbf{n}\rvert\lvert\mathbf{k}\rvert} = \dfrac{6}{2\sqrt{x^2+y^2+9}}$ $\Rightarrow x^2+y^2 = \ldots$	**M1**	Equates the scalar product of their normal with $\mathbf{k}$, over the moduli, to $\cos\frac{1}{3}\pi$ or $\frac{1}{2}$, and solves. Condone a missing modulus or sign slips.
	$x^2+y^2 = 27$	**A1**	oe
	"27" $+ 6z = 54 \Rightarrow z = \ldots$	**M1**	Substitutes their x^2+y^2 into the equation of S to find z.
	$\Pi : z = \frac{9}{2}$	**A1**	cso. Needs a statement that every point of C has this z-coordinate.
		[5]	
8(b)	At $\left(p, q, \text{"}\frac{9}{2}\text{"}\right)$: $2px + 2qy + 6z = 2(p^2+q^2) + \text{"27"}$ $= \text{"81"}$	**M1**	Forms the tangent plane at a general point of C with their normal and their z, using their x^2+y^2 from (a) in its constant. Any parametrisation accepted.
	$x = y = 0 \Rightarrow$ common point $\left(0, 0, \frac{27}{2}\right)$	**A1**	cso. $\left(0, 0, \frac{27}{2}\right)$ from a plane whose constant is independent of the point of C, with a conclusion that every such tangent plane contains it.
		[2]	
	Total for Question 8	**7**	

Question	Scheme	Marks	Notes
9(a)	$\frac{\partial g}{\partial x} = 2x + 2z$	**B1**	
	$\frac{\partial g}{\partial y} = 4z - 2y$	**B1**	
	$\frac{\partial g}{\partial z} = 2x + 4y - 2z$ $\nabla g = \begin{pmatrix} 2x + 2z \\ 4z - 2y \\ 2x + 4y - 2z \end{pmatrix}$	**B1**	$\frac{\partial g}{\partial z}$ correct. Each B1 is for its own derivative, seen alone or as a component of their normal; a non-zero multiple of the whole vector is accepted.
		[3]	
9(b)	$\nabla g = \begin{pmatrix} 4 \\ -8 \\ 24 \end{pmatrix}$	**B1ft**	ft their normal vector from (a) evaluated at $(1, 6, 1)$, or any non-zero multiple.
	"1"x − "2"y + "6"z = "1"(1) − "2"(6) + "6"(1)	**M1**	Uses their normal and Q to form a plane, as Cartesian with constant found or $\mathbf{r} \cdot \mathbf{n} = \overrightarrow{OQ} \cdot \mathbf{n}$.
	$x - 2y + 6z = -5$	**A1**	Cartesian form, or any non-zero multiple.
		[3]	
9(c)	$4z - 2y = 0,\ 2x + 4y - 2z = 0$ $\Rightarrow y = \ldots,\ x = \ldots$	**M1**	Sets the y- and z-components of their normal to zero, or equivalent with the normal a multiple of $\mathbf{i}$.
	$y = 2z,\ x = -3z$	**A1**	oe, two variables in terms of the third.
	$(-3z)^2 + 2(-3z)z - (2z)^2$ $+ 4(2z)z - z^2 + 10 = 0$	**M1**	Substitutes their expressions for two variables in terms of the third into the equation of S.
	$6z^2 + 10 = 0$ $6z^2 + 10 > 0$ for all real z, so no such point	**A1 AG**	Correct $6z^2 + 10 = 0$ oe from correct substitution, with a reason it has no real roots and a conclusion.
		[4]	
9(d)	$\begin{pmatrix} x + z \\ 2z - y \\ x + 2y - z \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$	**M1**	Sets their normal equal to a multiple of $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$, or equates ratios of components, giving equations in the coordinates.
	$x = \lambda - z,\ y = 2z - 2\lambda$ $\Rightarrow (\lambda - z) + 2(2z - 2\lambda) - z = -2\lambda$ $\Rightarrow z = \ldots$	**dM1**	Solves the equations together to express x, y, z in terms of the multiplier or of one coordinate. Condone sign slips.

Question 9 continued

Question	Scheme	Marks	Notes
	$x = \frac{1}{2}\lambda,\ y = -\lambda,\ z = \frac{1}{2}\lambda$	**A1**	oe, such as $x = z$, $y = -2z$.
	$\frac{1}{4}\lambda^2 + \frac{1}{2}\lambda^2 - \lambda^2 - 2\lambda^2 - \frac{1}{4}\lambda^2 + 10 = 0 \Rightarrow \lambda^2 = \ldots$	**M1**	Substitutes their coordinates into the equation of S and solves for their parameter.
	$\lambda = 2$ or $\lambda = -2$	**A1**	Both values, or the equivalent for their parameter.
	$(1, -2, 1)$ and $(-1, 2, -1)$	**A1**	
	$k = 1 + 2(-2) - 2(1)$ $k = -1 + 2(2) - 2(-1)$	**M1**	Substitutes at least one of their points into $x + 2y - 2z$ to find k.
	$k = -5$ at $(1, -2, 1)$ $k = 5$ at $(-1, 2, -1)$	**A1**	Both values, each paired with its correct point.
		[8]	
Total for Question 9		**18**	

Question	Scheme	Marks	Notes
10(a)	$g(1,2,3) = 1+12+27-4+24-6-54 = 0$	**B1**	Substituted values or terms shown, with $= 0$ and a conclusion that P lies on S.
	$\frac{\partial g}{\partial x} = 2x-2y-2z$, $\frac{\partial g}{\partial y} = 6y-2x+4z$ $\frac{\partial g}{\partial z} = 6z+4y-2x$ $\nabla g = \begin{pmatrix} 2-4-6 \\ 12-2+12 \\ 18+8-2 \end{pmatrix}$	**M1**	Attempts all three partial derivatives, each with a power decreased by 1, and substitutes the coordinates of P.
	$\nabla g = \begin{pmatrix} -8 \\ 22 \\ 24 \end{pmatrix}$	**A1**	Or any non-zero multiple, such as -4, 11, 12.
	"-4"x + "11"y + "12"z = "-4"(1) + "11"(2) + "12"(3)	**M1**	Forms a plane with their normal and substitutes P for its constant, or uses $\mathbf{r}\cdot\mathbf{n} = \overrightarrow{OP}\cdot\mathbf{n}$.
	$-4x+11y+12z = 54$	**A1**	Or any multiple, in the form $ax+by+cz=d$.
		[5]	
10(b)	$2x-2y-2z=0$, $6y-2x+4z=0$	**M1**	Sets their $\frac{\partial g}{\partial x}$ and $\frac{\partial g}{\partial y}$ equal to zero.
	$z=-2y$, $x=-y$	**A1**	oe, any two variables in terms of the third.
	$y^2+3y^2+12y^2+2y^2-8y^2-4y^2 = 54$ $\Rightarrow y = \ldots$	**M1**	Substitutes their expressions into $g(x,y,z)=0$, obtaining an equation in one variable, and solves it.
	$6y^2 = 54 \Rightarrow y = \pm 3$	**A1**	Both values, or the equivalent for their variable.
	$(-3,3,-6)$ and $(3,-3,6)$	**A1**	
	$z=-6$ and $z=6$	**B1ft**	ft their points: both planes, each z equal to its point's z-coordinate.
		[6]	
10(c)	$x-y-z=4k$, $-x+3y+2z=4k$ $-x+2y+3z=-3k$	**M1**	Sets their three partial derivatives proportional to 4, 4, -3, with a multiplier or equal ratios. Condone the factor 2 absorbed.
	$2y+z=8k$, $y+2z=k \Rightarrow y = \ldots$	**dM1**	Eliminates to find x, y, z in terms of the multiplier, or two coordinates in terms of the third. Condone arithmetic slips.

Question 10 continued

Question	Scheme	Marks	Notes
	$(7k, 5k, -2k)$	**A1**	oe, such as $x = \frac{7}{5}y$, $z = -\frac{2}{5}y$.
	$49k^2 + 75k^2 + 12k^2 - 70k^2 - 40k^2 + 28k^2 = 54$ $\Rightarrow k = \ldots$	**M1**	Substitutes their point into $g(x, y, z) = 0$, obtaining an equation in one unknown, and solves it. Or substitutes their point into the plane, solves, and verifies $g = 0$ there; that route earns the remaining marks equivalently.
	$54k^2 = 54 \Rightarrow k = \pm 1$	**A1**	Both values, or the equivalent for their parameter.
	$k = 1$: $(7, 5, -2)$	**A1**	Selected as the only point of contact, by the constant 54 or otherwise; A0 if $(-7, -5, 2)$ is also given.
	$4(\text{“}7\text{”}) + 4(\text{“}5\text{”}) - 3(\text{“}-2\text{”}) = \ldots$	**M1**	Substitutes their point into $4x + 4y - 3z$, or into a tangent plane formed with their normal, to find its constant.
	$4x + 4y - 3z = 54$ is the tangent plane at $(7, 5, -2)$	**A1 AG**	Correct substituted working leading without error to 54, with a conclusion that the plane is a tangent plane to S.
		[8]	
Total for Question 10		**19**	