OPTIONAL PURE / RECURRENCE RELATIONS

Modelling with Recurrence Relations

Mark scheme 10 questions · 120 marks

EXAM BOARDS Edexcel FP2 9FM0/4A · OCR A Y545 · OCR MEI Extra Pure Y435

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	Total	**120**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last accuracy mark of the working that reaches a printed result: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	$P_5 = \frac{3\times405.03}{6} + \frac{4\times4}{405.03}$	**M1**	Substitutes $n = 4$ and $P_4 = 405.03$ into both terms of the recurrence relation. Condone a slip in one term.
	$P_5 = 202.55$	**A1**	awrt 202.55
	$P_6 = \frac{3P_5}{7} + \frac{4\times5}{P_5} = 86.91$	**A1ft**	ft their P_5 in $\frac{3P_5}{7} + \frac{20}{P_5}$, to 2 d.p.; awrt 86.91 from a correct P_5, rounded or unrounded.
		[3]	
1(b)(i)	Rises to about 900 000 on days 1 and 2 Falls rapidly to a minimum near 6250 on day 9 Rises slightly to about 7460 on day 10	**E1**	States the initial rise, the rapid fall to a minimum on day 9 and the slight rise after. Values need not be quoted; condone values in thousands.
		[1]	
1(b)(ii)	$P_8 = 11.81 \geq 10$, $P_9 = 6.25 < 10$ 9 days	**B1**	
		[1]	
1(c)(i)	$\frac{a\times400^k}{a\times100^k} = 4^k = \frac{40.13}{20.25} \Rightarrow k = \ldots$	**M1**	Uses P_{100} and P_{400} in an^k to eliminate a, by a ratio or by logarithms, and finds k. Condone arithmetic slips.
	$k = \frac{1}{2}$ $a\sqrt{100} = 20.25 \Rightarrow a = 2$	**A1**	$k = \frac{1}{2}$ with $a = 2$ or awrt 2.0
		[2]	
1(c)(ii)	$2\sqrt{n} = 10 \Rightarrow n = 25$ (days)	**B1ft**	ft their $f(n) = 10$ solved for n, given as a value or the next whole number of days.
		[1]	
1(c)(iii)	$P_{21} = \frac{3\times9.51}{22} + \frac{4\times20}{9.51} = \ldots$	**M1**	Uses the recurrence relation from $P_{20} = 9.51$ with the correct n to find at least P_{21}. Condone a slip in one term.
	$P_{21} = 9.71$, $P_{22} = 9.92$, $P_{23} = 10.11$ $P_{22} < 10 \leq P_{23} \Rightarrow 23$ days	**A1**	P_{22} awrt 9.92 and P_{23} awrt 10.1, concluding 23 days.
		[2]	

Question 1 continued

Question	Scheme	Marks	Notes
1(d)	Further treatment would start once $P_n \geq 10$, which the model does not include	**E1**	States one valid reason linked to large n, such as further treatment not modelled or unlimited growth being unrealistic. Not for "models are approximate" alone.
		[1]	
	Total for Question 1	**11**	

Question	Scheme	Marks	Notes
2(a)	$6000 = 2.5 \times 6000(1 - 6000k)$ $\Rightarrow k = \ldots$	**M1**	Substitutes 6000 for both P_n and P_{n+1} and solves for k. Condone sign slips.
	$k = 0.0001$	**A1**	oe, such as $\frac{1}{10\,000}$.
		[2]	
2(b)	$P_1 = 6250,\ P_2 \approx 6109.4,\ P_3 \approx 6192.3,\ \ldots$ $P_n \to 6162$	**B1**	Limit stated, awrt 6160.
	Oscillates about the limit, with decreasing amplitude	**E1**	States both that the values lie alternately above and below the limit and that the swings decrease, oe such as damped oscillation.
		[2]	
2(c)(i)	$P_{n+1} = 2.5P_n(1 - 0.0001P_n) - 1440$	**B1**	oe, such as $2.5P_n - 0.00025P_n^2 - 1440$.
		[1]	
2(c)(ii)	$P_1 = 4560,\ P_2 = 4761.6,\ P_3 \approx 4795.8,\ \ldots$ Falls to 4560, then rises without oscillating to 4800	**E1**	States a first-year fall, then a rise without oscillation, settling at awrt 4800. Condone 4560 not stated.
		[1]	
2(c)(iii)	$P = 2.5P(1 - 0.0001P) - 1440$	**M1**	Sets $P_{n+1} = P_n = P$ in their relation from (c)(i), of the form $2.5P(1 - 0.0001P) \pm c$, and rearranges to a three-term quadratic. Condone sign slips.
	$P^2 - 6000P + 5\,760\,000 = 0$	**A1**	Any multiple, oe.
	$(P - 1200)(P - 4800) = 0 \Rightarrow P = 1200,\ 4800$	**A1**	Both values. The algebraic method the question asks for is the equation in P, in place of iteration; the quadratic itself may be solved by calculator.
	Iterates from $P_0 = 6000$ tend to 4800, so 4800	**E1**	Chooses 4800, justified by the iterates from $P_0 = 6000$ tending to it, or another valid reason. Not for 4800 without a reason.
		[4]	
2(c)(iv)	$6000 = 2.5 \times 6000(1 - 6000q) - 1440$ $\Rightarrow q = \ldots$	**M1**	Substitutes 6000 for both P_n and P_{n+1} in the relation with -1440 and an unknown coefficient, and solves for it. Condone sign slips.

Question 2 continued

Question	Scheme	Marks	Notes
	$q = 0.000084$	**A1**	oe, such as 8.4×10^{-5}.
		[2]	
	Total for Question 2	**12**	

Question	Scheme	Marks	Notes
3(a)	$0.0008I_n^2 - 0.6I_n + 200 > I_n$ $\Rightarrow I_n^2 - 2000I_n + 250\,000 > 0 \Rightarrow I_n = \ldots$	**M1**	Sets $I_{n+1} > I_n$, or $=$, and solves the resulting three-term quadratic for its critical values by any means. Condone sign slips.
	$I_n = 1000 \pm 500\sqrt{3}$ $[\approx 133.97,\ 1866.03]$	**A1**	Both values, exact or awrt 134.0 and 1866.0.
	Rules out starts below the lower value: $0 < \alpha < 1000 - 500\sqrt{3}$ $\Rightarrow 1000 - 500\sqrt{3} < I_1 < 200 \Rightarrow I_2 < I_1$	**dM1**	Shows a start below the lower value makes the value fall in a later year, by bounding I_1 or iterating an example. An example alone earns this mark but not the A1.
	$\alpha > 1000 + 500\sqrt{3} \Rightarrow I_{n+1} > I_n > 1000 + 500\sqrt{3}$ Minimum sum £1867	**A1**	cso. 1867, with every start below the lower value ruled out and starts above the upper value shown to increase every year.
		[4]	
3(b)(i)	$I_{n+1} = 10\,\mathrm{INT}\left(\frac{0.0008I_n^2 - 0.6I_n + 200}{10}\right)$	**B1**	Any notation for rounding down to a multiple of 10, such as floor brackets or words. $I_0 = \alpha$ need not be restated.
		[1]	
3(b)(ii)	$I_1 = 0.0008 \times 1900^2 - 0.6 \times 1900 + 200$ $[= 1948]$ $I_2 = \ldots$ $[= 2066.9632]$, $\quad I_3 = \ldots$	**M1**	Substitutes 1900 into the original relation and iterates to a value for I_3. Condone arithmetic slips.
	$I_3 = 2377.6916\ldots$	**A1**	awrt 2377.69.
	Rounds down to a multiple of 10 each year: $1948 \Rightarrow I_1 = \ldots$ $0.0008I_1^2 - 0.6I_1 + 200 = \ldots \Rightarrow I_2 = \ldots$	**M1**	Uses the relation repeatedly from 1900, rounding each value down to a multiple of 10 before the next step, at least twice. Condone arithmetic slips.
	$I_1 = 1940, \quad I_2 = 2040$	**A1**	
	$2305.28 \Rightarrow I_3 = 2300$	**A1**	
	$2377.69\ldots - 2300 = 77.69$	**A1**	awrt £77.69.
		[6]	
	Total for Question 3	**11**	

Question	Scheme	Marks	Notes
4(a)	$u_{n+1} = (1-r)u_n + k$	**B1**	oe, such as $u_{n+1} = u_n - ru_n + k$.
	$A(1-r)^n$	**B1**	Any letter for the constant. May be seen in the general solution.
	$p = (1-r)p + k \Rightarrow p = \ldots$	**M1**	Substitutes a constant into their recurrence relation of the form $u_{n+1} = au_n + b$ and solves for it. Condone sign slips.
	$p = \frac{k}{r}$	**A1**	
	$u_n = A(1-r)^n + \text{“}\frac{k}{r}\text{”}$ $60\,000 = A + \text{“}\frac{k}{r}\text{”} \Rightarrow A = \ldots$	**M1**	Substitutes $n = 0$, $u_0 = 60\,000$ into their general solution, $A(1-r)^n$ plus their constant particular solution, and finds A.
	$u_n = \left(60\,000 - \frac{k}{r}\right)(1-r)^n + \frac{k}{r}$	**A1**	oe, in terms of n, r and k only.
		[6]	
4(b)	$30\,000(0.96)^n + 30\,000 < 32\,000$	**M1**	Substitutes $r = 0.04$ and $k = 1200$ into their u_n from (a) and sets it below 32 000. Condone an equation or $\leq$.
	$(0.96)^n < \frac{1}{15}$ $n \ln 0.96 < \ln \frac{1}{15} \Rightarrow n > \ldots$	**dM1**	Rearranges to isolate the power and takes logarithms of both sides, in any base, reaching $n = \ldots$ or $n > \ldots$. Condone an equation or a reversed sign.
	$n > 66.3 \Rightarrow m = 67$	**A1**	cao. Condone an equation in place of the inequality.
	$30\,000(0.96)^n > 0 \Rightarrow u_n > 30\,000$ for all n	**E1**	States that $30\,000(0.96)^n > 0$ for every n, oe, so $u_n > 30\,000$. Not for $u_n \to 30\,000$ alone.
		[4]	
	Total for Question 4	**10**	

Question	Scheme	Marks	Notes
5(a)(i)	$T_0 = 40\,000, \quad T_{k+1} = (1-r)T_k, \quad 0 \leq k \leq 7$	**B1**	Needs $T_0 = 40\,000$ and a correct range of k; oe such as $T_k = (1-r)T_{k-1}$, $1 \leq k \leq 8$.
	$T_1 = 40\,000(1-r),\ T_2 = 40\,000(1-r)^2,\ \ldots$	**M1**	Applies the recurrence at least twice from T_0, or identifies a geometric sequence with first term 40 000 and ratio $1-r$.
	$T_k = 40\,000(1-r)^k,\ 0 \leq k \leq 8$	**A1 AG**	Correct recurrence and a general pattern or geometric sequence leading without error to the printed result.
		[3]	
5(a)(ii)	$(1-r)^8 = \frac{5006}{50\,000} \Rightarrow 1-r = \ldots$	**M1**	Forms $(1-r)^8 = \frac{5006}{50\,000}$ oe and takes the eighth root, or evaluates $(1-r)^8$ at $r = 0.2495$ and $r = 0.2505$.
	$1-r = 0.10012^{\frac{1}{8}} = 0.750006\ldots$ $\Rightarrow r = 0.24999\ldots = 0.250$	**A1 AG**	cso, with r or $1-r$ seen to at least 4 s.f. before 0.250, or both bounds shown either side of 0.10012 with a conclusion.
		[2]	
5(b)(i)	$T_8 = 40\,000 \times 0.10012 = 4004.8$	**B1**	Or 4004.5 from $r = 0.25$; soi.
	"4004.8" $p \times \left(1 - \frac{1}{5} \times 0.25\right)^n$	**M1**	Multiplies their $T_8\,p$, or their T_8, by $1 - \frac{1}{5}r$ for each week. Condone a wrong number of weeks.
	$4004.8p \times 0.95^{18}$ $[\approx 1590.8p]$	**A1**	Correct expression over 18 weeks, oe.
	$1590.8p \geq 200 \Rightarrow p \geq 0.1257\ldots$ $p = 0.126$	**A1**	awrt 0.126. Condone an equation in place of the inequality.
		[4]	
5(b)(ii)	The weekly drop-out factor stays $\frac{1}{5}r$ for all 18 weeks after the course	**E1**	States one assumption the result relies on, such as the factor staying constant, similar courses' completion rate applying, or no tutors joining later.
		[1]	
5(c)	$200 \times 150 = 30\,000$	**B1**	
	$30\,000 < 40\,000$, so the course shrinks from one run to the next	**E1**	Compares their second-run total with 40 000 and concludes that 200 tutors do not sustain the course at its original size.

Question 5 continued

Question	Scheme	Marks	Notes
	$150N \geq 40\,000 \Rightarrow N \geq 266\frac{2}{3}$ $N = 267$	**B1**	cao, as a whole number of tutors.
		[3]	
	Total for Question 5	**13**	

Question	Scheme	Marks	Notes
6(a)	$A_{n+1} = 1.04\,(A_n - F)$	**B1**	oe, such as $A_{n+1} = 1.04A_n - 1.04F$.
		[1]	
6(b)	$(12\,000 - 26F) \times 1.04^0 + 26F = 12\,000 = A_1$	**B1**	Shows the formula gives 12 000 at $n = 1$. Not for "true for $n = 1$" alone.
	Assumes $A_k = (12\,000 - 26F)\,1.04^{k-1} + 26F$ $A_{k+1} = 1.04\,(A_k - F)$	**M1**	States the result for $n = k$ as an assumption and applies their relation from (a) to A_k. Condone no mention of $A_k \geq F$.
	$A_{k+1} = 1.04\left((12\,000 - 26F)\,1.04^{k-1} + 26F - F\right)$	**M1**	Substitutes the hypothesis into their relation and multiplies out. Condone sign slips.
	$= (12\,000 - 26F)\,1.04^k + 1.04 \times 25F$	**A1**	Correct expression, oe, need not be simplified.
	$= (12\,000 - 26F)\,1.04^{(k+1)-1} + 26F$	**A1**	cso. The printed form with $n = k + 1$; exponent written k accepted.
	True for $n = 1$, and true for $n = k$ $\Rightarrow$ true for $n = k + 1$, so true for all $n \geq 1$	**E1**	States true for $n = 1$, true for $n = k$ implies true for $n = k + 1$, so true for all $n \geq 1$. Condone the payment condition omitted.
		[6]	
6(c)	$26\,000 - 14\,000 \times 1.04^{n-1} < 1000$	**M1**	Sets their part (b) expression with $F = 1000$ below 1000. Condone an equation or $\leq$.
	$1.04^{n-1} > \frac{25}{14} \Rightarrow n - 1 > \frac{\ln\left(\frac{25}{14}\right)}{\ln 1.04}$ $\Rightarrow n > \ldots$ $[= 15.78]$	**dM1**	Rearranges to isolate the power and solves by logarithms, or by trying integer values either side. Condone sign slips.
	$n = 16$	**A1**	Year 16 identified as the first year with $A_n < 1000$.
	$A_{16} = 26\,000 - 14\,000 \times 1.04^{15}$	**M1**	Substitutes their integer year into their expression for A_n, or iterates the relation to that year.
	16 payments, final payment £786.79	**A1**	Both, with awrt 786.79.
		[5]	
	Total for Question 6	**12**	

Question	Scheme	Marks	Notes
7(a)	$u_{n+1} = \frac{4}{5}u_n + 20n + k$	**B1**	oe, such as $u_{n+1} = u_n - \frac{1}{5}u_n + k + 20n$.
	$A\left(\frac{4}{5}\right)^n$	**B1**	Any equivalent form, such as $A(0.8)^n$ or $A\left(\frac{4}{5}\right)^{n-1}$.
	$a(n+1) + b = \frac{4}{5}(an + b) + 20n + k$	**M1**	Substitutes $an + b$ into their recurrence relation of the form $u_{n+1} = pu_n + qn + r$. Condone slips.
	$a = \frac{4}{5}a + 20 \Rightarrow a = \ldots$ $a + b = \frac{4}{5}b + k \Rightarrow b = \ldots$	**dM1**	Equates coefficients of n and constant terms, or substitutes two values of n, and solves for a and b.
	$a = 100$	**A1**	
	$b = 5k - 500$	**A1**	oe
	$\frac{4}{5}A +$ "100" + "$5k - 500$" $= k \Rightarrow A = \ldots$	**M1**	Uses $u_1 = k$, or another correct term, in a general solution $A\left(\frac{4}{5}\right)^n +$ their particular solution, and solves for A.
	$u_n = (500 - 5k)\left(\frac{4}{5}\right)^n + 100n + 5k - 500$	**A1**	Any equivalent expression in n and k.
		[8]	
7(b)	$(500 - 5k)\left(\frac{4}{5}\right)^{12} + 1200 + 5k - 500 \geq 2500$ $\Rightarrow k \geq \ldots$	**M1**	Sets their u_{12}, from (a) or the recurrence relation, equal to or at least 2500 and solves for k. Condone the inequality sign.
	£379.19	**A1**	cao, rounded up to the nearest penny.
		[2]	
	Total for Question 7	**10**	

Question	Scheme	Marks	Notes
8(a)(i)	$N = \frac{200N^2}{6400+N^2} \Rightarrow N\left(6400+N^2\right) = 200N^2$	**M1**	Sets $N_{t+1} = N_t$ and multiplies through by $6400 + N^2$. Condone slips in expanding.
	$N^3 - 200N^2 + 6400N = 0$	**A1**	oe
	$N\left(N^2 - 200N + 6400\right) = 0 \Rightarrow N = \ldots$	**dM1**	Takes out the factor N, or divides by it, and solves their three-term quadratic by any method.
	$N = 0,\ 40,\ 160$	**A1**	All three values and no others.
		[4]	
8(a)(ii)	$N_{t+1} - N_t = \frac{200N_t^2 - N_t\left(6400+N_t^2\right)}{6400+N_t^2}$ $= -\frac{N_t(N_t-40)(N_t-160)}{6400+N_t^2}$	**M1**	Writes $N_{t+1} - N_t$ as one fraction and factorises its numerator, using (a)(i) or otherwise, or forms $N_{t+1} > N_t$, multiplies by $6400 + N_t^2$ and factorises. Condone sign slips.
	$N_t > 0,\ N_t - 40 > 0,\ N_t - 160 < 0,\ 6400 + N_t^2 > 0$ $\Rightarrow N_{t+1} - N_t > 0$	**A1**	Correct factorised form, or the equivalent inequality $(N_t - 40)(N_t - 160) < 0$, with the sign of each factor and of the denominator given for $40 < N_t < 160$.
	$160 - N_{t+1} = \frac{160\left(6400+N_t^2\right) - 200N_t^2}{6400+N_t^2}$ $= \frac{40(160-N_t)(160+N_t)}{6400+N_t^2}$	**M1**	Writes $160 - N_{t+1}$, or $N_{t+1} - 160$, as one fraction and factorises its numerator, or forms $N_{t+1} < 160$, multiplies by $6400 + N_t^2$ and reaches $N_t^2 < 25\,600$. Condone sign slips.
	$160 - N_{t+1} > 0 \Rightarrow N_{t+1} - N_t < 160 - N_t$ $\Rightarrow 0 < N_{t+1} - N_t < 160 - N_t$	**A1 AG**	Correct factorised form, or the equivalent inequality $N_t^2 < 25\,600$, with its sign justified for $40 < N_t < 160$, and both inequalities combined into the printed result without error.
		[4]	
8(a)(iii)	The number of birds increases every year and stays below 160	**E1**	States that the number increases every year and stays below 160.
	$N_t \to 160$, never reaching 160	**E1**	States that the number tends to 160 and never reaches it. Not for "tends to a limit" alone.
		[2]	
8(b)(i)	Model I: $N_3 = 139.8$	**B1**	awrt 139.8

Question 8 continued

Question	Scheme	Marks	Notes
	Model II: $N_1 = 100,\ N_2 = 121 \Rightarrow N_3 = 139$	**B1**	cao
		[2]	
8(b)(ii)	$\text{INT}\left(\frac{200\times159^2}{6400+159^2}\right) = \text{INT}(159.6) = 159$	**B1**	Evaluates with $N_t = 159$, showing the value before the integer part is taken, awrt 159.6 or an exact fraction.
	Model II reaches 159 after finitely many years and stays there Model I tends to 160 but never reaches it	**E1**	States that Model II becomes constant at 159, while Model I tends to 160. Both models and both values needed.
		[2]	
	Total for Question 8	**14**	

Question	Scheme	Marks	Notes
9(a)	$H_{n+2} - H_{n+1} = p(H_{n+1} - H_n)$ $\Rightarrow H_{n+2} = (1+p)H_{n+1} - pH_n$	**E1**	Equates the increase $H_{n+2} - H_{n+1}$ to p times the previous increase $H_{n+1} - H_n$ and rearranges to the printed relation.
	$2.2 = 1.8(1+p) - 1.2p \Rightarrow p = \ldots$	**M1**	Uses H_1, H_2 and H_3 in the relation, or in $H_3 - H_2 = p(H_2 - H_1)$, to form an equation in p. Condone sign slips.
	$0.6p = 0.4 \Rightarrow p = \frac{2}{3}$	**A1 AG**	Correct substituted equation leading without error to $p = \frac{2}{3}$.
		[3]	
9(b)	$n = 1$ and $n = 2$ $3 - 1.8\left(\frac{2}{3}\right)^0 = 1.2 = H_1$ $3 - 1.8\left(\frac{2}{3}\right)^1 = 1.8 = H_2$	**B1**	Both base cases checked against the given heights.
	Assumes true for $n = k$ and $n = k+1$ $H_{k+2} = \frac{5}{3}H_{k+1} - \frac{2}{3}H_k$	**M1**	Assumes the result for two consecutive values and expresses H_{k+2} by the recurrence with $p = \frac{2}{3}$.
	$H_{k+2} = \frac{5}{3}\left(3 - 1.8\left(\frac{2}{3}\right)^k\right)$ $\quad - \frac{2}{3}\left(3 - 1.8\left(\frac{2}{3}\right)^{k-1}\right)$ $\quad = 5 - 3\left(\frac{2}{3}\right)^k - 2 + 1.2\left(\frac{2}{3}\right)^{k-1}$	**M1 A1**	**M1** Substitutes both assumed expressions into the recurrence and expands. Condone sign slips. **A1** Correct expression in powers of $\frac{2}{3}$, unsimplified.
	$H_{k+2} = 3 - 0.8\left(\frac{2}{3}\right)^{k-1}$ $\quad = 3 - 1.8\left(\frac{2}{3}\right)^{k+1}$	**A1**	cso. Reaches $3 - 1.8\left(\frac{2}{3}\right)^{k+1}$, or $3 - 1.8\left(\frac{2}{3}\right)^{(k+2)-1}$, by combining the powers.
	True for $n = 1$ and $n = 2$ True for $n = k$, $n = k+1 \Rightarrow$ true for $n = k+2$ So true for all $n \geq 1$	**E1**	States the result true for $n = 1$ and $n = 2$, that truth for $n = k$ and $n = k+1$ implies truth for $n = k+2$, so true for all $n \geq 1$.
		[6]	
9(c)	$1.8\left(\frac{2}{3}\right)^{n-1} > 0$ for all n $\Rightarrow H_n < 3$	**E1**	States that $1.8\left(\frac{2}{3}\right)^{n-1}$ is positive for every n, so $H_n < 3$. Not for checking some values alone.
	$H_n \to 3$, so 3 metres	**B1**	
		[2]	

Question 9 continued

Question	Scheme	Marks	Notes
9(d)	$1.2 + \frac{0.6}{1-p} = 4 \Rightarrow p = \ldots$	**M1**	Forms the long-term height as a starting height plus the sum to infinity of the geometric increases, oe, and sets it equal to 4. Condone slips.
	$p = \frac{11}{14} = 78.6\%$	**A1**	awrt 78.6%. $\frac{11}{14}$ or 0.786 alone does not score.
		[2]	
	Total for Question 9	**13**	

Question	Scheme	Marks	Notes
10(a)	$A\alpha^n$	**B1**	
	$a(n+1)\alpha^{n+1} = \alpha \times an\alpha^n - \beta\alpha^n$	**M1**	Substitutes a trial particular solution of the form $an\alpha^n$, or $an\alpha^{n-1}$, into $(*)$. Condone sign slips.
	$a\alpha^{n+1} = -\beta\alpha^n \Rightarrow a = \ldots \ [= -\frac{\beta}{\alpha}]$	**dM1**	Cancels the terms in n and solves for a. Condone sign slips.
	$u_n = A\alpha^n - \beta n\alpha^{n-1}$	**A1**	oe, such as $u_n = A\alpha^n - \frac{\beta}{\alpha}n\alpha^n$.
		[4]	
10(b)	$u_0 = L \Rightarrow A = L$	**B1**	
	$L\alpha^T - \beta T\alpha^{T-1} = 0 \Rightarrow \beta = \ldots$	**M1**	Substitutes $n = T$ and $u_T = 0$ into their general solution of the form $A\alpha^n + kn\alpha^{n-1}$ and rearranges for β.
	$\beta = \frac{L\alpha}{T}$	**A1**	oe
	$u_n = L\alpha^n - \frac{L\alpha}{T}\, n\alpha^{n-1}$ $= L\alpha^n\left(1 - \frac{n}{T}\right)$	**A1 AG**	Substitutes $A = L$ and $\beta = \frac{L\alpha}{T}$ into the correct general solution, reaching the printed result without error.
		[4]	
10(c)(i)	$\alpha = 1.06$	**B1**	
		[1]	
10(c)(ii)	β is Jordan's first annual repayment	**E1**	States that β is the first repayment, or the repayment at the end of the first year. Not for "a repayment" or "the annual repayment" alone.
		[1]	
10(c)(iii)	$\beta = \frac{1.06L}{25} = 0.0424L$	**B1ft**	ft their α in $\frac{\alpha L}{25}$. Accept $\frac{53}{1250}L$ oe.
		[1]	
10(d)	$L(1.06)^n\left(1 - \frac{n}{25}\right) < L$	**M1**	Compares the result of (b), with their α and $T = 25$, with L, as an equation or an inequality.
	$n = 13: \ (1.06)^{13} \times \frac{12}{25} \approx 1.024$ $n = 14: \ (1.06)^{14} \times \frac{11}{25} \approx 0.995$	**dM1**	Evaluates the expression, or u_n, at $n = 13$ and $n = 14$, or solves the equation numerically to find n between 13 and 14 ($n \approx 13.8$).

Question 10 continued

Question	Scheme	Marks	Notes
	14 (complete years)	**A1**	cao
		[3]	
	Total for Question 10	**14**	