OPTIONAL PURE / MISCELLANEOUS OPTIONAL PURE

Further Loci in the Argand Diagram

Mark scheme 12 questions · 105 marks

EXAM BOARDS Edexcel FP2 9FM0/4A

Questions

Question	Marks by part	Marks	Page
Question 1		4	4
Question 2	(a) 6 (b) 3	9	5
Question 3	(a) 6 (b) 2 (c) 2	10	6
Question 4	(a) 6 (b) 4 (c) 2	12	8
Question 5	(a) 3 (b) 3	6	9
Question 6	(a) 3 (b) 4	7	10
Question 7	(a) 8 (b) 2	10	11
Question 8	(a) 3 (b) 1 (c) 2 (d) 3	9	12
Question 9	(a) 5 (b) 4	9	14
Question 10	(a) 3 (b) 6	9	15
Question 11	(a) 2 (b) 1 (c) 4 (d) 3	10	17
Question 12	(a) 3 (b) 7	10	19
	Total	**105**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last accuracy mark of the working that reaches a printed result: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

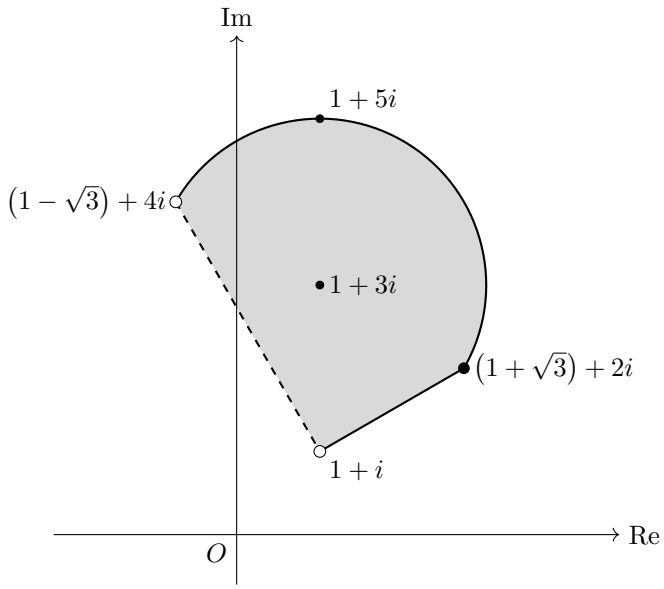
- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

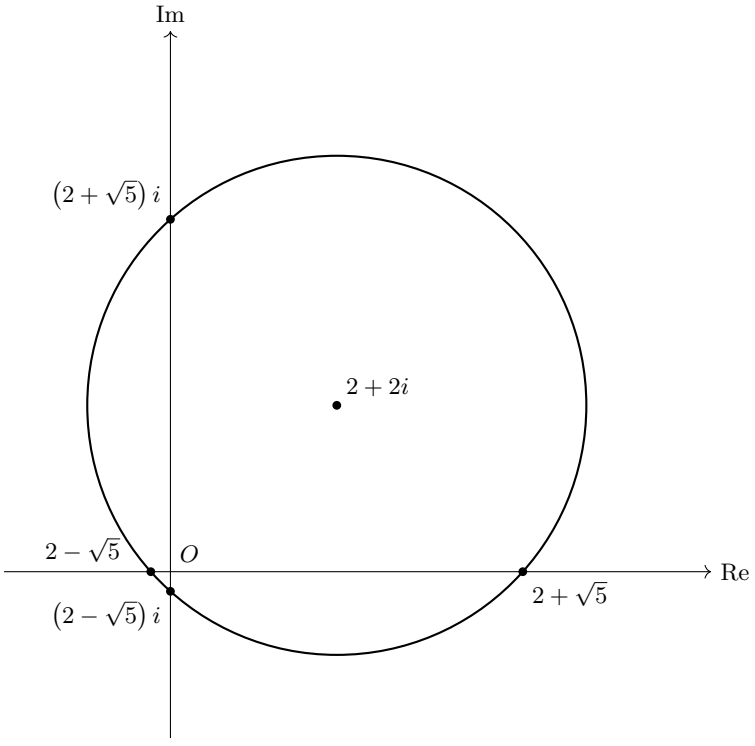
- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

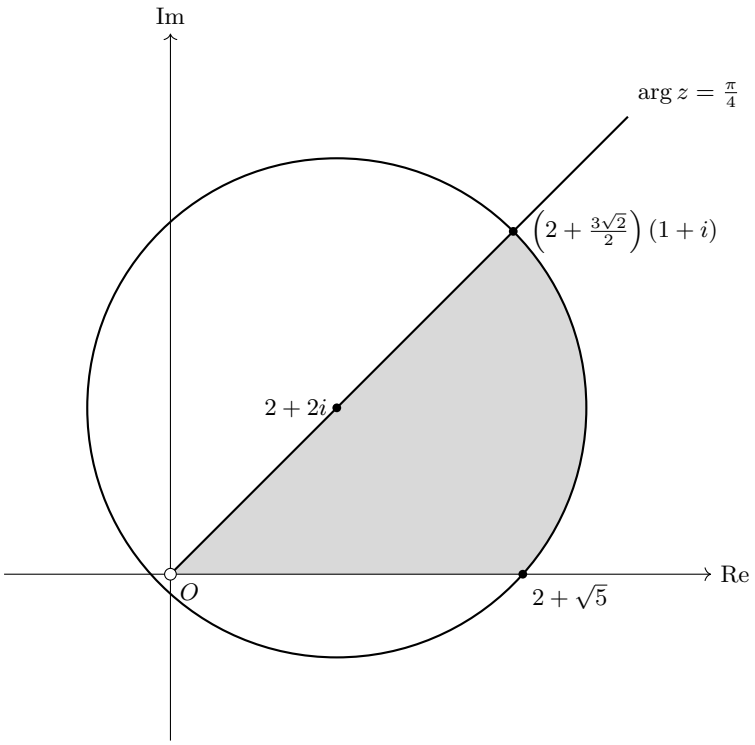
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1			
	Half-line	**B1**	Two lines from $1+i$ at $\frac{\pi}{6}$ and $\frac{2\pi}{3}$ to the positive real direction, shown by angles, points on them or clearly correct directions. Segments to the circle suffice.
	Centre and radius	**B1**	Circle or arc with centre $1+3i$ and radius 2, through $1+i$, shown by labels, a scale or a point such as $1+5i$.
	Shading	**B1**	Only the part of the disc between the two lines, containing $1+3i$, shaded.
	Half-line	**B1**	Line at $\frac{\pi}{6}$ and arc solid, line at $\frac{2\pi}{3}$ dashed. Needs the correct region. Condone end points not shown as open circles.
	Total for Question 1	**4**	

Question	Scheme	Marks	Notes
2(a)	$\lvert(x+3)+iy\rvert = 2\lvert x+(y-3)i\rvert$	**M1**	Substitutes $z = x+iy$ in both moduli and groups real and imaginary parts. Condone sign slips; implied by the next line.
	$(x+3)^2+y^2 = 4\left[x^2+(y-3)^2\right]$	**M1 A1**	**M1** Forms an equation with each squared modulus as a sum of squares of real and imaginary parts. Condone sign slips and the 2 not squared. **A1** Any correct equation in x and y, expanded or not.
	$x^2+y^2-2x-8y+9=0$ $\Rightarrow (x-1)^2+(y-4)^2 = \ldots$	**M1**	Completes the square in x and y on their equation with equal x^2 and y^2 coefficients, or reads the centre from its general form. Condone sign slips.
	Circle, centre $(1,4)$, radius $2\sqrt{2}$	**A1 A1**	**A1** Correct centre, $(1,4)$ or $1+4i$, or correct radius, $\sqrt{8}$ oe or awrt 2.83. **A1** cso. Centre and radius both correct.
		[6]	
2(b)	$\sqrt{(4-1)^2+(1-4)^2} \pm$ "$2\sqrt{2}$"	**M1**	Finds the distance from $(4,1)$ to their centre and adds or subtracts their radius. One of the two is enough.
	Greatest $= 5\sqrt{2}$	**A1**	$5\sqrt{2}$ oe, or awrt 7.07.
	Least $= \sqrt{2}$	**A1**	$\sqrt{2}$ oe, or awrt 1.41.
		[3]	
	Total for Question 2	**9**	

Question	Scheme	Marks	Notes
3(a)	$\lvert(x-2)+(y+7)i\rvert = 3\lvert(x-2)+(y-1)i\rvert$	**M1**	Substitutes $z = x + iy$ in both moduli, grouping real and imaginary parts. Condone sign slips; implied by the next line.
	$(x-2)^2+(y+7)^2 = 9\left[(x-2)^2+(y-1)^2\right]$	**M1 A1**	**M1** Squares both moduli as sums of squares of real and imaginary parts. Condone sign slips and the 3 not squared. **A1** Correct equation, oe, need not be simplified.
	$(x-2)^2+y^2-4y-5=0$ $\Rightarrow (x-2)^2+(y-2)^2 = \ldots$	**M1**	Collects terms and completes the square in x and y, or reads centre and radius from the general form of a circle. Condone arithmetic slips.
	$(x-2)^2+(y-2)^2=9$ Centre $2+2i$, radius 3	**A1 A1**	**A1** Correct centre or correct radius. **A1** cso. Centre and radius both correct; centre as $(2,2)$ or $2+2i$.
		[6]	
3(b)			
	Shape	**B1**	Circle with centre in the first quadrant, enclosing the origin.
	Centre and radius	**B1**	Centre $2+2i$ labelled, with radius 3 shown or the axis intercepts $2\pm\sqrt{5}$ and $(2\pm\sqrt{5})\,i$ marked.
		[2]	

Question 3 continued

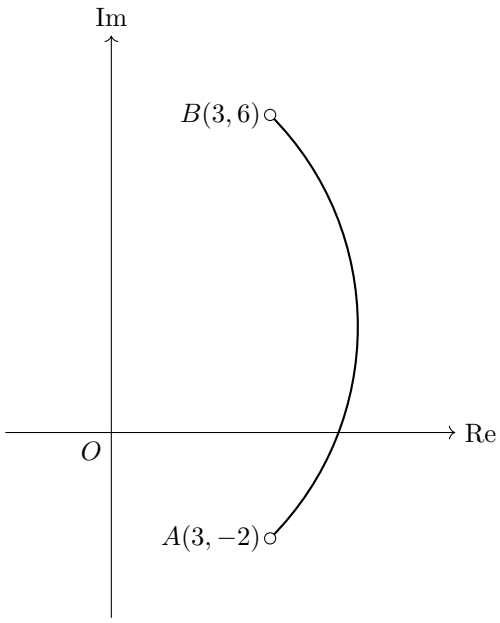
Question	Scheme	Marks	Notes
3(c)			
	Half-line	**B1**	Half-line from O at angle $\frac{\pi}{4}$ to the positive real axis, passing through $2+2i$.
	Shading	**B1ft**	Only the region inside their circle from (b), between the positive real axis and the half-line, shaded. ft their circle from (b).
		[2]	
	Total for Question 3	**10**	

Question	Scheme	Marks	Notes
4(a)	$\lvert(x-5)+(y+1)i\rvert = 2\lvert(x+1)+(y-2)i\rvert$	**M1**	Substitutes $z = x+iy$ in both moduli and groups real and imaginary parts. Condone sign slips; implied by the next line.
	$(x-5)^2+(y+1)^2 = 4\left[(x+1)^2+(y-2)^2\right]$	**M1 A1**	**M1** Squares both moduli as sums of squares of real and imaginary parts. Condone sign slips and the 2 not squared. **A1** Any correct equation in x and y, expanded or not.
	$x^2+y^2+6x-6y-2=0$ $\Rightarrow (x+3)^2+(y-3)^2 = \ldots$	**M1**	Completes the square in both x and y on their equation with equal square coefficients, or reads the centre and radius from its general form.
	C: $-3+3i$, radius $\sqrt{20} = 2\sqrt{5}$	**A1 A1**	**A1** Centre or radius correct; $(-3,3)$ accepted here. **A1** Both, with the centre as the complex number $-3+3i$ and the radius exact, $2\sqrt{5}$ or $\sqrt{20}$.
		[6]	
4(b)	$y = x+4,\ x > -7$	**B1**	Line oe, such as $y+3 = x+7$; no restriction needed.
	$(x+3)^2+(x+1)^2 = 20$ $\Rightarrow x^2+4x-5=0 \Rightarrow x = \ldots$	**M1**	Substitutes their line of the form $y = x+k$ into their circle to form a quadratic in one variable and solves it by any means.
	$x = 1,\ x = -5$	**A1**	Or $y = 5$, $y = -1$.
	$1+5i$ and $-5-i$	**A1**	Both, as complex numbers, and no others; no check against the half-line or a sketch needed.
		[4]	
4(c)	$RS = 6\sqrt{2},\ CM = \sqrt{2}$ $\frac{1}{2}\times 6\sqrt{2}\times\sqrt{2} = \ldots$	**M1**	Uses a complete area method with their C, R, S from (a) and (b), such as base RS with perpendicular height from C, the sine formula or a coordinate formula.
	6	**A1**	cao
		[2]	
	Total for Question 4	**12**	

Question	Scheme	Marks	Notes
5(a)	$\lvert(-1-3i)-(x+i)\rvert = \lvert(6+4i)-(x+i)\rvert$ $\Rightarrow (x+1)^2+4^2=(6-x)^2+3^2 \Rightarrow x=\ldots$	**M1**	Equates distances or squared distances from $x+i$ to z_1 and z_3, or uses the perpendicular bisector of z_1z_3, to find x. Condone sign slips.
	Centre $2+i$	**A1**	As a complex number, not $(2,1)$.
	$4^2+(a-1)^2=25 \Rightarrow a=-2$	**B1**	$a=-2$ only, from the radius or from z_2 and z_3 symmetric about $\text{Im}(z)=1$; B0 if $a=4$ is kept.
		[3]	
5(b)	$z_1-(2+i)=-3-4i,\ z_2-(2+i)=4-3i$, perpendicular $\Rightarrow$ angle at circumference $=\frac{1}{2}\times\frac{\pi}{2}$	**M1**	Finds the angle chord z_1z_2 subtends at the centre and halves it, or finds the angle $z_1z_3z_2$ at z_3 directly.
	$\frac{\pi}{4}$	**A1**	
	$\arg\left(\frac{z-6+2i}{z+1+3i}\right)=\frac{\pi}{4}$	**A1**	oe, such as $\arg(z-6+2i)-\arg(z+1+3i)=\frac{\pi}{4}$. A0 for the arguments in the reverse order.
		[3]	
Total for Question 5		**6**	

Question	Scheme	Marks	Notes
6(a)	$a = -1$	**B1**	
	$p = \frac{\pi}{4},\ q = \frac{3\pi}{4}$	**B1**	Both values needed.
	$r = 2\sqrt{2}$	**B1**	Accept $\sqrt{8}$.
		[3]	
6(b)	Perpendicular from $3 - i$ to $y = x + 1$: $y + 1 = -(x - 3)$ $x + 1 = -x + 2 \Rightarrow x = \ldots$	**M1**	Attempts the perpendicular from $3 - i$ to the edge line $y = x + 1$, by gradient -1 and intersection, vectors or a distance formula. Condone sign slips.
	$F = \frac{1}{2} + \frac{3}{2}i$	**A1**	Correct foot, or implied by a correct perpendicular distance found directly.
	$\lvert w - z\rvert = \left\lvert(3 - i) - \text{“}\left(\frac{1}{2} + \frac{3}{2}i\right)\text{”}\right\rvert = \ldots$	**dM1**	Finds the distance from $3 - i$ to their foot on $y = x + 1$, or evaluates a perpendicular distance formula.
	$\lvert w - z\rvert_{\min} = \frac{5\sqrt{2}}{2}$	**A1**	Accept $\frac{5}{\sqrt{2}}$.
		[4]	
Total for Question 6		**7**	

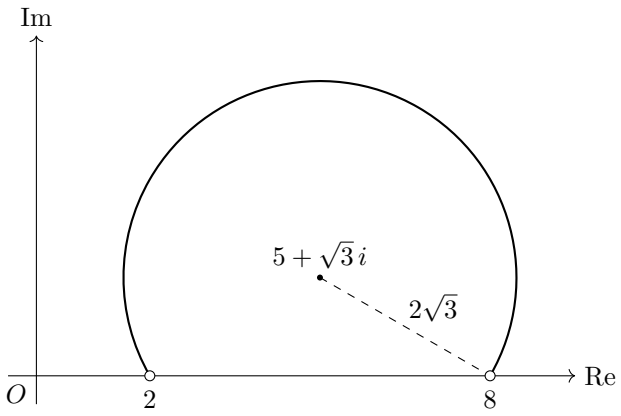
Question	Scheme	Marks	Notes
7(a)	$\lvert(x-3a)+iy\rvert = 2\lvert x+(y+a)i\rvert$	**M1**	Substitutes $z = x+iy$ into both moduli, with real and imaginary parts grouped. Condone sign slips; implied by the next line.
	$(x-3a)^2+y^2 = 4\left[x^2+(y+a)^2\right]$	**M1 A1**	**M1** Squares both moduli to an equation in x and y with no i. Condone sign slips and the 2 not squared. **A1** Any correct equation in x and y, expanded or not.
	$3x^2+3y^2+6ax+8ay-5a^2=0$ $\Rightarrow (x+a)^2+\left(y+\frac{4a}{3}\right)^2 = \ldots$	**M1**	Makes the coefficients of x^2 and y^2 one and completes the square in both, or uses the general centre and radius formulae. Condone sign slips.
	Centre $\left(-a,-\frac{4a}{3}\right)$, $r^2=\frac{40a^2}{9}$	**A1 A1**	**A1** Correct centre or correct radius. **A1** Both correct; radius as $r^2=\frac{40a^2}{9}$, or $r=\frac{2\sqrt{10}}{3}\lvert a\rvert$ oe, condoning the modulus omitted.
	$\pi\times$ "$\frac{40a^2}{9}$" $= 40\pi \Rightarrow a = \ldots$	**M1**	Sets π times their r^2, of the form ka^2, equal to 40π and solves for a.
	$a=\pm 3$, $-a>0 \Rightarrow a=-3$	**A1**	cso. $a=-3$ only, with $a=3$ rejected by reference to the centre's quadrant.
		[8]	
7(b)	"$2\sqrt{10}$" $-\sqrt{\text{"3"}^2+\text{"4"}^2}$	**M1**	Finds the difference between their radius and the distance from O to their centre, for their a.
	$2\sqrt{10}-5$	**A1**	cao; $\sqrt{40}-5$ oe.
		[2]	
	Total for Question 7	**10**	

Question	Scheme	Marks	Notes
8(a)			
	Labelled point	**B1**	Arc of a circle with ends at $3-2i$ and $3+6i$, labelled or as $(3,-2)$ and $(3,6)$.
	Shape	**B1**	Minor arc, clearly less than a semicircle, wholly to the right of $\text{Re}(z)=3$.
	Labelled point	**B1**	Both end points excluded, by open circles or stated.
		[3]	
8(b)	Centre on the perpendicular bisector of the chord AB: $y=\frac{1}{2}(-2+6)=2$	**E1**	States the centre lies on the perpendicular bisector of the chord joining the end points, or is equidistant from them, giving $y=2$.
		[1]	
8(c)	Angle at centre $=2\left(\pi-\frac{3\pi}{4}\right)=\frac{\pi}{2}$ $r^2+r^2=8^2\Rightarrow r=\ldots$	**M1**	Uses circle theorems for the angle at the centre with Pythagoras or the cosine rule on the chord, or uses $AB=2r\sin\frac{3\pi}{4}$. Condone a slip in the angle.
	$r=4\sqrt{2}$	**A1**	Exact, oe such as $\sqrt{32}$.
		[2]	
8(d)	$(3-h)^2+(-2-2)^2=$ "32" $\Rightarrow h=\ldots$	**M1**	Uses their radius with centre $(h,2)$, by distance to an end point or Pythagoras with the half-chord, to find h. Condone sign slips.

Question 8 continued

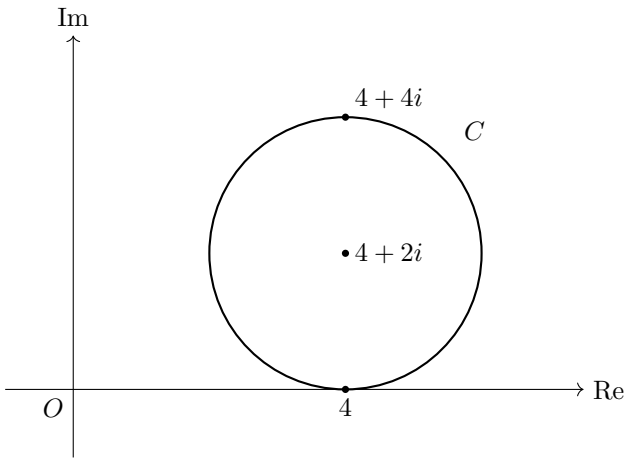
Question	Scheme	Marks	Notes
	$h = -1$ [rejecting $h = 7$]	**A1**	-1 only; $h = 7$ rejected if found.
	$\max \mathrm{Re}(z) =$ "-1" + "$4\sqrt{2}$" $= 4\sqrt{2} - 1$ $[\approx 4.66]$	**A1ft**	ft their h plus their r. Exact or awrt 4.66.
		[3]	
	Total for Question 8	**9**	

Question	Scheme	Marks	Notes
9(a)	$\frac{(x-6)+iy}{x+(y+2)i} \times \frac{x-(y+2)i}{x-(y+2)i}$	**M1**	Substitutes $z = x + iy$ and multiplies by the conjugate of the denominator, or uses tan of $\arg(z-6) - \arg(z+2i)$. Condone sign slips.
	Re : $\frac{x^2+y^2-6x+2y}{x^2+(y+2)^2}$ Im : $\frac{-2x+6y+12}{x^2+(y+2)^2}$	**A1**	Both correct, oe; the denominator may be omitted.
	$-2x + 6y + 12 = -\left(x^2 + y^2 - 6x + 2y\right)$	**dM1**	Equates their imaginary part to minus their real part, oe from $\tan\frac{3\pi}{4} = -1$.
	$x^2 + y^2 - 8x + 8y + 12 = 0$ $a = -8,\ b = 8,\ c = 12$	**A1**	
	$(x-4)^2 + (y+4)^2 = 20$ centre $4 - 4i$, radius $2\sqrt{5}$	**A1**	Centre as $4 - 4i$ or $(4, -4)$, and radius $2\sqrt{5}$ or $\sqrt{20}$.
		[5]	
9(b)	$\sqrt{(8 - \text{“4”})^2 + (-8 + \text{“4”})^2} + \text{“}2\sqrt{5}\text{”} = \ldots$	**M1**	Finds the distance from $8 - 8i$ to their centre and adds their radius.
	$4\sqrt{2} + 2\sqrt{5}$	**A1**	oe exact, such as $\sqrt{32} + \sqrt{20}$.
	Tests the furthest point of the circle: $F = \left(4 - \sqrt{10},\ -4 + \sqrt{10}\right)$ $-2x + 6y + 12 = \ldots$ at F	**M1**	Finds the point of the circle on the line through $8 - 8i$ and their centre, beyond the centre, and tests it lies on the arc by any valid test.
	$8\sqrt{10} - 20 > 0$, so F is on C	**A1**	Correct point and test, with a conclusion that F lies on C.
		[4]	
	Total for Question 9	**9**	

Question	Scheme	Marks	Notes
10(a)			
	Shape	**B1**	Major arc of a circle above the real axis, meeting it at both ends; no part below the real axis.
	Intercepts	**B1**	Ends at 2 and 8 marked and shown excluded, by open circles or stated.
	Centre and radius	**B1**	Centre $5+\sqrt{3}\,i$ and radius $2\sqrt{3}$, shown or stated; both needed.
		[3]	
10(b)	$(x-5)^2+\left(mx-\sqrt{3}\right)^2=12$	**M1**	Substitutes $y=mx$ into their circle's equation, or forms another tangency condition for a line through O, such as distance from centre equal to radius.
	$\left(1+m^2\right)x^2-\left(10+2\sqrt{3}\,m\right)x+16=0$	**A1**	Correct quadratic in x, oe; or an equivalent correct tangency equation.
	$\left(10+2\sqrt{3}\,m\right)^2-64\left(1+m^2\right)=0$ $13m^2-10\sqrt{3}\,m-9=0\Rightarrow m=\ldots$	**dM1**	Sets their discriminant to zero, or uses their tangency condition, and solves the quadratic in m by any means.
	$m=\sqrt{3}\Rightarrow$ maximum $\arg z=\frac{\pi}{3}$	**A1**	Exact $\frac{\pi}{3}$ from $m=\sqrt{3}$. A0 if $-\frac{3\sqrt{3}}{13}$ is chosen as giving the maximum.
	$x=\frac{10+2\sqrt{3}\,m}{2(1+m^2)}$ with $m=$ "$\sqrt{3}$" $=\ldots,\quad y=mx=\ldots$	**M1**	Uses their positive m to find the point of contact, by the repeated root or the foot of the perpendicular from the centre, reaching x and y.

Question 10 continued

Question	Scheme	Marks	Notes
	$z = 2 + 2\sqrt{3}\,i$	**A1**	Or $a = 2$, $b = 2\sqrt{3}$; exact.
		[6]	
	Total for Question 10	**9**	

Question	Scheme	Marks	Notes
11(a)			
	Shape	**B1**	Circle in the first quadrant touching the real axis, crossing neither axis.
	Centre and radius	**B1**	Centre $4+2i$ and radius 2, shown by labels such as the centre, 4 and $4+4i$, or stated.
		[2]	
11(b)	$y=0 \Rightarrow (\alpha^2-1)(x-4)^2 = 4-\alpha^2$ One solution only if $4-\alpha^2=0 \Rightarrow \alpha = 2$	**E1**	Shows $\alpha = 2$ makes the real axis touch C: one root of the $y=0$ equation using $\alpha > 1$, the centre's height equal to the radius, or $2 = \alpha \times 1$ at the touching point 4.
		[1]	
11(c)	$y = mx:\ (x-4)^2 + (mx-2)^2 = 4$	**M1**	Substitutes $y = mx$, or $y = x\tan\beta$, into their Cartesian equation of C, or uses the symmetry of the tangents from O about the line to the centre.
	$(1+m^2)x^2 - (8+4m)x + 16 = 0$	**A1**	oe, or $\arg(4+2i) = \arctan\frac{1}{2}$.
	$(8+4m)^2 - 64(1+m^2) = 0 \Rightarrow m = \ldots$	**dM1**	Sets the discriminant of their quadratic to zero and solves for nonzero m, or doubles their $\arg(4+2i)$. Condone slips.
	$\beta = \arctan\frac{4}{3}$	**A1**	Or any exact equivalent, such as $\arcsin\frac{4}{5}$, $\arccos\frac{3}{5}$ or $2\arctan\frac{1}{2}$.
		[4]	

Question 11 continued

Question	Scheme	Marks	Notes
11(d)	$x = \frac{8+4m}{2(1+m^2)},\ m = \text{“}\frac{4}{3}\text{”} \Rightarrow x = \ldots$	**M1**	Solves their quadratic with their nonzero m by any method, uses the foot of the perpendicular from the centre to the tangent, or uses $\lvert z\rvert = 4$ from equal tangents from O.
	$x = \frac{12}{5}$	**A1**	
	$z = \frac{12}{5} + \frac{16}{5}i$	**A1**	oe in the form $x + iy$, such as $2.4 + 3.2i$.
		[3]	
	Total for Question 11	**10**	

Question	Scheme	Marks	Notes
12(a)	$(x+7)^2+(y-2)^2=9\left[(x-1)^2+(y-2)^2\right]$	**M1 A1**	**M1** Substitutes $z=x+iy$ and writes both squared moduli as sums of squares of real and imaginary parts. Condone sign slips and the 3 not squared. **A1** Correct equation, oe.
	$8x^2+8y^2-32x-32y-8=0$ $\Rightarrow x^2+y^2-4x-4y-1=0$	**A1 AG**	Correct expansion of a correct equation leading without error to the printed equation; collected steps accepted.
		[3]	
12(b)	$y=\sqrt{3}x+a, \quad x>0$	**B1**	Gradient $\sqrt{3}$ through $(0,a)$ with $x>0$. Condone $x\geq 0$.
	$x^2+\left(\sqrt{3}x+a\right)^2-4x$ $-4\left(\sqrt{3}x+a\right)-1=0$	**M1**	Substitutes their line into the circle's equation. Equivalent marks for the perpendicular distance from $(2,2)$ to the line set equal to 3.
	$4x^2+\left(2\sqrt{3}a-4-4\sqrt{3}\right)x$ $+a^2-4a-1=0$	**A1**	Correct quadratic, or correct distance equation, oe; terms need not be collected.
	$\left(2\sqrt{3}a-4-4\sqrt{3}\right)^2$ $-16\left(a^2-4a-1\right)=0\Rightarrow a=\ldots$	**dM1**	Sets their discriminant to zero, or their distance equal to 3, and solves for both values of a.
	$a=8-2\sqrt{3}$ or $a=-4-2\sqrt{3}$	**A1**	Both values, exact, oe.
	$x=-\frac{2\sqrt{3}a-4-4\sqrt{3}}{8}$ $a=8-2\sqrt{3}\Rightarrow x=2-\frac{3\sqrt{3}}{2}<0$, reject $a=-4-2\sqrt{3}\Rightarrow x=2+\frac{3\sqrt{3}}{2}>0$	**M1**	Finds the point of contact for at least one value of their a, or uses the half-line's direction, and tests $x>0$ to decide which value is valid.
	$a=-4-2\sqrt{3}$ $\left(2+\frac{3\sqrt{3}}{2}\right)+\frac{1}{2}i$	**A1**	cso. Only $a=-4-2\sqrt{3}$ retained, and the point in the form $p+iq$ with exact p and q, oe. Not for coordinates alone.
		[7]	
	Total for Question 12	**10**	