OPTIONAL PURE / NUMBER THEORY

Linear and Simultaneous Congruences

Questions by topic 10 questions · 75 marks

EXAM BOARDS Edexcel FP2 9FM0/4A · OCR A Y545

Questions

Question 1	*5 marks*	2
Question 2	*7 marks*	3
Question 3	*5 marks*	4
Question 4	*9 marks*	5
Question 5	*9 marks*	6
Question 6	*7 marks*	7
Question 7	*7 marks*	8
Question 8	*8 marks*	9
Question 9	*11 marks*	10
Question 10	*7 marks*	11

1. **(a)** Evaluate 17×11 modulo 37. **[1]**

(b) Hence solve the linear congruence $17x \equiv 5 \pmod{37}$, giving your answer in the form $x \equiv a \pmod{37}$, where $0 \leq a < 37$. **[4]**

2. **(a)** Explain why the congruence equation

$$24x \equiv 20 \pmod{54}$$

has no solutions. **[2]**

(b) The integer k satisfies $1 \leq k \leq 30$. Consider the congruence equation

$$15y \equiv 2k + 1 \pmod{40}$$

(i) Find every value of k for which this congruence equation has solutions. **[3]**

(ii) For each of these values of k, state how many solutions the congruence equation has modulo 40, giving a reason for your answer. **[2]**

3. **(a)** Solve $13x \equiv 6 \pmod{31}$. **[2]**

(b) Show that the simultaneous congruences

$$6x \equiv 4 \pmod{16}, \qquad x \equiv 7 \pmod{12}$$

have no solution. **[3]**

4. **(a)** Use the Euclidean algorithm to show that the highest common factor of 83 and 30 is 1. **[2]**

(b) Hence find integers p and q such that

$$30p + 83q = 1$$

[3]

(c) Hence solve the linear congruence $30x \equiv 11 \pmod{83}$, giving your answer in the form $x \equiv a \pmod{83}$, where $0 \leq a < 83$. **[2]**

(d) Use your answer to part (b) to solve the linear congruence $83y \equiv 4 \pmod{30}$, giving your answer in the form $y \equiv c \pmod{30}$, where $0 \leq c < 30$. **[2]**

5. **(a)** Find all integers x such that $21x \equiv 4 \pmod{89}$. **[4]**

(b) Hence, or otherwise, solve

(i) $68x \equiv 8 \pmod{89}$ **[2]**

(ii) $21x \equiv 9 \pmod{178}$ **[3]**

6. **(a)** Solve the linear congruence $5x \equiv 8 \pmod{17}$, giving your answer in the form $x \equiv a \pmod{17}$, where $0 \leq a < 17$. **[2]**

(b) Hence, or otherwise, find all integers x with $1000 < x < 2000$ that satisfy the simultaneous linear congruences

$$5x \equiv 8 \pmod{17}, \qquad 9x \equiv 4 \pmod{23}$$ **[5]**

7. A circular track has 17 marker posts, equally spaced and numbered 0 to 16 in order clockwise. A robot starts at post 0 and travels clockwise around the track. It makes x short moves, each taking it forward 3 posts, and y long moves, each taking it forward 8 posts, in some order. When it has made all of its moves, the robot is at post n, where $0 \leq n \leq 16$.

(a) Write down a congruence for n in the form $n \equiv ax + by \pmod{c}$, where a, b and c are integers. **[1]**

It is now given that the robot makes exactly 45 moves in total and finishes at post 5.

(b) Use your congruence to find all the possible values of the number of long moves made by the robot. **[6]**

8. Consider the simultaneous linear congruences

$$4x \equiv 6 \pmod{14}, \qquad 5x \equiv 2 \pmod{12}$$

(a) The first congruence, including its modulus, is divided through by the highest common factor of 4 and 14. Explain why the resulting pair of congruences has exactly one solution modulo 84. **[2]**

(b) Hence find all integers x with $0 < x < 500$ that satisfy both of the original congruences. **[6]**

9. **(a)** Use the Euclidean algorithm to find the highest common factor h of 270 and 114. **[3]**

(b) Hence find integers a and b such that

$$270a + 114b = h$$ **[3]**

(c) Hence find all integers x with $0 \leq x < 114$ that satisfy

$$270x \equiv 24 \pmod{114}$$ **[5]**

10. Determine all integers x that satisfy the simultaneous congruences

$$4x \equiv 3 \pmod{9}, \qquad x \equiv 7 \pmod{11}, \qquad 3x \equiv 2 \pmod{13}$$

Give your answer in the form $x = qn + r$, where n is any integer and q, r are integers with $q > 0$ and $0 \leq r < q$. **[7]**