OPTIONAL PURE / SURFACES & PARTIAL DIFFERENTIATION

Partial Derivatives, Sections and Contours

Mark scheme

15 questions · 141 marks

EXAM BOARDS OCR A Y545 · OCR MEI Extra Pure Y435

Questions

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	Total	**141**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last accuracy mark of the working that reaches a printed result: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

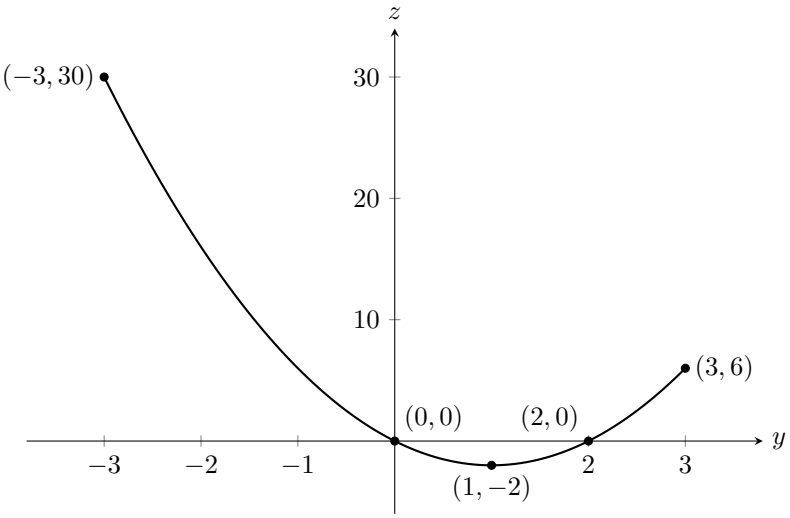
- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

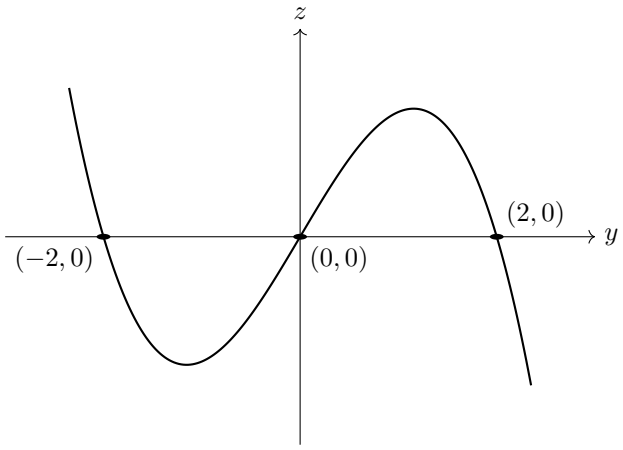
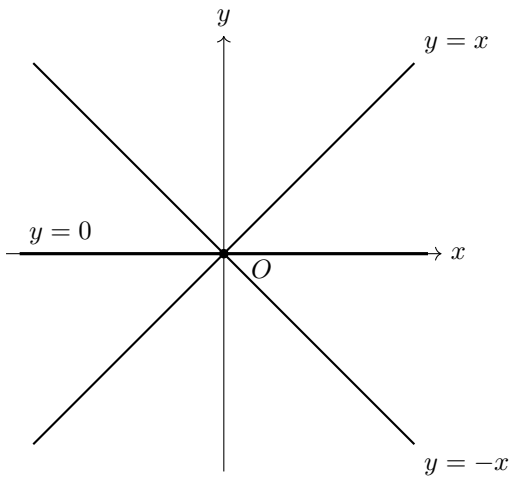
Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)(i)	$z = 2y^2 - 4y$	**B1**	oe, such as $z = 2(y-1)^2 - 2$.
		[1]	
1(a)(ii)			
	Shape	**B1**	U-shaped parabola, drawn only for $-3 \leq y \leq 3$, its left end higher than its right.
	Labelled point	**B1**	Vertex $(1, -2)$ and points $(0, 0)$ and $(2, 0)$, all labelled. Condone the intercepts shown as 0 and 2 on the axis.
		[2]	
1(b)	$z = 2 \times 1^2 - 4 \times 1 = -2$ $xy^2 - 4y =$ "-2"	**M1**	Finds z at $x = 2$, $y = 1$ and sets $xy^2 - 4y$ equal to their value. Condone sign slips.
	$x = \frac{4y-2}{y^2}$	**A1**	oe, such as $x = \frac{4}{y} - \frac{2}{y^2}$, with x the subject.
		[2]	
1(c)	$\frac{\partial z}{\partial x} = y^2$	**B1**	
	$\frac{\partial z}{\partial y} = 2xy - 4$	**B1**	
	$\frac{\partial z}{\partial x} = 1, \quad \frac{\partial z}{\partial y} = 0$	**B1ft**	Both values, ft their partial derivatives evaluated at $x = 2$, $y = 1$.
		[3]	
	Total for Question 1	**8**	

Question	Scheme	Marks	Notes
2(a)			
	Shape	**B1**	Negative cubic, z positive for large negative y and negative for large positive y, with a minimum at negative y and a maximum at positive y.
	Intercepts	**B1**	-2 and 2 marked on the y-axis, as values or as coordinates.
	Labelled point	**B1**	Curve through the origin, labelled $(0,0)$, O or 0.
		[3]	
2(b)			
	Shape	**B1**	Three straight lines through the origin, one of them along the x-axis.
	Labelled point	**B1**	$y = x$ and $y = -x$, each against its correct line, and $y = 0$ against the x-axis where it is drawn as part of the section.
		[2]	
2(c)	$\frac{\partial z}{\partial x} = 2xy$	**B1**	

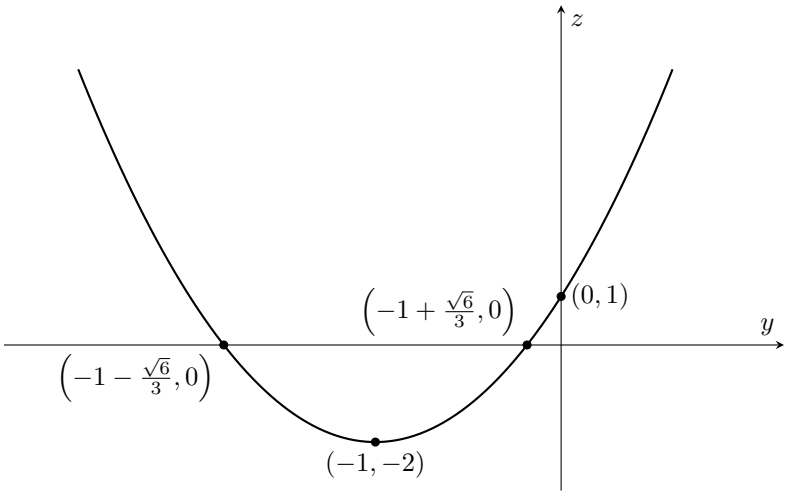
Question 2 continued

Question	Scheme	Marks	Notes
	$\frac{\partial z}{\partial y} = x^2 - 3y^2$	**B1**	
	$x = 2,\ y = 2 \Rightarrow \frac{\partial z}{\partial y} = 2^2 - 3 \times 2^2 = -8$	**B1ft**	ft their $\frac{\partial z}{\partial y}$, a function of x and y, evaluated at $x = 2$, $y = 2$.
		[3]	
	Total for Question 2	**8**	

Question	Scheme	Marks	Notes
3(a)	$f_x = 2e^{2x}\cos 2y + 2y$	**B1**	
	$f_y = -2e^{2x}\sin 2y + 2x$	**B1**	
		[2]	
3(b)	$f_{xx} = 4e^{2x}\cos 2y$ $f_{yy} = -4e^{2x}\cos 2y$	**M1 A1**	**M1** Differentiates their f_x or f_y from (a) again, with at least one term of the correct form. Condone sign and coefficient slips. **A1** Both correct.
	$f_{xy} = f_{yx} = -4e^{2x}\sin 2y + 2$	**A1**	Both correct.
	$f_{xx} + f_{yy} = 4e^{2x}\cos 2y - 4e^{2x}\cos 2y$ $= 0$	**A1 AG**	Correct f_{xx} and f_{yy} added, reaching 0 without error. A brief conclusion, such as "as required", is enough.
		[4]	
3(c)	$x = 0 \Rightarrow z = \cos 2y$	**B1**	
	Stretch parallel to the y-axis, scale factor $\frac{1}{2}$	**B1**	Needs both the direction and the scale factor. Condone "in the y-direction".
		[2]	
	Total for Question 3	**8**	

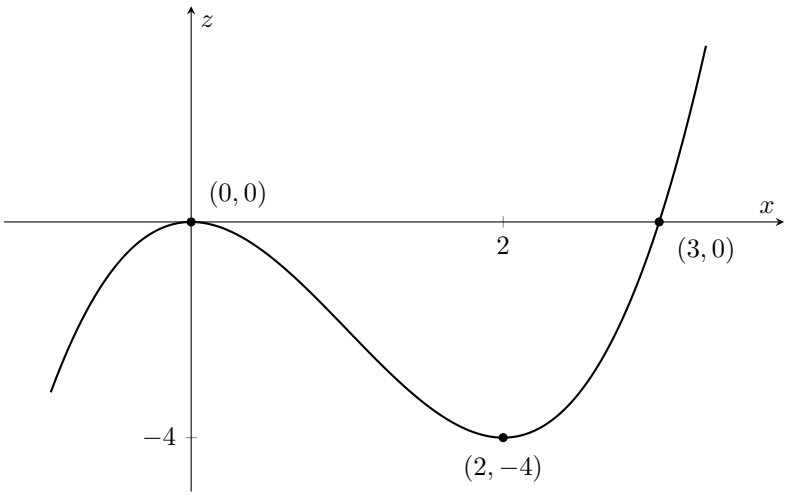
Question	Scheme	Marks	Notes
4(a)	$\frac{\partial z}{\partial x} = 2y - 6$	**B1**	
	$\frac{\partial z}{\partial y} = 2x + 8$	**B1**	
		[2]	
4(b)	$z = 2ay - 6a + 8y = (2a + 8)y - 6a$ $\Rightarrow 2a + 8 = -4, \quad b = -6a$	**M1**	Substitutes $x = a$ and equates the coefficient of y to -4, or the constant term to b. Condone sign slips.
	$a = -6$	**A1**	
	$b = 36$	**A1ft**	ft $-6\times$ their a.
		[3]	
4(c)	$f(1,1) = 2 - 6 + 8 = 4$ $2xy - 6x + 8y =$ "4" $\Rightarrow y = \ldots$	**M1**	Evaluates $f(1,1)$ and sets $f(x,y)$ equal to their value. Condone one arithmetic slip.
	$y = \frac{3x+2}{x+4}$	**A1**	oe, such as $y = \frac{6x+4}{2x+8}$. Must be y in terms of x.
		[2]	
4(d)	$x = -4$	**B1**	
	$y = 3$	**B1**	
		[2]	
	Total for Question 4	**9**	

Question	Scheme	Marks	Notes
5(a)	$\frac{\partial g}{\partial x} = 2(x-z)^3 + 3(2x+y)(x-z)^2$	**M1**	Differentiates with respect to x, holding y and z constant, by the product rule with two terms or by expanding first. Condone coefficient slips.
	$= (x-z)^2\left[2(x-z)+3(2x+y)\right]$ $= (x-z)^2(8x+3y-2z)$	**A1 AG**	Correct derivative leading without error to the printed result; combined steps accepted.
		[2]	
5(b)	$\frac{\partial g}{\partial y} = (x-z)^3$	**B1**	
	$\frac{\partial g}{\partial z} = -3(2x+y)(x-z)^2$	**B1**	
	$(x-z)^2(8x+3y-2z)+k(x-z)^3-3(2x+y)(x-z)^2$ $= (x-z)^2\left[2(x-z)+k(x-z)\right]$	**M1**	Substitutes their three partial derivatives and simplifies towards an equation in k, by a common factor, expanding or comparing coefficients. Condone sign slips.
	$(k+2)(x-z)^3 = 0 \Rightarrow k = -2$	**A1**	cso.
		[4]	
	Total for Question 5	**6**	

Question	Scheme	Marks	Notes
6(a)	$\frac{\partial z}{\partial x} = 2x + 2y - 6$	**B1**	
	$\frac{\partial z}{\partial y} = 2x + 6y$	**B1**	
		[2]	
6(b)	$2x + 2c - 6 = 0 \Rightarrow x = \ldots$	**M1**	Sets their $\frac{\partial z}{\partial x}$ with $y = c$ equal to zero, or completes the square in x, or uses the vertex of the quadratic in x.
	$a = 3 - c$	**A1**	oe, such as $x = 3 - c$ or $c = 3 - a$.
	$z = \text{“}3 - c\text{”}^2 + 2c\text{“}3 - c\text{”} + 3c^2 - 6\text{“}3 - c\text{”} + 10$ $= 21$	**M1**	Substitutes their a, linear in c, and $y = c$ into S, or eliminates c instead, and sets $z = 21$. Need not be simplified.
	$2c^2 + 6c + 1 = 21 \Rightarrow c = \ldots$	**dM1**	Solves their three-term quadratic by factorising, the formula or completing the square; a calculator is allowed.
	$a = 1,\ c = 2$ or $a = 8,\ c = -5$	**A1**	Both pairs, each a with its c, and no others.
		[5]	
6(c)			
	Shape	**B1**	U-shaped parabola, its vertex below the y-axis and to the left of the z-axis.
	Turning points	**B1**	Minimum point $(-1, -2)$, stated or labelled on the sketch.

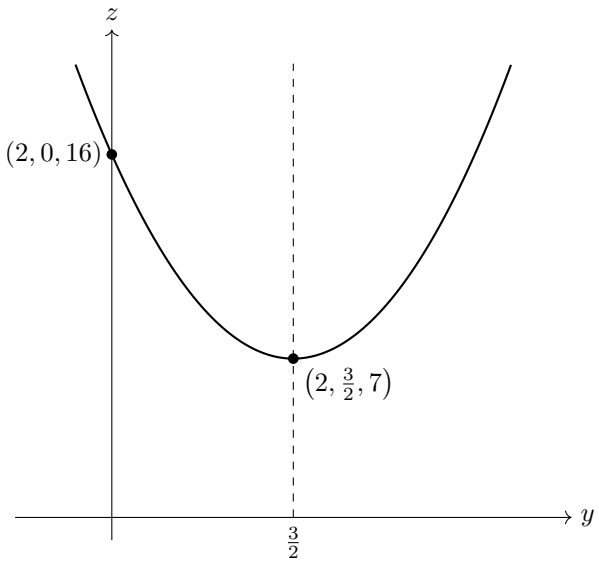
Question 6 continued

Question	Scheme	Marks	Notes
	Intercepts	**B1**	$(0, 1)$ and both y-intercepts, $-1 \pm \frac{\sqrt{6}}{3}$ oe or awrt -1.82 and -0.184, stated or labelled.
		[3]	
	Total for Question 6	**10**	

Question	Scheme	Marks	Notes
7(a)	$\frac{\partial z}{\partial x} = 3x^2 + 3y^2 - 6x$ $\frac{\partial z}{\partial y} = 6xy - 6y$	**M1 A1**	**M1** Differentiates partially in both x and y, holding the other variable constant, with the power of at least one term decreased by 1 in each. **A1** Both correct.
	$3x^2 + 3y^2 - 6x = 6xy - 6y$ $\Rightarrow x^2 - 2xy + y^2 - 2x + 2y = 0$	**M1**	Sets their two partial derivatives equal and collects all terms on one side. Condone sign slips; need not be simplified.
	$(x-y)^2 - 2(x-y) = 0$ $\Rightarrow (x-y)(x-y-2) = 0$	**dM1**	Factorises their equation as a quadratic in $x-y$, or solves it as a quadratic in y or in x by any method.
	$y = x$ or $y = x - 2$	**A1 AG**	Correct equation from equal derivatives, factorised or solved without error, leading to both $y = x$ and $y = x - 2$.
		[5]	
7(b)	$x = 1$: $z = 1 + 3y^2 - 3 - 3y^2 = -2$, a straight line	**B1**	Substitutes $x = 1$, shows the y^2 terms cancel to $z = -2$, and states that the section is a straight line.
	In the plane $x = 1$, parallel to the y-axis, through $(1, 0, -2)$	**E1**	States the line is parallel to the y-axis and at $z = -2$, or through $(1, 0, -2)$, oe.
		[2]	
7(c)			
	Shape	**B1**	Positive cubic touching the x-axis at the origin and crossing it once to the right.
	Intercepts	**B1**	$(0, 0)$ and $(3, 0)$, as coordinates or values marked on the axis.

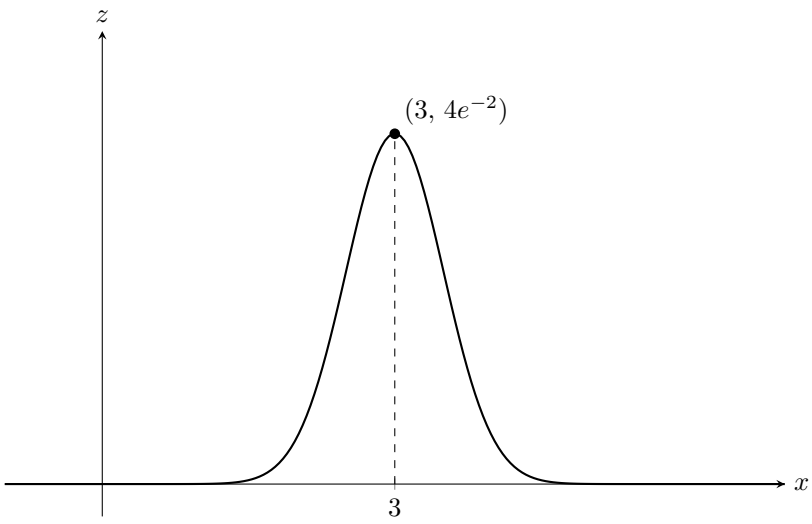
Question 7 continued

Question	Scheme	Marks	Notes
	Turning points	**B1**	Maximum at $(0, 0)$ and minimum at $(2, -4)$, as coordinates or values marked on the axes.
		[3]	
	Total for Question 7	**10**	

Question	Scheme	Marks	Notes
8(a)	$z = 4y^2 - 12y + 16$	**B1**	oe, in the form $z = g(y)$.
	$z = 4\left(y - \frac{3}{2}\right)^2 + \ldots$ or $\frac{\mathrm{d}z}{\mathrm{d}y} = 8y - 12 = 0 \Rightarrow y = \ldots$	**M1**	Finds the vertex of their quadratic in y by completing the square, differentiating and setting to zero, or $y = -\frac{b}{2a}$. Condone slips.
	$\left(2, \frac{3}{2}, 7\right)$	**A1**	All three coordinates, oe as $x = 2$, $y = \frac{3}{2}$, $z = 7$.
			
	Shape	**B1**	U-shaped parabola with its minimum right of the z-axis, meeting the z-axis at $z = 16$; $(2, 0, 16)$ stated or labelled, oe as y and z values.
	$\mathbf{r} = \begin{pmatrix} 2 \\ \frac{3}{2} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	**B1**	Any point $\left(2, \frac{3}{2}, t\right)$ and any nonzero multiple of $\mathbf{k}$, in vector form. Not for $x = 2$, $y = \frac{3}{2}$ alone.
		[5]	
8(b)	$\frac{\partial z}{\partial x} = 6x^2 - 6xy$	**B1**	
	$\frac{\partial z}{\partial y} = -3x^2 + 8y$	**B1**	
		[2]	
8(c)	$-3k^2 + 8y = 0 \Rightarrow y = \ldots$	**M1**	Sets their $\frac{\partial z}{\partial y}$, of the form $ax^2 + by$, equal to zero with $x = k$, leading to $y = \ldots$.
	$y = \frac{3k^2}{8}$	**A1**	

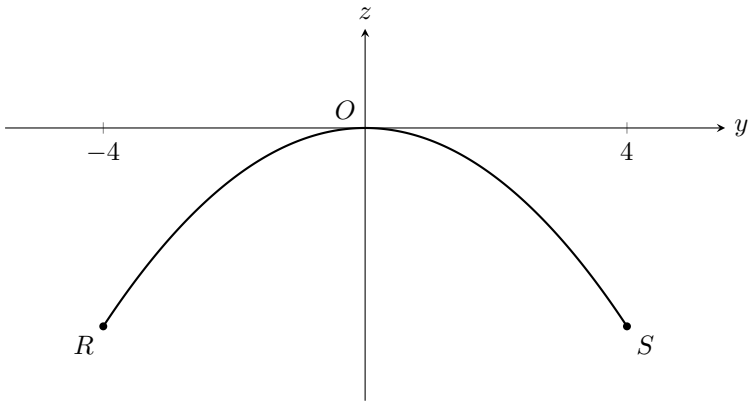
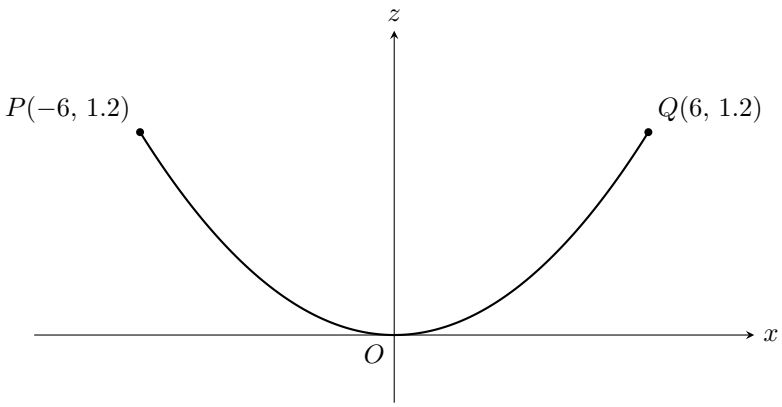
Question 8 continued

Question	Scheme	Marks	Notes
	$z = 2k^3 - 3k^2\left(\frac{3k^2}{8}\right) + 4\left(\frac{3k^2}{8}\right)^2$ $= 2k^3 - \frac{9k^4}{16}$ $\left(k, \frac{3k^2}{8}, 2k^3 - \frac{9k^4}{16}\right)$	**A1**	All three coordinates, with z in any equivalent form.
		[3]	
	Total for Question 8	**10**	

Question	Scheme	Marks	Notes
9(a)	$\frac{\partial f}{\partial x} = -y^3(2x-6)e^{-y(x^2-6x+10)}$	**M1 A1**	**M1** Differentiates, treating y as a constant, and reaches the form $y^3(px+q)e^{-y(x^2-6x+10)}$. Condone sign and coefficient slips. **A1** Correct, oe; need not be simplified.
	$\frac{\partial f}{\partial y} = 2ye^{-y(x^2-6x+10)}$ $\quad - y^2(x^2-6x+10)e^{-y(x^2-6x+10)}$	**M1 A1**	**M1** Differentiates, treating x as a constant, with the product rule, reaching terms in $ye^{-y(x^2-6x+10)}$ and $y^2(x^2-6x+10)e^{-y(x^2-6x+10)}$. Condone sign and coefficient slips. **A1** Correct, oe; need not be simplified.
		[4]	
9(b)	$z = 4e^{-2(x^2-6x+10)} = 4e^{-2}e^{-2(x-3)^2}$ Stretch parallel to the x-axis, scale factor $\frac{1}{\sqrt{2}}$	**B1**	Accept scale factor $\frac{\sqrt{2}}{2}$ or awrt 0.707.
	Then translation of 3 in the positive x-direction	**B1**	Translation by 3 after the x-stretch, or by $3\sqrt{2}$ before it; a vector form is accepted.
	Stretch parallel to the z-axis, scale factor $4e^{-2}$	**B1**	At any point in the sequence. Accept $\frac{4}{e^2}$, awrt 0.541, or two z-stretches with product $4e^{-2}$.
		[3]	
9(c)			
	Shape	**B1**	Single hump symmetric about $x = 3$, above the x-axis, with $z \to 0$ at both ends.

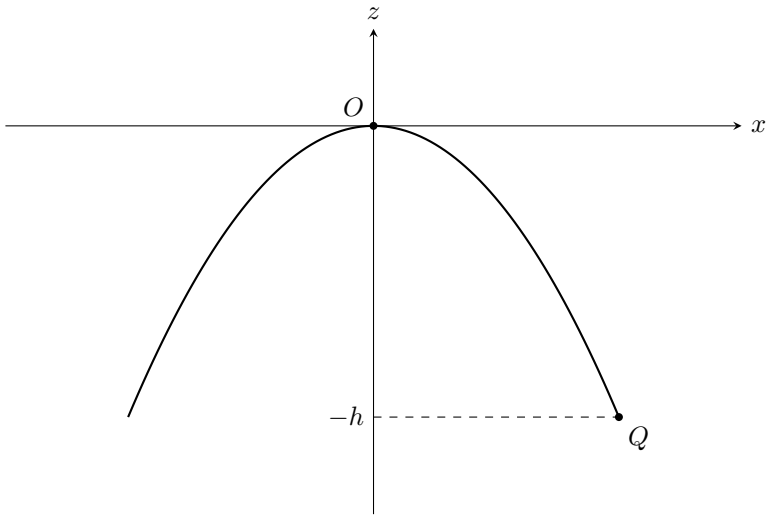
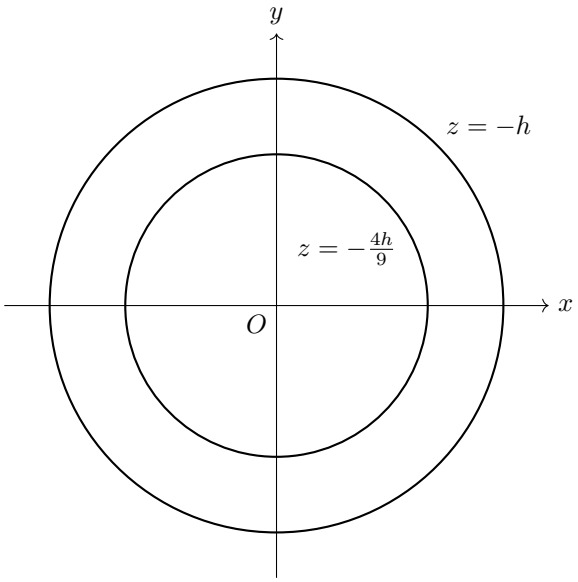
Question 9 continued

Question	Scheme	Marks	Notes
	Turning points	**B1**	Exact highest point $(3, 4e^{-2})$ oe, stated or labelled; accept $(3, 2, 4e^{-2})$.
		[2]	
	Total for Question 9	**9**	

Question	Scheme	Marks	Notes
10(a)			
	Shape	**B1**	∩-shaped parabola in y and z, symmetric about the z-axis, with its maximum at O. R and S need not be marked; accept the full curve.
			
	Shape	**B1**	∪-shaped parabola in x and z, symmetric about the z-axis, with its minimum at O. P and Q need not be marked; accept the full curve.
		[2]	
10(b)	$f(x, y) = ax^2 - by^2$, $a, b > 0$	**B1**	Form $ax^2 - by^2$ with $a, b > 0$, oe such as $ax^2 + by^2$ with $b < 0$ found later. Implied by combining two section equations into f.
	$36a = 1.2 \Rightarrow a = \ldots$	**M1**	Substitutes $x = \pm 6$, $y = 0$, $z = 1.2$ into their model and solves for its constant.
	$a = \frac{1}{30}$	**A1**	oe, such as $a = 0.0\dot{3}$ or $\frac{1.2}{36}$.
	$\frac{\partial f}{\partial y} = -2by$ $-2b \times 4 = -0.8 \Rightarrow b = \ldots$	**M1**	Differentiates their f, or the section $z = -by^2$, with respect to y and sets the gradient at $y = 4$ equal to -0.8. Condone sign slips.

Question 10 continued

Question	Scheme	Marks	Notes
	$b = \frac{1}{10}$ $f(x,y) = \frac{x^2}{30} - \frac{y^2}{10}$	**A1**	Correct function, oe such as $z = \frac{x^2}{30} - 0.1y^2$.
		[5]	
10(c)	$\frac{x^2}{30} - \frac{y^2}{10} = 0 \Rightarrow x^2 = 3y^2$ $\Rightarrow y = \ldots$	**M1**	Sets their f equal to 0 and solves for y, or factorises into two linear factors.
	$y = \frac{x}{\sqrt{3}}$ and $y = -\frac{x}{\sqrt{3}}$ $(z = 0)$	**A1**	Both lines, oe such as $x = \pm\sqrt{3}\,y$. $z = 0$ need not be stated.
	$\arctan \frac{1}{\sqrt{3}} = \frac{\pi}{6}$ $\Rightarrow$ angle $= 2 \times \frac{\pi}{6} = \frac{\pi}{3}$ $(= 60°)$	**A1ft**	ft the acute angle between their two lines through O; the correct angle is $\frac{\pi}{3}$, $60°$ or awrt 1.05.
		[3]	
	Total for Question 10	**10**	

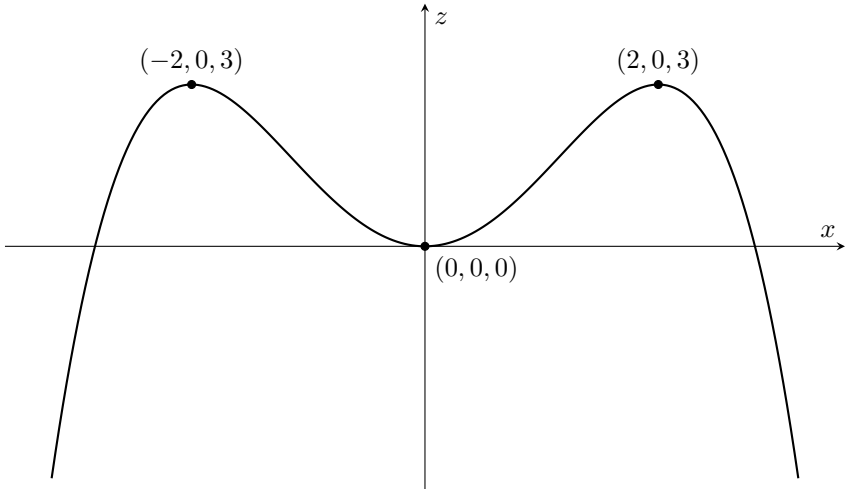
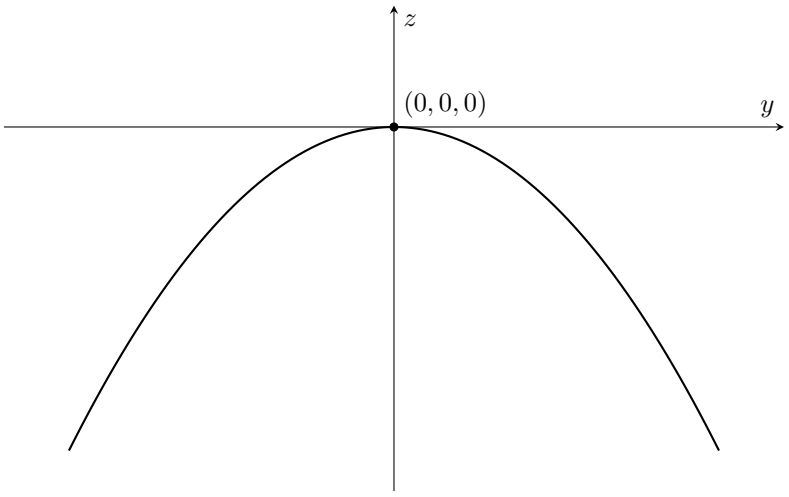
Question	Scheme	Marks	Notes
11(a)(i)			
	Shape	**B1**	∩-shaped parabola, symmetric about the z-axis, with vertex at O and Q marked on the curve below the positive x-axis.
		[1]	
11(a)(ii)	$z = -kx^2$, $k > 0$	**B1**	oe, such as $z = ax^2$ with $a < 0$, with any letter for the constant. Needs the constant's sign shown or stated.
		[1]	
11(b)(i)			
	Centre and radius	**B1**	Two circles centred on O, each identified by its plane, the $z = -\frac{4h}{9}$ contour the smaller.
		[1]	

Question 11 continued

Question	Scheme	Marks	Notes
11(b)(ii)	$z = -k\left(x^2 + y^2\right)$ $x^2 + y^2 = \frac{h}{k}$, $x^2 + y^2 = \frac{4h}{9k}$	**M1**	Replaces x^2 by $x^2 + y^2$ in their section equation, or finds each radius from it, and uses $z = -h$ and $z = -\frac{4h}{9}$. Condone sign slips.
	Radii $\sqrt{\frac{h}{k}}$ and $\sqrt{\frac{4h}{9k}} = \frac{2}{3}\sqrt{\frac{h}{k}}$	**A1 AG**	Both contour equations correct, leading without error to both radii and the ratio $\frac{2}{3}$, with a conclusion.
		[2]	
11(c)	$y = 0$: $z = ax^2$, $x = 0$: $z = by^2 \Rightarrow a = b$	**B1**	$a = b$ with a reason, such as the sections in $x = 0$ and $y = 0$ being the same curve, or symmetry about the z-axis.
	$x = y = t$: $z = (2a + c)t^2$ $z = a\left(\sqrt{2}\,t\right)^2 = 2at^2 \Rightarrow c = \ldots$	**M1**	Compares z at two points equidistant from the z-axis, one off the axes, such as (t, t) and $(\sqrt{2}\,t, 0)$, or equates coefficients with $z = a\left(x^2 + y^2\right)$.
	$c = 0$	**A1**	
	$a < 0$, $b < 0$: roof curves down from its highest point O	**E1**	States a and b negative with a reason, such as O the highest point, the roof curving downwards, or $a = -k$ from (a)(ii).
		[4]	
11(d)	$-\frac{1}{2} = a\left(2^2\right) \Rightarrow a = \ldots$	**M1**	Substitutes distance 2 from the axis and $z = -\frac{1}{2}$ into $z = a\left(x^2 + y^2\right)$ or the section $z = ax^2$, and solves. Condone a sign slip.
	$a = b = -\frac{1}{8}$, $c = 0$	**A1**	All three, oe such as -0.125.
	$-h = -\frac{1}{8} \times 4^2 \Rightarrow h = 2$	**A1**	
		[3]	
11(e)	$\frac{\partial z}{\partial x} = -\frac{x}{4}$, $x = 4$: $\frac{\partial z}{\partial x} = \ldots$	**M1**	Differentiates their $z = a\left(x^2 + y^2\right)$ partially in x, reducing the power of x by 1, and substitutes their x-coordinate of Q.
	$\frac{\partial z}{\partial x} = -1$	**A1**	

Question 11 continued

Question	Scheme	Marks	Notes
	$\tan\theta = 1 \Rightarrow \theta = 45°$	**A1ft**	ft arctan of the modulus of their $\frac{\partial z}{\partial x}$. Accept $\frac{\pi}{4}$ or awrt 0.785 radians.
		[3]	
	Total for Question 11	**15**	

Question	Scheme	Marks	Notes
12(a)			
	Shape	**B1**	Continuous M-shape symmetric about the z-axis, with two equal maxima either side of a minimum on the z-axis.
	Labelled point	**B1**	Both maxima labelled $(2,0,3)$ and $(-2,0,3)$, or $x=\pm 2$ and $z=3$ marked on the axes.
	Intercepts	**B1**	Minimum at the origin, touching the x-axis from above, labelled $(0,0,0)$ or O.
	End behaviour	**B1**	Curve crosses $z=0$ beyond each maximum, with $z<0$ for all larger $\lvert x\rvert$.
		[4]	
12(b)			
	Shape	**B1**	Inverted U-shape symmetric about the z-axis, falling as $\lvert y\rvert$ increases.

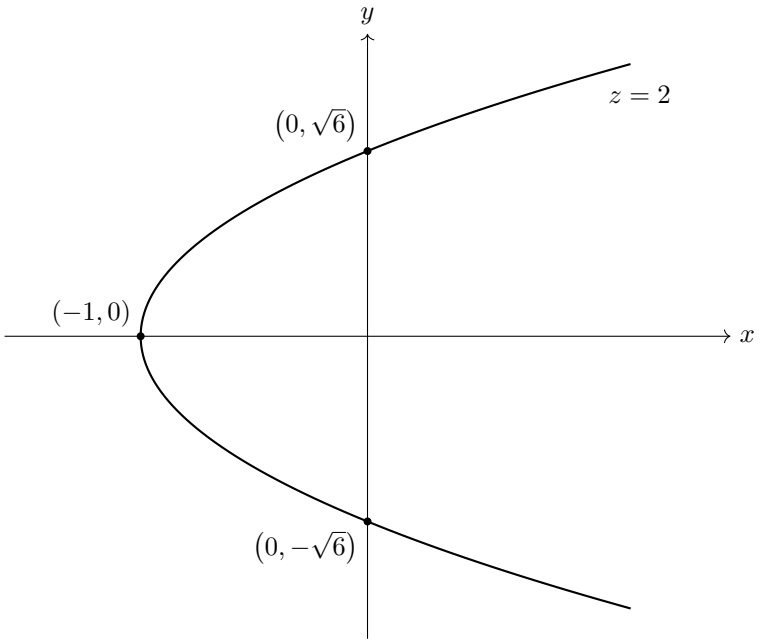
Question 12 continued

Question	Scheme	Marks	Notes
	Labelled point	**B1**	Highest point at the origin, labelled $(0,0,0)$ or O, with no part of the curve above $z=0$.
	$0 < a \leq 3$: contour $z=a$ misses $x=0$ $z=0$ meets $x=0$ only at O $\Rightarrow f(0,y) < 0$ for $y \neq 0$, highest point O	**E1**	States that no contour $z=a$ with $a>0$ meets $x=0$, and that $z=0$ meets it only at the origin. Both needed.
		[3]	
	Total for Question 12	**7**	

Question	Scheme	Marks	Notes
13(a)	$f_x = 3x^2e^{y/x} - \frac{y}{x^2}\left(x^3+y^3\right)e^{y/x}$	**M1 A1**	**M1** Uses the product rule, with $e^{y/x}$ differentiated to a multiple of $\frac{y}{x^2}e^{y/x}$ or $\frac{1}{x}e^{y/x}$, for f_x or f_y. Condone sign and coefficient slips.
	$f_y = 3y^2e^{y/x} + \frac{1}{x}\left(x^3+y^3\right)e^{y/x}$	**A1**	
	$xf_x + yf_y = 3x^3e^{y/x} - \frac{y}{x}\left(x^3+y^3\right)e^{y/x}$ $+ 3y^3e^{y/x} + \frac{y}{x}\left(x^3+y^3\right)e^{y/x}$	**dM1**	Multiplies their f_x by x and their f_y by y, adds, and collects terms towards a multiple of f.
	$= 3\left(x^3+y^3\right)e^{y/x} = 3f$ $n = 3$	**A1**	cso. Either $xf_x + yf_y = 3f$ or $n = 3$ earns the mark.
		[5]	
13(b)	$f_x + xf_{xx} + yf_{yx} = 3f_x$	**M1**	Differentiates their result of (a) partially with respect to x or y, using the product rule on xf_x or yf_y. Condone one slip.
	$xf_{xx} + yf_{xy} = 2f_x$	**A1**	oe, with f_{yx} accepted for f_{xy}.
	$xf_{xy} + f_y + yf_{yy} = 3f_y$ $\Rightarrow xf_{xy} + yf_{yy} = 2f_y$	**A1**	oe, with f_{yx} accepted for f_{xy}.
	$x^2f_{xx} + 2xyf_{xy} + y^2f_{yy} = 2\left(xf_x + yf_y\right)$ $= 2 \times$ "3" f	**dM1**	Multiplies their two equations by x and by y, adds, and replaces $xf_x + yf_y$ by their nf from (a).
	$k = 6$	**A1**	cso.
		[5]	
	Total for Question 13	**10**	

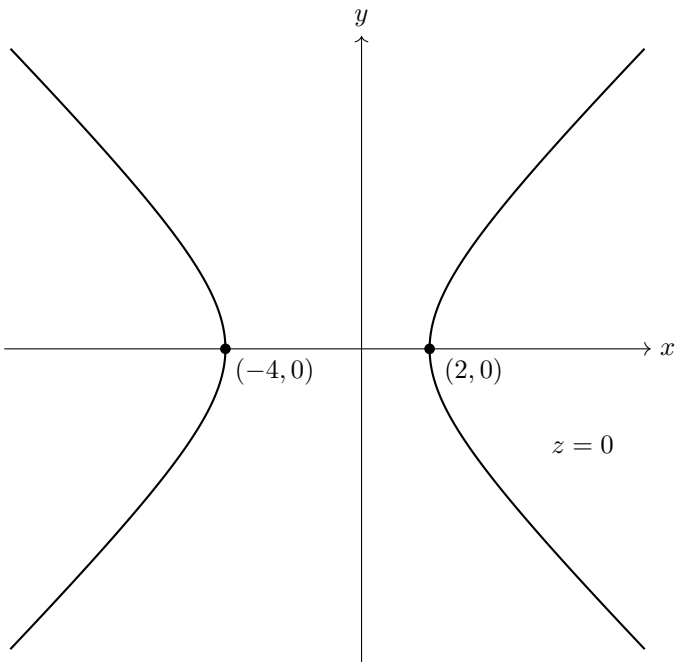
Question	Scheme	Marks	Notes
14(a)	$3z^2\frac{\partial z}{\partial x}+3z+3x\frac{\partial z}{\partial x}=0$	**M1**	Differentiates implicitly in x, with the chain rule on z^3 and the product rule on $3xz$. Condone sign slips.
	$\frac{\partial z}{\partial x}=-\frac{z}{z^2+x}$	**A1**	oe
	$3z^2\frac{\partial z}{\partial y}+3x\frac{\partial z}{\partial y}=2y \Rightarrow \frac{\partial z}{\partial y}=\ldots$	**M1**	Differentiates implicitly in y, with the chain rule on z^3 and x treated as constant. Condone sign slips.
	$\frac{\partial z}{\partial y}=\frac{2y}{3(z^2+x)}$ $\text{LHS}=2y\left(-\frac{z}{z^2+x}\right)+3z\times\frac{2y}{3(z^2+x)}$ $=0$	**A1 AG**	Correct $\frac{\partial z}{\partial y}$, and both derivatives substituted into the LHS, reaching 0 without error.
		[4]	
14(b)	$\frac{\partial^2 z}{\partial x\,\partial y}=-\frac{2y\left(2z\frac{\partial z}{\partial x}+1\right)}{3(z^2+x)^2}$	**M1**	Differentiates their $\frac{\partial z}{\partial y}$ in x, or their $\frac{\partial z}{\partial x}$ in y, using the chain rule on z^2. Condone sign and coefficient slips.
	$=-\frac{2y}{3(z^2+x)^2}\left(1-\frac{2z^2}{z^2+x}\right)$	**dM1**	Substitutes their $\frac{\partial z}{\partial x}$, or their $\frac{\partial z}{\partial y}$, from (a) as their route needs, and attempts a single fraction.
	$=-\frac{2y(x-z^2)}{3(z^2+x)^3}$ $=\frac{2y(z^2-x)}{3(z^2+x)^3}$	**A1 AG**	Printed result reached from a correct derivative without error; collected steps accepted.
		[3]	
14(c)	$2^3+3x\times 2=y^2+2 \Rightarrow y^2=6x+6$	**B1**	oe, such as $y^2=6(x+1)$.

Question 14 continued

Question	Scheme	Marks	Notes
			
	Shape	**B1**	Parabola symmetric about the x-axis, opening in the positive x-direction, vertex on the negative x-axis.
	Intercepts	**B1**	$(-1, 0)$ and $(0, \pm\sqrt{6})$ stated or marked, or awrt ± 2.45.
		[3]	
	Total for Question 14	**10**	

Question	Scheme	Marks	Notes
15(a)	$2xe^z + x^2e^z\frac{\partial z}{\partial x} + 2 = 8\frac{\partial z}{\partial x}$	**M1**	Differentiates implicitly with respect to x, the product rule on x^2e^z giving a $\frac{\partial z}{\partial x}$ term, or uses the ratio of partial derivatives. Condone sign slips.
	$\frac{\partial z}{\partial x} = -\frac{5}{4}$	**A1**	
	$x^2e^z\frac{\partial z}{\partial y} - 8y = 8\frac{\partial z}{\partial y}$	**M1**	Differentiates implicitly with respect to y, with $\frac{\partial z}{\partial y}$ terms from both x^2e^z and $8z$, or uses the ratio of partial derivatives. Condone sign slips.
	$\frac{\partial z}{\partial y} = 2$	**A1**	
		[4]	
15(b)	$2e^z + 4xe^z\frac{\partial z}{\partial x} + x^2e^z\left(\frac{\partial z}{\partial x}\right)^2 + x^2e^z\frac{\partial^2 z}{\partial x^2} = 8\frac{\partial^2 z}{\partial x^2}$	**M1 A1**	**M1** Differentiates their x-equation from (a) again with respect to x, with the product rule on each e^z term, or the quotient rule on their $\frac{\partial z}{\partial x}$. Condone coefficient slips. **A1** Correct equation in any arrangement, or a correct unsimplified expression for $\frac{\partial^2 z}{\partial x^2}$.
	$2 + 16\left(\text{“}-\frac{5}{4}\text{”}\right) + 16\left(\text{“}-\frac{5}{4}\text{”}\right)^2 + 16\frac{\partial^2 z}{\partial x^2} = 8\frac{\partial^2 z}{\partial x^2}$ $\Rightarrow \frac{\partial^2 z}{\partial x^2} = \ldots$	**dM1**	Substitutes $x = 4$, $z = 0$ and their $\frac{\partial z}{\partial x}$ from (a), and solves for $\frac{\partial^2 z}{\partial x^2}$.
	$\frac{\partial^2 z}{\partial x^2} = -\frac{7}{8}$	**A1**	
		[4]	
15(c)	$x^2 + 2x - 4y^2 = 8$ $\Rightarrow (x+1)^2 - 1 - 4y^2 = 8$	**M1**	Sets $z = 0$ and completes the square in x, or expands the printed equation to compare. Condone sign slips.
	$(x+1)^2 - 4y^2 = 9$	**A1 AG**	Correct equation $x^2 + 2x - 4y^2 = 8$ leading without error to the printed result, or the expanded printed form shown identical to it.

Question 15 continued

Question	Scheme	Marks	Notes
			
	Shape	**B1**	Two-branch curve, symmetric about the x-axis and opening left and right, with $(2,0)$ and $(-4,0)$ stated or marked.
		[3]	
	Total for Question 15	**11**	