OPTIONAL PURE / RECURRENCE RELATIONS

1st Order Recurrence Relations

Mark scheme 12 questions · 123 marks

EXAM BOARDS Edexcel FP2 9FM0/4A · OCR A Y545 · OCR MEI Extra Pure Y435

Questions

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	Total	**123**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last accuracy mark of the working that reaches a printed result: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	$4a + b = -5, \quad -5a + b = 13$ $\Rightarrow a = \ldots, \ b = \ldots$	**M1**	Forms two equations in a and b from pairs of consecutive given terms and solves them by any method. Condone sign slips.
	$a = -2, \quad b = 3$	**A1**	
		[2]	
1(b)	$A(-2)^n$	**B1**	Any constant multiple of $(-2)^n$, oe such as $A(-2)^{n-1}$.
	$k = -2k + 3 \Rightarrow k = \ldots$	**M1**	Substitutes a constant into their recurrence relation from (a) and solves for it.
	$k = 1$	**A1**	
	$u_n = A(-2)^n + \text{“1”}$ $-2A + 1 = 4 \Rightarrow A = \ldots$	**M1**	Uses $u_1 = 4$, or another given term, in a general solution of the form $A(-2)^n +$ their constant to find A.
	$u_n = 1 - \frac{3}{2}(-2)^n$	**A1**	oe, such as $1 + 3(-2)^{n-1}$.
		[5]	
1(c)	$1 + 3(-2)^{n-1} = 769$ $\Rightarrow (-2)^{n-1} = 256 \Rightarrow n = \ldots$	**M1**	Sets their u_n equal to 769 and rearranges to a power of -2 equal to a constant, then solves for n.
	$n = 9$	**A1**	
		[2]	
	Total for Question 1	**9**	

Question	Scheme	Marks	Notes
2(a)	Repays £P: $U_{n-1} - P$ owed	**E1**	States that the repayment leaves $U_{n-1} - P$ owed, in words or symbols.
	Adds 1% interest: $U_n = 1.01\,(U_{n-1} - P)$ $\quad = 1.01U_{n-1} - 1.01P$	**E1**	Explains that the interest multiplies the amount after repayment by 1.01, and reaches the printed relation.
		[2]	
2(b)	$A(1.01)^n$	**B1**	
	$k = 1.01k - 1.01P \Rightarrow k = \ldots$	**M1**	Substitutes a constant $U_n = k$ into the recurrence relation and solves for k. Condone sign slips.
	$k = 101P$	**A1**	
	$U_n = A(1.01)^n +$ "$101P$" $6000 = A +$ "$101P$" $\Rightarrow A = \ldots$	**M1**	Uses $U_0 = 6000$ in a general solution of the form $A(1.01)^n +$ their constant to find A.
	$U_n = (6000 - 101P)(1.01)^n + 101P$	**A1**	oe, such as $6000(1.01)^n - 101P\,((1.01)^n - 1)$.
		[5]	
2(c)	Sets $U_{24} = 0$: $(6000 - 101P)(1.01)^{24} + 101P = 0$	**M1**	Substitutes $n = 24$ into their U_n from (b), in terms of P, and sets it equal to 0.
	$101P\left((1.01)^{24} - 1\right) = 6000(1.01)^{24}$ $\Rightarrow P = \ldots$	**dM1**	Collects the terms in P and makes P the subject. Condone sign slips.
	$P = 279.64$	**A1**	awrt 279.64.
		[3]	
Total for Question 2		**10**	

Question	Scheme	Marks	Notes
3(a)	Stage 1 is the single original square: $u_1 = 1$ Each of the u_n squares becomes 4: $4u_n$	**E1**	Explains that each of the u_n squares is replaced by 4 squares, giving $4u_n$, with $u_1 = 1$ as stage 1 is the one original square.
	One square added at the centre: $u_{n+1} = 4u_n + 1$	**E1**	Explains that one further square is added at the centre, reaching the printed recurrence relation.
		[2]	
3(b)	$A \times 4^n$	**B1**	oe, such as $A \times 4^{n-1}$.
	$k = 4k + 1 \Rightarrow k = \ldots$	**M1**	Substitutes a constant k for both u_{n+1} and u_n in the recurrence relation and solves. Condone sign slips.
	$k = -\frac{1}{3}$	**A1**	
	$u_1 = 4A + \text{“}-\frac{1}{3}\text{”} = 1 \Rightarrow A = \ldots$	**M1**	Uses $u_1 = 1$ in a general solution of the form $A \times 4^n$ (or $A \times 4^{n-1}$) plus their constant, and solves for A.
	$u_n = \frac{4^n - 1}{3}$	**A1**	oe, such as $\frac{1}{3} \times 4^n - \frac{1}{3}$ or $\frac{4}{3} \times 4^{n-1} - \frac{1}{3}$.
		[5]	
3(c)	Side $\frac{9}{3^{n-1}}$, area $\left(\frac{27}{3^n}\right)^2 = \frac{729}{9^n}$	**B1**	Any correct expression for the area of one square in stage n, such as $81\left(\frac{1}{9}\right)^{n-1}$.
	$\text{“}\frac{4^n - 1}{3}\text{”} \times \text{“}\frac{729}{9^n}\text{”}$	**M1**	Multiplies their u_n from (b) by their area of one square, both expressions in n.
	$243\left(\left(\frac{4}{9}\right)^n - \left(\frac{1}{9}\right)^n\right)$	**A1**	Required form only; $\frac{243(4^n-1)}{9^n}$, or $p = 243$ stated alone, scores A0.
		[3]	
	Total for Question 3	**10**	

Question	Scheme	Marks	Notes
4(a)	$v_1 = 0.5 \times 60 = 30$	**B1**	Needs the first 60 mg halved; $v_1 = 30$ stated alone is not enough.
	$v_{n+1} = 0.5(v_n + 20) = 0.5v_n + 10$	**E1**	States that 20 is added to v_n and the total then halves, reaching the printed relation.
		[2]	
4(b)	$A(0.5)^n$	**B1**	oe, such as $A(0.5)^{n-1}$.
	$k = 0.5k + 10 \Rightarrow k = \ldots$	**M1**	Substitutes a constant $v_n = k$ into the recurrence relation and solves. Condone sign slips.
	$k = 20$	**A1**	
	$v_n = A(0.5)^n + \text{“20”}$ $0.5A + \text{“20”} = 30 \Rightarrow A = \ldots$	**M1**	Uses $v_1 = 30$ in a solution of the form $A(0.5)^n +$ their constant, or their equivalent form, to find A.
	$v_n = 20 + 20(0.5)^n$	**A1**	oe, such as $20 + 10(0.5)^{n-1}$ or $20 + \frac{20}{2^n}$.
		[5]	
4(c)	Greatest amounts immediately after injections: 60 and $v_n + 20 = 40 + 20(0.5)^n \leq 50$	**M1**	Considers the amounts immediately after injections, the first and the later ones using their v_n, as the greatest.
	$60 \leq 65$, so safe	**A1**	Greatest amount 60, with later amounts after injections shown not to exceed 65, and safe concluded.
	Least amounts immediately before injections: $20 + 20(0.5)^n \geq 22 \Rightarrow n \leq \ldots$	**M1**	Compares their v_n with 22, by an inequality or by evaluating terms until one falls below 22.
	$v_4 = 21.25 < 22$, so not effective Not safe and effective	**A1**	cso. $v_4 = 21.25$, or $v_n < 22$ for $n \geq 4$, and concludes the course is not safe and effective.
		[4]	
Total for Question 4		**11**	

Question	Scheme	Marks	Notes
5(a)	$A \times 3^n$	**B1**	
	$k \times 2^{n+1} = 3k \times 2^n + 2^{n+2}$	**M1**	Substitutes a trial $k \times 2^n$, or an equivalent such as $k \times 2^{n+2}$, into the recurrence relation. Condone slips in the powers of 2.
	$2k = 3k + 4 \Rightarrow k = \ldots$	**dM1**	Divides through by a power of 2, or equates coefficients, and solves for k.
	$u_n = A \times 3^n - 2^{n+2}$	**A1**	oe, such as $A \times 3^n - 4 \times 2^n$.
		[4]	
5(b)	$3A - 2^3 = 1 \Rightarrow A = \ldots$	**M1**	Substitutes $n = 1$, $u_1 = 1$ into their general solution of the form $A \times 3^n +$ their particular solution and solves for A.
	$u_n = 3^{n+1} - 2^{n+2}$	**A1**	oe, such as $3 \times 3^n - 4 \times 2^n$.
	3^{n+1} is odd and 2^{n+2} is even $\Rightarrow u_n$ is odd	**A1 AG**	Correct u_n, a reason why each term is odd or even, and the conclusion. Not for checking particular values of n.
		[3]	
	Total for Question 5	**7**	

Question	Scheme	Marks	Notes
6(a)	$2000k + 300 = 1900 \Rightarrow k = 0.8$ $F_0 = 2000,\ F_{n+1} = 0.8F_n + 300$	**B1**	Both $F_0 = 2000$ and the relation, oe such as $F_n = 0.8F_{n-1} + 300$.
		[1]	
6(b)(i)	Number added at end of year $n+1$: $300 + 50n$ $F_0 = 2000,\ F_{n+1} = 0.8F_n + 50n + 300$	**E1**	Explains that $0.8F_n$ remain and $300 + 50n$ are added at the end of year $n+1$, reaching the printed system.
		[1]	
6(b)(ii)	$A(0.8)^n$	**B1**	
	$a(n+1) + b = 0.8(an + b) + 50n + 300$	**M1**	Substitutes a trial $an + b$ into both sides of the recurrence relation, with $n+1$ on the left. Condone slips.
	$a = 0.8a + 50 \Rightarrow a = \ldots$ $a + b = 0.8b + 300 \Rightarrow b = \ldots$	**dM1**	Equates coefficients of n and constant terms and solves for both constants. Condone sign slips.
	$a = 250,\ b = 250$	**A1**	
	$F_n = A(0.8)^n +$ "$250n + 250$" $A +$ "250" $= 2000 \Rightarrow A = \ldots$	**M1**	Uses $F_0 = 2000$, or another correct term, in a solution of the form $A(0.8)^n +$ their linear particular solution to find A.
	$F_n = 1750(0.8)^n + 250n + 250$	**A1**	oe, such as $1750\left(\frac{4}{5}\right)^n + 250(n+1)$.
		[6]	
6(b)(iii)	$250n \to \infty$, so F_n grows without limit The lake can support only a limited number of fish	**E1**	States that the model predicts unbounded growth, or ever-increasing restocking, and why this is unrealistic. Not for "unrealistic" alone.
		[1]	
	Total for Question 6	**9**	

Question	Scheme	Marks	Notes
7	$u_n = \dfrac{3}{n+1} \times \dfrac{3}{n} \times \cdots \times \dfrac{3}{2} \times u_0$	**M1**	Applies the recurrence repeatedly, showing at least three factors $\frac{3}{m}$, or substitutes $\frac{ak^n}{(n+b)!}$ into the relation and compares. Condone slips in the denominators.
	$u_n = \dfrac{4 \times 3^n}{(n+1) \times n \times \cdots \times 2}$	**A1**	Correct expression for u_n, oe, with the denominator as a product down to 2 or 1, or as $(n+1)!$; or $k = 3$ and $b = 1$ from the comparison.
	$u_n = \dfrac{4 \times 3^n}{(n+1)!}$ $a = 4,\ k = 3,\ b = 1$	**A1**	
	$u_0 = \dfrac{4 \times 3^0}{(0+1)!} = 4$	**B1**	Substitution of $n = 0$ into the correct formula shown.
	$u_{n+1} = \dfrac{4 \times 3^{n+1}}{(n+2)!} = \dfrac{3}{n+2} \times \dfrac{4 \times 3^n}{(n+1)!}$	**M1**	Replaces n by $n+1$ in their formula, with their a, k and b, and uses $(n+2)! = (n+2)(n+1)!$, or forms $\frac{u_{n+1}}{u_n}$. Not for the general-form comparison already credited.
	$= \dfrac{3u_n}{n+2}$	**A1 AG**	From the correct formula, reaching $\frac{3u_n}{n+2}$ or showing LHS = RHS, without error.
	Total for Question 7	**6**	

Question	Scheme	Marks	Notes
8(a)	Ak^n	**B1**	
	$p = kp + k + 1 \Rightarrow p = \ldots$	**M1**	Substitutes a constant $u_n = p$ into the recurrence relation and solves for p. Condone sign slips.
	$u_n = Ak^n + \frac{1+k}{1-k}$	**A1**	oe, such as $Ak^n - \frac{k+1}{k-1}$. Needs the arbitrary constant.
		[3]	
8(b)	$u_0 = \frac{1+k}{1-k}$	**B1ft**	ft their constant particular solution in terms of k from (a), oe.
	$\frac{1+k}{1-k} = 3 \Rightarrow k = \frac{1}{2}$	**B1**	
		[2]	
8(c)	$u_1 = 2k + 1$ $u_2 = k(2k + 1) + k + 1$	**M1**	Finds u_2 in terms of k from $u_0 = 1$, by the recurrence relation twice or the general solution with $n = 2$. Condone slips.
	$u_2 = 2k^2 + 2k + 1$	**A1**	
	$2k^2 + 2k + 1 = 13$ $\Rightarrow k^2 + k - 6 = 0 \Rightarrow k = \ldots$	**M1**	Sets their quadratic u_2 equal to 13 and solves by factorising, the formula or completing the square; a calculator is allowed.
	$k = 2$ or $k = -3$	**A1**	Both values, and no others.
		[4]	
	Total for Question 8	**9**	

Question	Scheme	Marks	Notes
9(a)	$A \times 3^n$	**B1**	Or $A \times 3^{n-1}$ oe.
	$u_n = an^2 + bn + c$: $a(n+1)^2 + b(n+1) + c = 3(an^2 + bn + c) - 4n^2$	**M1**	Substitutes a trial of the form $an^2 + bn + c$ into the recurrence relation. Condone slips in expanding.
	$an^2 + (2a + b)n + (a + b + c)$ $\quad = (3a - 4)n^2 + 3bn + 3c$ $\Rightarrow a = \ldots,\ b = \ldots,\ c = \ldots$	**dM1**	Equates coefficients of n^2, n and the constants, or substitutes three values of n, and solves for a, b and c.
	$a = 2$	**A1**	
	$b = 2,\ c = 2$	**A1**	
	$u_n = A \times 3^n +$ "$2n^2 + 2n + 2$" $3A + 6 = 4 \Rightarrow A = \ldots$	**M1**	Substitutes $n = 1$, $u_1 = 4$ into their general solution, of the form $A \times 3^n$ (or $A \times 3^{n-1}$) plus their quadratic, and solves for A.
	$u_n = 2n^2 + 2n + 2 - 2 \times 3^{n-1}$	**A1**	oe, such as $-\frac{2}{3} \times 3^n$ for the last term.
		[7]	
9(b)	Takes the 3^n term as zero: $v_n =$ "$2n^2 + 2n + 2$" $\Rightarrow k = v_1 = \ldots$	**M1**	Sets the coefficient of 3^n in their general solution from (a) to zero, or substitutes a quadratic v_n into the relation, and finds v_1.
	$k = 6$	**A1**	
		[2]	
	Total for Question 9	**9**	

Question	Scheme	Marks	Notes
10(a)	$A \times 3^n$	**B1**	
	$2p \times 2^n + qn + q + r$ $= 3p \times 2^n + 3qn + 3r + 2^n - 4n$	**M1**	Substitutes a trial $p \times 2^n + qn + r$ for both t_n and t_{n+1} in the relation. Condone sign slips.
	Equates coefficients of 2^n, n and constants: $2p = 3p + 1,\ q = 3q - 4,\ q + r = 3r$ $\Rightarrow p = \ldots,\ q = \ldots,\ r = \ldots$	**dM1**	Equates coefficients of 2^n, n and the constant terms, or uses three values of n, and solves. Condone sign slips.
	$p = -1$	**A1**	
	$q = 2,\ r = 1$	**A1**	
	$t_n = A \times 3^n - 2^n + 2n + 1$ $3A - 2 + 2 + 1 = 4 \Rightarrow A = \ldots$	**M1**	Uses $t_1 = 4$ in their $A \times 3^n$ plus their particular solution and solves for A. Not in the complementary function alone.
	$t_n = 3^n - 2^n + 2n + 1$	**A1**	oe. Not for $A = 1$ alone.
		[7]	
10(b)(i)	States the multiplier of u_n is not constant: $u_{n+1} = \frac{2n}{n+1}u_n + \cdots$	**E1**	States that the coefficient of u_n, or of u_n relative to u_{n+1}, depends on n. Not for a comment on the right-hand side alone.
		[1]	
10(b)(ii)	$(n+1)u_{n+1} = a(n+1)^3 + b$ $2nu_n = 2an^3 + 2b$	**M1**	Substitutes $an^2 + \frac{b}{n}$ for u_n and $a(n+1)^2 + \frac{b}{n+1}$ for u_{n+1} into the LHS. Condone expansion slips.
	$(n+1)u_{n+1} - 2nu_n$ $= -an^3 + 3an^2 + 3an + a - b$	**A1**	Correct expanded expression, oe.
	Equates coefficients with $-2n^3 + 6n^2 + 6n - 1$: $-a = -2,\ a - b = -1 \Rightarrow a = \ldots,\ b = \ldots$	**dM1**	Equates coefficients of at least two powers of n, or uses two values of n, and solves for a and b. Condone sign slips.
	$a = 2,\ b = 3$, and $3a = 6$ for n^2 and n	**A1**	Both values, with every coefficient shown to agree, or the identity shown for all n.
	$u_1 = 2 \times 1^2 + \frac{3}{1} = 5$	**B1**	
		[5]	
	Total for Question 10	**13**	

Question	Scheme	Marks	Notes
11(a)	$L_n + 0.005L_n = 1.005L_n$	**B1**	Interest added to L_n gives $1.005L_n$, in words or symbols.
	Payment in month $n+1$ is $M + 2n$	**B1**	Or payment in month k is $M + 2(k-1)$, with $k = n+1$ used.
	$L_{n+1} = 1.005L_n - (M + 2n)$ $L_0 = P$, the amount borrowed	**E1**	States the payment is made after the interest, so is subtracted from $1.005L_n$, and that $L_0 = P$ is the amount borrowed.
		[3]	
11(b)	$A(1.005)^n$	**B1**	
	$L_n = an + b$ $a(n+1) + b = 1.005(an + b) - M - 2n$	**M1**	Substitutes $L_n = an + b$ and $L_{n+1} = a(n+1) + b$ into the recurrence relation. Condone sign slips.
	$a = 1.005a - 2 \Rightarrow a = \ldots$ $a + b = 1.005b - M \Rightarrow b = \ldots$	**dM1**	Equates coefficients of n and constant terms and solves for both. Condone sign slips.
	$a = 400$	**A1**	
	$b = 80\,000 + 200M$	**A1**	oe
	$P = A + 80\,000 + 200M \Rightarrow A = \ldots$	**M1**	Uses $n = 0$, $L_0 = P$ in their general solution, $A(1.005)^n$ plus their linear particular solution, to find A.
	$L_n = (P - 80\,000 - 200M)(1.005)^n$ $+ 400n + 80\,000 + 200M$	**A1**	oe, in terms of n, M and P.
		[7]	
11(c)	$(12\,000 - 80\,000 - 200M)(1.005)^{48}$ $+ 400 \times 48 + 80\,000 + 200M = 0$	**M1**	Substitutes $P = 12\,000$ and $n = 48$ into their L_n from (b), with terms in M, and sets it equal to 0.
	$200M\left((1.005)^{48} - 1\right)$ $= 99\,200 - 68\,000(1.005)^{48} \Rightarrow M = \ldots$	**dM1**	Collects the terms in M and rearranges to $M = \ldots$. Condone sign and arithmetic slips.
	$M = 237$	**A1**	
	$M + 2 \times 47 =$ "237" $+ 94 = 331$	**B1ft**	ft their M plus 94.
		[4]	
	Total for Question 11	**14**	

Question	Scheme	Marks	Notes
12(a)	$L_{n+1} = \left(1 + \frac{\alpha}{100}\right)(L_n - R)$	**M1**	Subtracts R from L_n, then multiplies by an interest factor of the form $1 + \frac{\alpha}{100}$. Condone a slip in the factor.
	$a = 1 + \frac{\alpha}{100}$ $b = -\left(1 + \frac{\alpha}{100}\right)R$	**A1**	Both, oe, such as $a = \frac{100+\alpha}{100}$, $b = -\frac{(100+\alpha)R}{100}$.
		[2]	
12(b)	Da^n	**B1**	
	$k = ak + b \Rightarrow k = \ldots$	**M1**	Substitutes a constant $L_n = k$ into the recurrence relation and attempts to solve for k.
	$k = \frac{b}{1-a}$	**A1**	oe, such as $-\frac{b}{a-1}$.
	$C = D + \text{“}\frac{b}{1-a}\text{”} \Rightarrow D = \ldots$	**M1**	Sets $n = 0$ in a general solution of the form $Da^n +$ their constant and solves for D.
	$L_n = \left(C - \frac{b}{1-a}\right)a^n + \frac{b}{1-a}$	**A1**	oe in a, b, C and n.
		[5]	
12(c)	$\frac{b}{1-a} = \frac{-(100+\alpha)R/100}{-\alpha/100}$ $= \frac{(100+\alpha)R}{\alpha}$	**M1**	Substitutes their a and b from (a) into $\frac{b}{1-a}$ and simplifies to a single expression in α and R. Condone sign slips.
	Coefficient of a^n negative, else $L_n > R$ always: $C - \text{“}\frac{(100+\alpha)R}{\alpha}\text{”} < 0$	**M1**	Argues that the coefficient of a^n in their L_n must be negative, as otherwise L_n never falls to R; oe argument that L_n must decrease.
	$C < \frac{(100+\alpha)R}{\alpha} \Rightarrow R > \frac{\alpha C}{100+\alpha}$	**A1 AG**	Correct inequality from a complete argument, with $a^n > 0$ and $\frac{(100+\alpha)R}{\alpha} > R$ used, or L_n shown to decrease, rearranged without error to the printed result.
		[3]	
12(d)(i)	$L_n = (20\,000 - 42\,000) \times (1.05)^n + 42\,000$ $42\,000 - 22\,000 \times (1.05)^n \leq 2000$	**M1**	Substitutes values of a, b and C into their L_n from (b) and sets it ≤ 2000 or $= 2000$.

Question 12 continued

Question	Scheme	Marks	Notes
	$(1.05)^n \geq \frac{20}{11} \Rightarrow n \geq \frac{\ln\left(\frac{20}{11}\right)}{\ln 1.05}$ $\Rightarrow n \geq \ldots$ $[\approx 12.25]$	**dM1**	Rearranges to $(1.05)^n$ against a constant and uses logarithms to reach a value of n. Condone $=$ or a wrong inequality sign. Trial of values alone scores nothing.
	$n = 13$, so 14 repayments	**A1**	cao. Needs $n = 13$ read as the amount before the 14th repayment.
		[3]	
12(d)(ii)	(Total $=$) $13 \times 2000 +$ "L_{13}"	**M1**	Adds 2000 times one fewer than their number of repayments to their final repayment, their L_n at that n.
	$L_{13} = 42\,000 - 22\,000 \times (1.05)^{13} \approx 515.72$	**A1**	awrt 516, soi.
	(Total $=$) 26 515.72	**A1**	awrt 26 500 (3 s.f.), such as 26 516.
		[3]	
	Total for Question 12	**16**	