OPTIONAL PURE / MISCELLANEOUS OPTIONAL PURE

Further Loci in the Argand Diagram

Questions by topic 12 questions · 105 marks

EXAM BOARDS Edexcel FP2 9FM0/4A

Questions

Question 1	*4 marks*	2
Question 2	*9 marks*	3
Question 3	*10 marks*	4
Question 4	*12 marks*	5
Question 5	*6 marks*	6
Question 6	*7 marks*	7
Question 7	*10 marks*	8
Question 8	*9 marks*	9
Question 9	*9 marks*	10
Question 10	*9 marks*	11
Question 11	*10 marks*	12
Question 12	*10 marks*	13

1. Sketch on an Argand diagram the region defined by

$$\left\{ z \in \mathbb{C} \;\middle|\; \frac{\pi}{6} \leq \arg(z - 1 - i) < \frac{2\pi}{3} \right\} \cap \left\{ z \in \mathbb{C} \;\middle|\; |z - 1 - 3i| \leq 2 \right\}$$

On your sketch

- shade the part of the diagram that is included in the region
- use solid lines to show the parts of the boundary that are included in the region
- use dashed lines to show the parts of the boundary that are not included in the region

[4]

2. A complex number $z = x + iy$ is represented by the point P in an Argand diagram, where

$$|z + 3| = 2|z - 3i|$$

(a) Show that the locus of P is a circle, and find its centre and radius. **[6]**

(b) Hence find the greatest and least values of $|z - 4 - i|$. **[3]**

3. A complex number z is represented by the point P in an Argand diagram, where

$$|z - 2 + 7i| = 3|z - 2 - i|$$

(a) Show that the locus of P is a circle, and find its centre and radius. **[6]**

(b) Sketch on an Argand diagram the locus of P as z varies. **[2]**

(c) On your sketch, shade the region whose points satisfy both

$$|z - 2 + 7i| \geq 3|z - 2 - i| \quad \text{and} \quad 0 \leq \arg z \leq \frac{\pi}{4}$$ **[2]**

4. The complex number z is represented on an Argand diagram by the point P, where

$$|z - 5 + i| = 2\,|z + 1 - 2i|$$

The locus of P is a circle with centre C.

(a) Find the complex number represented by C and the exact radius of the circle. **[6]**

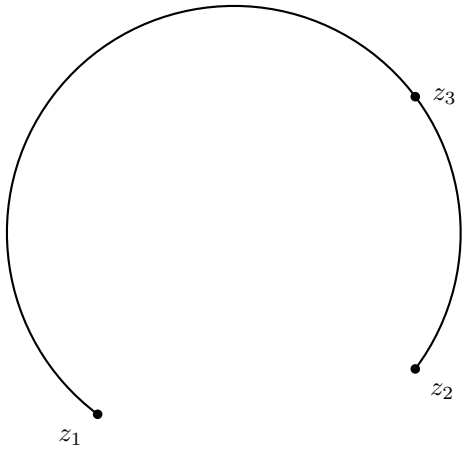
The complex number w is represented on the same Argand diagram by the point Q, where

$$\arg(w + 7 + 3i) = \frac{\pi}{4}$$

The locus of Q meets the locus of P at two distinct points R and S.

(b) Find the complex numbers represented by R and S. **[4]**

(c) Find the area of triangle CRS. **[2]**

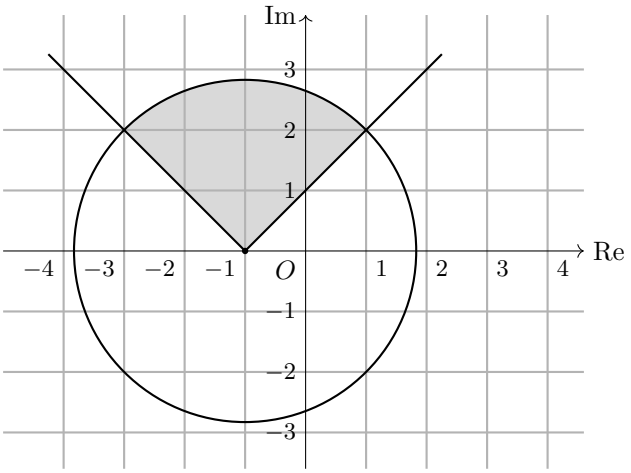


5. The diagram above shows a locus in the complex plane.
The locus is an arc of a circle from the point represented by $z_1 = -1 - 3i$ to the point represented by $z_2 = 6 + ai$, where a is a constant, $a \neq 4$.
Given that

- the point $z_3 = 6 + 4i$ also lies on the locus
- the centre of the circle has imaginary part equal to 1

(a) determine the complex number representing the centre of the circle and the value of a. **[3]**

(b) Hence determine a complex equation for the locus, giving any angles in the equation as positive values. **[3]**



6. The diagram above shows a shaded region in an Argand diagram.
The set of points, A, that lie within the shaded region, including its boundaries, is defined by

$$A = \{z \in \mathbb{C} \mid p \leq \arg(z - a) \leq q\} \cap \{z \in \mathbb{C} \mid |z - a| \leq r\}$$

where a is a real constant and p, q and r are positive constants.

(a) Write down the values of a, p, q and r. **[3]**

Given that $w = 3 - i$ and $z \in A$,

(b) find the minimum value of $|w - z|$, giving your answer in an exact simplified form. **[4]**

7. The locus of points z satisfies

$$|z - 3a| = 2|z + ai|$$

where a is a non-zero integer.
The locus is a circle whose centre lies in the first quadrant and which encloses a region of area 40π.

(a) Find the value of a. **[8]**

(b) Find the exact least value of $|z|$ for points z on the circle. **[2]**

8. The locus of points $z = x + iy$ that satisfy

$$\arg\left(\frac{z-3+2i}{z-3-6i}\right) = \frac{3\pi}{4}$$

is an arc of a circle C.

(a) On an Argand diagram, sketch the locus of z. **[3]**

(b) Explain why the centre of C has y-coordinate 2. **[1]**

(c) Find the exact radius of C. **[2]**

(d) Find the x-coordinate of the centre of C and hence find the greatest value of $\text{Re}(z)$ for points on the locus. **[3]**

9. In an Argand diagram, the curve C is given by

$$\arg\left(\frac{z-6}{z+2i}\right) = \frac{3\pi}{4}$$

and represents an arc of a circle.
Given that $z = x + iy$, where x and y are real numbers,

(a) show that this circle has equation

$$x^2 + y^2 + ax + by + c = 0$$

where a, b and c are integers to be determined, and hence find the centre and the exact radius of the circle. **[5]**

(b) determine the exact maximum value of $|z - 8 + 8i|$ for points z on C. **[4]**

10. A complex number z is represented by the point P on an Argand diagram.
Given that

$$\arg\left(\frac{z-8}{z-2}\right) = \frac{\pi}{3}$$

(a) sketch the locus of P as z varies, **[3]**

(b) find the exact maximum value of $\arg z$ for points on this locus, and find, in the form $a + ib$, where a and b are exact real numbers, the value of z at which this maximum occurs. **[6]**

11. A circle C in the complex plane has equation

$$|z - (4 - 2i)| = \alpha\,|z - (4 + i)|$$

where α is a real constant with $\alpha > 1$.
Given that the real axis is a tangent to C,

(a) sketch, on an Argand diagram, the circle C, **[2]**

(b) explain why $\alpha = 2$. **[1]**

The circle C is contained in the region

$$R = \{z \in \mathbb{C} \mid 0 \leq \arg z \leq \beta\}$$

(c) Determine the least possible value of β, giving your answer in an exact form. **[4]**

(d) For this least value of β, find, in the form $x + iy$, the complex number z on C for which $\arg z = \beta$. **[3]**

12. The circle C has equation

$$|z + 7 - 2i| = 3\,|z - 1 - 2i|$$

where $z = x + iy$.

(a) Show that an equation for C is

$$x^2 + y^2 - 4x - 4y - 1 = 0 \qquad \textbf{[3]}$$

The half-line with equation

$$\arg(z - ai) = \frac{\pi}{3}$$

where a is a real constant, is a tangent to C.

(b) Determine the exact value of a, and find the complex number representing the point at which the half-line touches C, giving your answer in the form $p + iq$, where p and q are exact real numbers. **[7]**