OPTIONAL PURE / MISCELLANEOUS OPTIONAL PURE

Transformations of the Complex Plane

Mark scheme 12 questions · 115 marks

EXAM BOARDS Edexcel FP2 9FM0/4A

Questions

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	Total	**115**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last accuracy mark of the working that reaches a printed result: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

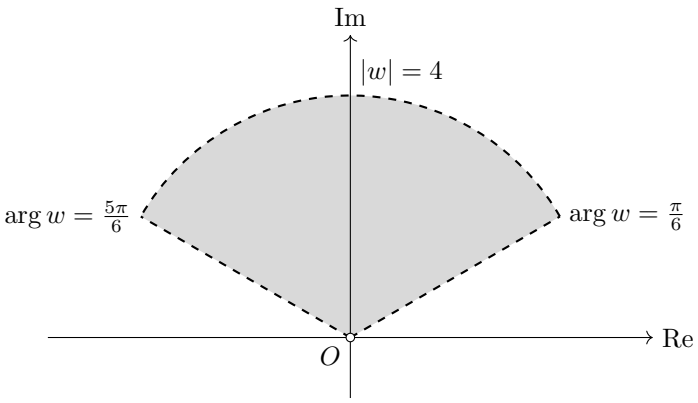
- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

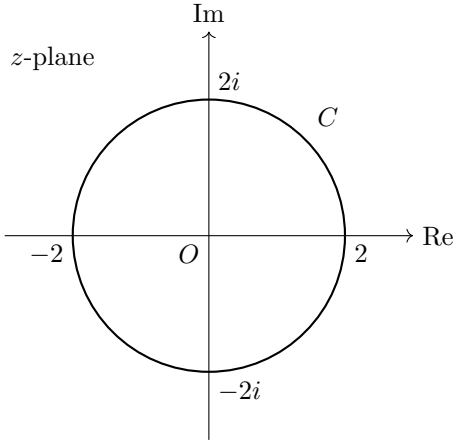
Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

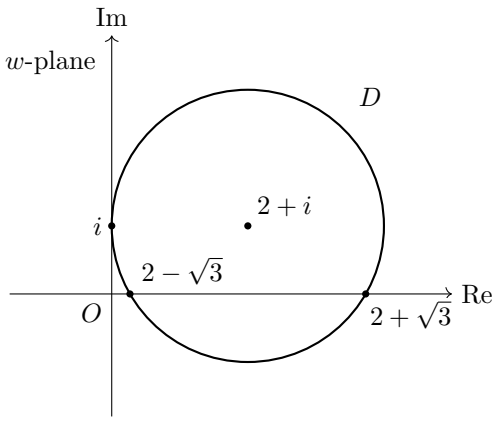
Question	Scheme	Marks	Notes
1(a)	Modulus squared: $\lvert w\rvert = \lvert z\rvert^2$	**B1**	In words or symbols.
	Argument doubled: $\arg w = 2\arg z + \ldots$	**B1**	In words or symbols.
	Rotation through $\frac{\pi}{2}$ anticlockwise about O	**B1**	Needs angle, direction and centre O, in words or as $\arg w = 2\arg z + \frac{\pi}{2}$; $90°$ accepted. Accept rotation through $\frac{\pi}{4}$ anticlockwise about O applied before the squaring.
		[3]	
1(b)			
	Shape	**B1**	Sector at O, symmetric about the positive imaginary axis, straight edges marked $\arg w = \frac{\pi}{6}$ and $\frac{5\pi}{6}$, or $30°$ and $150°$. Dashed edges not required.
	Centre and radius	**B1**	Curved boundary an arc centred at O, radius 4 marked.
		[2]	
	Total for Question 1	**5**	

Question	Scheme	Marks	Notes
2	$w(z-1)=2z+i \Rightarrow z=\ldots$	**M1**	Multiplies by $z-1$ and collects the z terms to make z the subject. Condone sign slips.
	$z=\frac{w+i}{w-2}$	**A1**	
	$z=\frac{(u+(v+1)i)((u-2)-iv)}{(u-2)^2+v^2}$	**M1**	Substitutes $w=u+iv$ into their $z=\frac{w+p}{w+q}$ and multiplies numerator and denominator by the conjugate of the denominator. Condone sign slips.
	$\mathrm{Re}(z)=\frac{u(u-2)+v(v+1)}{(u-2)^2+v^2}=1$ $\Rightarrow u^2-2u+v^2+v=u^2-4u+4+v^2$	**dM1**	Sets their real part equal to 1, clears the denominator and expands to an equation in u and v. Condone sign slips.
	$2u+v=4$	**A1**	In the form $au+bv=c$ with integers, or any integer multiple, such as $4u+2v=8$. Not for $v=4-2u$.
	Total for Question 2	**5**	

Question	Scheme	Marks	Notes
3(a)	$w = (x+iy)^2 + 2(x+iy) = \ldots$	**M1**	Substitutes $z = x+iy$ and expands $(x+iy)^2$ to at least three terms. Condone sign slips.
	$= x^2 + 2ixy - y^2 + 2x + 2iy$	**A1**	Correct expansion with $i^2 = -1$ used, real and imaginary terms need not be collected.
	$v = 2xy + 2y$	**A1**	oe, such as $2y(x+1)$. Must be free of i.
	$u = x^2 - y^2 + 2x$	**A1 AG**	Equates real parts of a correct expansion, reaching the printed result without error.
		[4]	
3(b)	$x = 1$: $u = 1 - y^2 + 2, \quad v = 2y + 2y$	**M1**	Substitutes $x = 1$ into u and their v from (a), or puts $z = 1 + iy$ into $w = z^2 + 2z$.
	$u = 3 - y^2, \quad v = 4y$	**A1**	
	$y = \frac{v}{4} \Rightarrow u = 3 - \left(\frac{v}{4}\right)^2$	**dM1**	Eliminates y between their u and v, as by substituting for y or y^2.
	$v^2 = 48 - 16u$	**A1**	In the form $v^2 = a + bu$. Not for $16u = 48 - v^2$, nor a and b stated alone.
		[4]	
	Total for Question 3	**8**	

Question	Scheme	Marks	Notes
4(a)	$w(z-1)=(1+i)z+2-i \Rightarrow z=\ldots$	**M1**	Multiplies by $z-1$ and collects the terms in z to make z the subject. Condone sign slips.
	$z=\frac{w+2-i}{w-(1+i)}$	**A1**	oe
	$\lvert w+2-i\rvert = 2\lvert w-(1+i)\rvert$	**dM1**	Sets the modulus of their z equal to 2, giving an equation in moduli of linear expressions in w.
	$(u+2)^2+(v-1)^2=4\left[(u-1)^2+(v-1)^2\right]$	**M1 A1**	**M1** Writes $w=u+iv$, or $x+iy$, and squares both moduli as sums of squares of real and imaginary parts. Condone sign slips. **A1** Correct equation, oe.
	$3u^2-12u+3(v-1)^2=0$ $\Rightarrow (u-\ldots)^2+(v-\ldots)^2=\ldots$	**dM1**	Expands and collects their equation, then completes the square to reach the form $(u-a)^2+(v-b)^2=k$, or reads the centre and radius from $u^2+v^2+2gu+2fv+k=0$. Condone arithmetic slips.
	$(u-2)^2+(v-1)^2=4$ Centre $2+i$, radius 2	**A1**	Correct equation, with centre $2+i$ or $(2,1)$ and radius 2 stated.
		[7]	
4(b)			
	Centre and radius	**B1**	Circle centred at O through ± 2 and $\pm 2i$, on its own Argand diagram.

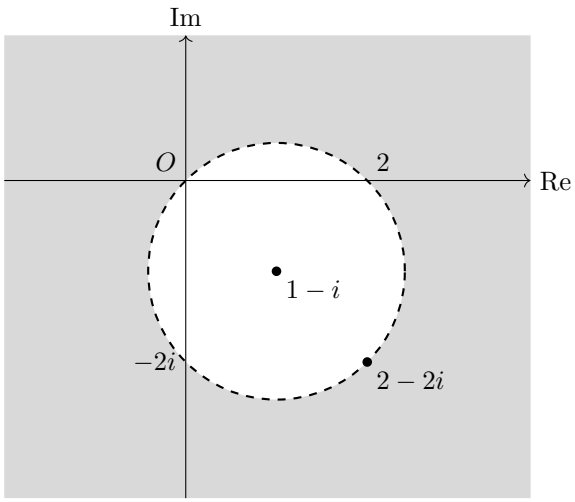
Question 4 continued

Question	Scheme	Marks	Notes
			
	Centre and radius	**B1**	Circle centred at $2+i$ with radius 2, touching the imaginary axis at i, on a separate Argand diagram.
		[2]	
	Total for Question 4	**9**	

Question	Scheme	Marks	Notes
5	$z(z+i) = 5z - p \Rightarrow z^2 + (i-5)z + p = 0$ $(2+2i)^2 + (i-5)(2+2i) + p = 0 \Rightarrow p = \ldots$	**M1**	Sets $w = z$, clears the fraction and substitutes $z = 2 + 2i$ to reach an equation in p. Condone slips in expanding.
	$p = 12$	**A1**	
	$(2+2i) + \beta = -(i-5) \Rightarrow \beta = \ldots$	**M1**	Uses the sum or product of roots of $z^2 + (i-5)z +$ "12" $= 0$, or solves it by any means, for the second root. Condone sign slips.
	$\beta = 3 - 3i$	**A1**	Accept $(3, -3)$.
	Total for Question 5	**4**	

Question	Scheme	Marks	Notes
6(a)	$w(z-i)=z+1 \Rightarrow z=\ldots$	**M1**	Multiplies out and collects the z terms to make z the subject. Condone sign slips.
	$z=\frac{1+iw}{w-1}$	**A1**	oe
	$\left\lvert\frac{1+iw}{w-1}-1\right\rvert=2$ $\Rightarrow \frac{\lvert(i-1)w+2\rvert}{\lvert w-1\rvert}=2$	**M1**	Substitutes their z into $\lvert z-1\rvert=2$ and takes moduli of a single fraction in w. Condone sign slips.
	$\frac{\lvert i-1\rvert\,\lvert w-(1+i)\rvert}{\lvert w-1\rvert}=2$ $\Rightarrow \lvert w-(1+i)\rvert=\sqrt{2}\,\lvert w-1\rvert$	**A1 AG**	Correct substituted relation leading without error to the printed result; combined steps accepted.
		[4]	
6(b)	$\lvert(u-1)+(v-1)i\rvert=\sqrt{2}\,\lvert(u-1)+vi\rvert$	**M1**	Substitutes $w=u+iv$ (any letters) into the result of (a), with real and imaginary parts grouped in each modulus.
	$(u-1)^2+(v-1)^2=2\left[(u-1)^2+v^2\right]$	**M1 A1**	**M1** Squares both sides, writing each squared modulus as a sum of two squares. Condone sign slips. **A1** oe, expanded or not.
	$u^2+v^2-2u+2v=0$ $\Rightarrow (u-1)^2+(v+1)^2=\ldots$	**M1**	Collects terms and completes the square in both u and v, or reads the centre and radius from the collected equation. Condone sign slips.
	$(u-1)^2+(v+1)^2=2$ Centre $1-i$	**A1**	Correct centre, oe such as $(1,-1)$, or radius $\sqrt{2}$ or awrt 1.41.
	Circle, radius $\sqrt{2}$	**A1**	cso. Correct centre and radius, with C stated to be a circle.
		[6]	

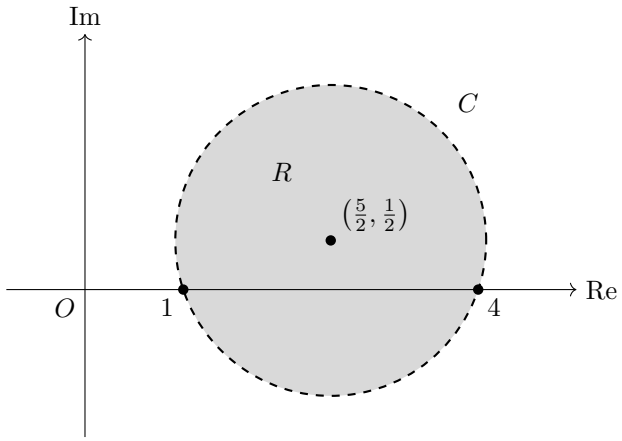
Question 6 continued

Question	Scheme	Marks	Notes
6(c)			
	Centre and radius	**B1ft**	Circle centred at their centre from (b), with the centre or points on it marked and the radius roughly in proportion; the correct circle passes through O.
	Shading	**B1ft**	Region outside their circle shaded, the inside left clear. ft their circle from (b).
		[2]	
	Total for Question 6	**12**	

Question	Scheme	Marks	Notes
7(a)	$w(z-2)=z+2i \Rightarrow z=\ldots$	**M1**	Multiplies out and collects the z terms to make z the subject. Condone sign slips.
	$z=\frac{2w+2i}{w-1}$	**A1**	oe.
	$\left\lvert\frac{2w+2i}{w-1}\right\rvert=2 \Rightarrow \lvert w+i\rvert=\lvert w-1\rvert$	**dM1**	Sets the modulus of their z equal to 2 and splits it into moduli of numerator and denominator.
	$u^2+(v+1)^2=(u-1)^2+v^2$	**M1**	Substitutes $w=u+iv$ (any letters) into their modulus equation and squares both moduli, or uses the perpendicular bisector of their two points. Condone sign slips.
	$v=-u$	**A1**	oe, such as $u+v=0$. Condone x and y for u and v.
		[5]	
7(b)	$w-i=\frac{z+2i}{z-2}-i$ $=\frac{z+2i-i(z-2)}{z-2}$	**M1**	Subtracts i from the transformation over a common denominator, or substitutes the transformation into $\lvert w-i\rvert=2$. Condone sign slips.
	$=\frac{(1-i)z+4i}{z-2}$	**A1**	oe.
	$\lvert(1-i)z+4i\rvert=2\lvert z-2\rvert$	**dM1**	Sets the modulus equal to 2 and splits it into moduli of numerator and denominator.
	$(x+y)^2+(y-x+4)^2$ $=4\left[(x-2)^2+y^2\right]$	**M1 A1**	**M1** Substitutes $z=x+iy$, separates real and imaginary parts on each side and squares both moduli. Condone sign slips. **A1** Correct equation, unsimplified or oe.
	$x^2+y^2-4x-4y=0$ $\Rightarrow (x-2)^2+(y-2)^2=\ldots$	**dM1**	Simplifies to a circle equation and completes the square in both variables, or reads the centre and radius from $x^2+y^2+2gx+2fy+k=0$.
	$a=2,\ b=2,\ c=2\sqrt{2}$	**A1**	All three; $c=\sqrt{8}$ accepted.
		[7]	
	Total for Question 7	**12**	

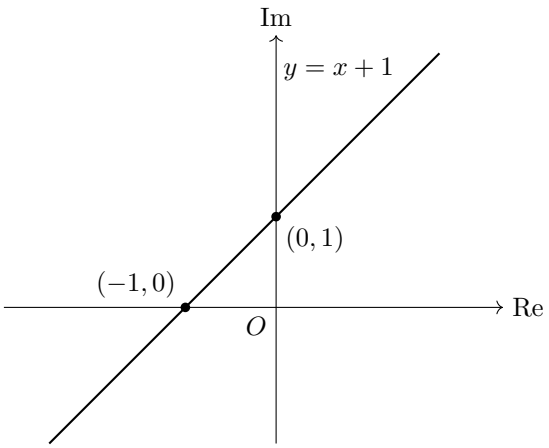
Question	Scheme	Marks	Notes
8(a)	$w = \frac{(x+iy)(x-(y-2)i)}{(x+(y-2)i)(x-(y-2)i)}$	**M1**	Substitutes $z = x + iy$ and multiplies numerator and denominator by the conjugate of their denominator, or otherwise makes the denominator real. Condone sign slips.
	$(x + (y-2)i)(x - (y-2)i) = x^2 + (y-2)^2$	**A1**	oe, such as $x^2 + y^2 - 4y + 4$.
	$x^2 - x(y-2)i + xyi - y(y-2)i^2 = \ldots$	**dM1**	Expands the numerator using $i^2 = -1$ and collects real and imaginary parts. Condone sign slips.
	$v = \frac{2x}{x^2+(y-2)^2}$	**A1**	oe; denominator need not be expanded.
	$x^2 + y^2 - 2y + 2xi$ $\Rightarrow u = \frac{x^2+y^2-2y}{x^2+(y-2)^2}$	**A1 AG**	Correct expanded numerator over the real denominator, leading without error to the printed u.
		[5]	
8(b)	$wz - z = 2iw \Rightarrow z = \ldots$	**M1**	Makes z the subject, collecting the z terms on one side. Condone sign slips.
	$z = \frac{2iw}{w-1}$	**A1**	oe, such as $\frac{2w}{i(1-w)}$.
	$z = \frac{(-2v+2ui)((u-1)-iv)}{(u-1)^2+v^2} = \ldots$	**M1**	Substitutes $w = u + iv$ into their z and makes the denominator real to equate real and imaginary parts. Condone sign slips.
	$x = \frac{2v}{(u-1)^2+v^2}$, $y = \frac{2(u^2+v^2-u)}{(u-1)^2+v^2}$	**A1**	Both correct, oe.
	Sets $y = x$: $u^2 + v^2 - u = v$ $\Rightarrow \left(u - \frac{1}{2}\right)^2 + \left(v - \frac{1}{2}\right)^2 = \ldots$	**dM1**	Equates their x and y and completes the square in both u and v, or reads the centre and radius from $u^2 + v^2 + 2gu + 2fv + k = 0$. Condone sign slips.
	$\left(u - \frac{1}{2}\right)^2 + \left(v - \frac{1}{2}\right)^2 = \frac{1}{2}$	**A1**	oe in completed-square form, or the correct general form $u^2 + v^2 - u - v = 0$.
	Centre $\frac{1}{2} + \frac{1}{2}i$, radius $\frac{\sqrt{2}}{2}$	**A1**	Centre as a point or complex number; radius $\frac{1}{\sqrt{2}}$ oe, or awrt 0.707. Both needed.
		[7]	
	Total for Question 8	**12**	

Question	Scheme	Marks	Notes
9(a)	$w(z+i) = z - 1 \Rightarrow z = \ldots$	**M1**	Multiplies up and collects the z terms to make z the subject. Condone sign slips.
	$z = \frac{1+iw}{1-w}$	**A1**	oe
	$z = \frac{1+i(1+iv)}{1-(1+iv)} = \frac{-1+(1-v)i}{v}$ $\Rightarrow x = -\frac{1}{v},\ y = \frac{1}{v} - 1$	**M1**	Substitutes $w = 1 + iv$ into their z in terms of w, makes the denominator real and equates real and imaginary parts. Condone sign slips.
	$\frac{1}{v} = -x \Rightarrow y = \ldots$	**dM1**	Eliminates v between their x and y to reach an equation in x and y only.
	$x + y + 1 = 0$	**A1**	Any equivalent Cartesian form, such as $y = -x - 1$.
		[5]	
9(b)(i)	$z = \frac{(1-v)+iu}{(1-u)-iv} \times \frac{(1-u)+iv}{(1-u)+iv}$	**M1**	Substitutes $w = u + iv$ into their z from (a) and multiplies numerator and denominator by the conjugate of the denominator. Condone sign slips.
	$x = \frac{1-u-v}{(1-u)^2+v^2}$ $y = \frac{u+v-u^2-v^2}{(1-u)^2+v^2}$	**A1**	Both correct; numerators need not be simplified.
	$y = x$: "$u + v - u^2 - v^2$" = "$1 - u - v$"	**dM1**	Sets their y equal to their x, or equates their numerators, to form an equation in u and v.
	$(u-1)^2 + (v-1)^2 = 1$	**A1**	Any correct form, such as $u^2 + v^2 - 2u - 2v + 1 = 0$.
	Centre $1 + i$, radius 1	**A1**	Both needed. Accept centre $(1, 1)$.
		[5]	
9(b)(ii)	$\lvert w\rvert_{\max} = \lvert 1 + i\rvert + 1 = \ldots$	**M1**	Adds their radius to the distance from the origin to their centre, or maximises $\lvert w\rvert$ on their circle by any valid method.
	$\sqrt{2} + 1$	**A1**	
		[2]	
	Total for Question 9	**12**	

Question	Scheme	Marks	Notes
10(a)	$(x-3)^2+(y-1)^2=(x-1)^2+(y+1)^2$	**M1**	Squares both moduli with $z=x+iy$, or uses the perpendicular bisector of $(3,1)$ and $(1,-1)$, or substitutes z as the subject into both moduli. Condone sign slips.
	$x+y=2$	**A1**	oe. Direct route, a correct modulus equation in w.
	$w(z-1)=z+2i \Rightarrow z(w-1)=w+2i$	**M1**	Rearranges to make z the subject, collecting the z terms on one side. Condone sign slips.
	$z=\frac{w+2i}{w-1}$	**A1**	
	$x+iy=\frac{u+(v+2)i}{(u-1)+iv}\times\frac{(u-1)-iv}{(u-1)-iv}$	**M1**	Substitutes $w=u+iv$ and multiplies numerator and denominator by the conjugate of their denominator, or on the direct route writes $w=u+iv$ and squares both moduli. Condone sign slips.
	$x=\frac{u^2-u+v^2+2v}{(u-1)^2+v^2}$ $y=\frac{2u-v-2}{(u-1)^2+v^2}$	**A1**	Both correct, oe; denominator need not be expanded. On the direct route, a correct squared equation in u and v.
	"u^2-u+v^2+2v" + "$2u-v-2$" $=2\left((u-1)^2+v^2\right)$	**dM1**	Substitutes their x and y into their line and clears the denominator, or on the direct route collects to a circle equation. Condone sign slips.
	$u^2-5u+v^2-v+4=0$	**A1**	oe, any correct equation in u and v.
	$\left(u-\frac{5}{2}\right)^2+\left(v-\frac{1}{2}\right)^2=\frac{5}{2}$ Centre $\left(\frac{5}{2},\frac{1}{2}\right)$, radius $\frac{\sqrt{10}}{2}$	**A1**	Both correct. Radius exact, $\sqrt{\frac{5}{2}}$ oe.
		[9]	
10(b)			

Question 10 continued

Question	Scheme	Marks	Notes
	Centre and radius	**B1ft**	Circle with centre in the correct position, marked or evident, and radius roughly in proportion. ft their centre and radius from (a).
	Shading	**B1**	Interior of the circle shaded and labelled R.
		[2]	
	Total for Question 10	**11**	

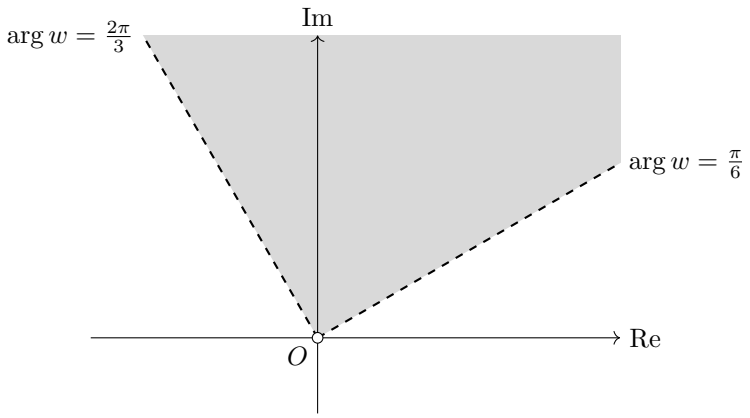
Question	Scheme	Marks	Notes
11(a)	$(x-1)^2+y^2=(x+1)^2+(y-2)^2$	**M1**	Substitutes $z=x+iy$ and squares both moduli, or uses the perpendicular bisector of $(1,0)$ and $(-1,2)$. Condone sign slips.
	$y=x+1$	**A1**	oe
			
	Intercepts	**B1ft**	ft their line from (a): a straight line with both axis intersections marked, correct for their equation.
		[3]	
11(b)	$w(z+i)=2z \Rightarrow z=\ldots$	**M1**	Rearranges $w=\frac{2z}{z+i}$ to make z the subject. Condone sign slips.
	$z=\frac{iw}{2-w}$	**A1**	oe
	$\left\lvert \text{“}\frac{iw}{2-w}\text{”}-1\right\rvert=\left\lvert \text{“}\frac{iw}{2-w}\text{”}+1-2i\right\rvert$	**dM1**	Substitutes their z in terms of w into both moduli of the locus equation, or makes the denominator of their z real to find x and y in terms of u and v.
	$\lvert(1+i)w-2\rvert=\lvert(-1+3i)w+2-4i\rvert$	**A1**	oe, such as $\lvert w-(1-i)\rvert=\sqrt{5}\left\lvert w-\left(\frac{7}{5}+\frac{1}{5}i\right)\right\rvert$, or x and y in terms of u and v both correct.
	$(u-1)^2+(v+1)^2$ $=5\left[\left(u-\frac{7}{5}\right)^2+\left(v-\frac{1}{5}\right)^2\right]$	**M1 A1**	**M1** Substitutes $w=u+iv$ into their modulus equation and squares, with real and imaginary parts on each side, or substitutes their x and y into their line from (a). Condone sign slips. **A1** Any correct equation in u and v, oe.

Question 11 continued

Question	Scheme	Marks	Notes
	$u^2+v^2-3u-v+2=0$ $\Rightarrow \left(u-\frac{3}{2}\right)^2+\left(v-\frac{1}{2}\right)^2=\ldots$	**dM1**	Collects to equal coefficients of u^2 and v^2 and completes the square in both variables, or reads the centre and radius from $u^2+v^2+2gu+2fv+k=0$.
	$\left\lvert w-\left(\frac{3}{2}+\frac{1}{2}i\right)\right\rvert=\frac{\sqrt{2}}{2}$	**A1**	oe, or $p=\frac{3}{2}$, $q=\frac{1}{2}$, $r=\frac{\sqrt{2}}{2}$ stated; accept $r=\frac{1}{\sqrt{2}}$ or awrt 0.707.
		[8]	
	Total for Question 11	**11**	

Question	Scheme	Marks	Notes
12(a)	$w = \frac{\left(-1+\sqrt{3}\,i\right)(iy-3i)}{iy+3i}$	**M1**	Substitutes a general point iy of the imaginary axis, in any letter, into T.
	$w = \left(-1+\sqrt{3}\,i\right)\frac{y-3}{y+3}$	**A1**	w as a real multiple of $-1+\sqrt{3}\,i$, or correct u and v in terms of y.
	$u = -\frac{y-3}{y+3}, \quad v = \sqrt{3}\,\frac{y-3}{y+3}$ $\Rightarrow v = -\sqrt{3}\,u$	**A1 AG**	Eliminates the parameter without error to $v = -\sqrt{3}\,u$, or argues that real multiples of $-1+\sqrt{3}\,i$ lie on that line.
		[3]	
12(b)	$w(z+3i) = \left(-1+\sqrt{3}\,i\right)(z-3i)$ $\Rightarrow z\left(w+1-\sqrt{3}\,i\right) = \ldots$	**M1**	Multiplies up and collects the terms in z to make z the subject. Condone sign slips.
	$z = \frac{-3i\left(w-1+\sqrt{3}\,i\right)}{w+1-\sqrt{3}\,i}$	**A1**	oe
	$\lvert z\rvert = 3 \Rightarrow \left\lvert w-1+\sqrt{3}\,i\right\rvert = \left\lvert w+1-\sqrt{3}\,i\right\rvert$	**dM1**	Applies $\lvert z\rvert = 3$ to their z, taking moduli of numerator and denominator, to form an equation in moduli of w.
	$(u-1)^2 + \left(v+\sqrt{3}\right)^2$ $= (u+1)^2 + \left(v-\sqrt{3}\right)^2$	**M1**	Writes $w = u+iv$ and squares both moduli as sums of squares, or identifies the perpendicular bisector of their two points. Condone sign slips.
	$v = \frac{u}{\sqrt{3}}$	**A1**	oe in u and v, such as $u = \sqrt{3}\,v$.
		[5]	
12(c)	$\lvert z\rvert > 3 \Rightarrow \left\lvert w-1+\sqrt{3}\,i\right\rvert > \left\lvert w+1-\sqrt{3}\,i\right\rvert$ $\Rightarrow (u-1)^2 + \left(v+\sqrt{3}\right)^2 > \ldots$	**M1**	Uses $\lvert z\rvert > 3$ with their z to form an inequality in u and v, or chooses the side of their line from (b) by testing a point.
	$v > \frac{u}{\sqrt{3}}$	**A1**	oe, with the inequality the right way round.
	$\mathrm{Re}(z) = \frac{6\left(\sqrt{3}\,u+v\right)}{\left\lvert w+1-\sqrt{3}\,i\right\rvert^2} > 0$	**M1**	Finds $\mathrm{Re}(z)$ in terms of u and v and sets it > 0, or chooses the side of the line from (a) by testing a point. Condone expansion slips.
	$v > -\sqrt{3}\,u$	**A1**	oe, with the inequality the right way round.
	$\left\{w \in \mathbb{C} \;\middle\vert\; \frac{\pi}{6} < \arg w < \frac{2\pi}{3}\right\}$	**A1**	Exact, strict inequalities, in the form asked for. $\alpha = \frac{\pi}{6}$, $\beta = \frac{2\pi}{3}$ stated alone does not score.

Question 12 continued

Question	Scheme	Marks	Notes
			
	Shading	**B1ft**	Sector from O between half-lines at their α and β, correctly shaded. ft their α, β.
		[6]	
	Total for Question 12	**14**	