OPTIONAL PURE / MISCELLANEOUS OPTIONAL PURE

Counting, Permutations and Combinations

Mark scheme 10 questions · 79 marks

EXAM BOARDS Edexcel FP2 9FM0/4A

Questions

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	Total	**79**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last accuracy mark of the working that reaches a printed result: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	$3 \times {}^{8}P_{3} = 3 \times 8 \times 7 \times 6$	**M1**	Multiplies 3 letter choices by the ordered choices of three from the remaining 8 characters, as ${}^{8}P_{3}$ or $8 \times 7 \times 6$.
	1008	**A1**	
		[2]	
1(b)	$9^4 = 6561$	**B1**	9^4 or 6561 seen; on the route by numbers of letters, one correct term instead, such as $4 \times 3 \times 6^3$.
	"6561" $- 6^4$	**M1**	Subtracts the digits-only codes, 6^4, from their total, or adds the counts of codes with exactly one, two, three and four letters.
	5265	**A1**	
		[3]	
	Total for Question 1	**5**	

Question	Scheme	Marks	Notes
2(a)	$\frac{8!}{3! \times 2!}$	**B1 M1**	**B1** 8! or 40 320 seen as the arrangements of 8 letters, soi. **M1** Divides a factorial by 3! and 2!, or by 6 and 2, for the repeated Ts and Os. Condone a wrong numerator.
	$= 3360$	**A1**	
		[3]	
2(b)	Treats TTT as one item: 6 items, TTT, A, O, O, E, D	**M1**	Takes the three Ts as a single item, soi by 6 items, a factor 6! or 5 or 6 places for the block.
	Block not first: 5 places	**B1**	5 places, or the arrangements with the block first subtracted from all those with the Ts together, as $\frac{6!}{2!} - \frac{5!}{2!}$.
	$\frac{5!}{2!}$ $[= 60]$	**M1**	Finds the arrangements of the five other letters, dividing 5! by 2! for the repeated Os. Not for 5! alone.
	$5 \times 60 = 300$	**A1**	
		[4]	
Total for Question 2		**7**	

Question	Scheme	Marks	Notes
3(a)	$\binom{19}{6} = 27\,132$	**B1**	
		[1]	
3(b)	$\binom{9}{3}$ [= 84]	**B1**	
	"84" $\times \left(\binom{9}{2} + \binom{9}{4}\right) = \ldots$	**M1**	Multiplies their adult count by a junior count, both twins in plus both out, or all choices less those with exactly one twin. Condone a slip in one combination.
	13 608	**A1**	
		[3]	
3(c)	$\binom{9}{2} \times \binom{11}{5}$ [= 16 632]	**M1**	Multiplies an adult and a junior combination for one case totalling 7 players, either an allowed case or an excluded case in a complement method.
	$16\,632 + 27\,720 + 20\,790 + 6930 = \ldots$	**dM1**	Adds the counts for all four cases with 2 to 5 adults, or subtracts all four excluded cases from $\binom{20}{7}$. Values need not be correct.
	72 072	**A1**	
		[3]	
	Total for Question 3	**7**	

Question	Scheme	Marks	Notes
4(a)	$8! = 40\,320$	**B1**	
		[1]	
4(b)	$5! \times 2^3 = \ldots$	**M1**	Multiplies 5!, for five units, by a factor for the orders within the couples. Condone 2×3 for 2^3.
	960	**A1**	
		[2]	
4(c)	$6 \times 5 \times 6! = \ldots$	**M1**	Multiplies 6×5, adults at the ends or children in non-end seats, by 6!, or subtracts arrangements with a child at an end from 8!. Condone $\binom{6}{2}$.
	21 600	**A1**	
		[2]	
4(d)	$3! \times 2^3$ [= 48]	**M1**	Multiplies the arrangements of the blocks by a factor for orders within couples, such as $3! \times 2^3$, or $4! \times 2^3$ with the children as one block.
	Places the children in different gaps: "48" $\times 4 \times 3 = \ldots$	**M1**	Uses 4×3 or ${}^4P_2$ for the children in the four gaps, or subtracts the arrangements with the children together from their (b).
	576	**A1**	
		[3]	
Total for Question 4		**8**	

Question	Scheme	Marks	Notes
5(a)	$\binom{7}{3} \times {}^6P_4 \times 5^3$	**M1**	Multiplies counts of letter positions, ordered distinct digits and letter choices, oe. Condone one incorrect factor.
	1 575 000	**A1**	
		[2]	
5(b)	Letters in different gaps around the 4 digits: $\binom{5}{3}$ [= 10]	**M1**	Finds the number of letter positions with no two letters adjacent, using the 5 gaps around the digits, or with two or more letters adjacent directly, oe such as listing.
	(35 − "10") × 360 × 125	**dM1**	Subtracts their count from 35 and multiplies by ${}^6P_4 \times 5^3$, multiplies their adjacent count by ${}^6P_4 \times 5^3$, or subtracts their non-adjacent codes from their total in (a).
	1 125 000	**A1**	
		[3]	
Total for Question 5		**5**	

Question	Scheme	Marks	Notes
6(a)	$4 \times 3 \times (6 \times 5 \times 4) = \ldots$	**M1**	Forms a product of choices for the two even end digits and ordered middle digits from the discs left, oe such as ${}^4P_2 \times {}^6P_3$. Condone one wrong factor.
	1440	**A1**	
		[2]	
6(b)	First digit 1: $3 \times (6 \times 5 \times 4) = \ldots$ First digit 2: $4 \times (6 \times 5 \times 4) = \ldots$	**M1**	Uses cases by first digit, counting at least one as their odd last-digit choices times ordered middle digits from the discs left. Condone one wrong factor.
	360 and 480	**A1**	Both, or the total 840.
	First digit 3, second digit 1, 2 or 4: $2 \times (5 \times 4) + 3 \times (5 \times 4) + 3 \times (5 \times 4)$ $= \ldots$	**M1**	Takes the second digit below 5 when the first is 3, counting at least one case as odd last-digit choices times ordered middle digits from the discs left. Condone one wrong factor.
	$40 + 60 + 60 = 160$	**A1**	
	$360 + 480 + 160 = 1000$	**A1**	
		[5]	
Total for Question 6		**7**	

Question	Scheme	Marks	Notes
7(a)	$\binom{8}{3} \times \binom{16}{2} = \ldots$	**M1**	Multiplies a choice of 3 from the 8 odd primes by a choice of 2 from the 16 non-primes. Need not be evaluated.
	6720	**A1**	
		[2]	
7(b)	$\binom{16}{3} \times \binom{9}{2}$ [= 20 160]	**M1**	Forms $\binom{16}{r} \times \binom{9}{5-r}$ for at least one value of r, or the matching product for primes in a complement method.
	$\binom{16}{3}\binom{9}{2} + \binom{16}{4}\binom{9}{1} + \binom{16}{5} = \ldots$	**dM1**	Adds the cases with 3, 4 and 5 non-primes, or subtracts those with 3, 4 and 5 primes from $\binom{25}{5}$. Condone one slip in a pool size.
	40 908	**A1**	
		[3]	
7(c)	$\{5, 17\},\ \{7, 19\},\ \{11, 23\}$	**B1**	All three pairs and no others.
	Two pairs: $\binom{3}{2} \times 21$ [= 63]	**M1**	Finds the number of selections holding two of their pairs, the fifth number from those left, oe, or the pairwise intersection terms of inclusion-exclusion.
	One pair: $3 \times \left(\binom{23}{3} - 2 \times 21\right)$ [= 5187]	**M1**	Finds the number of selections holding one pair, the other three from the 23 left, less those holding a second pair. Inclusion-exclusion, $3\binom{23}{3} - 3 \times 21$, accepted.
	$5187 + 63 = 5250$	**A1**	
		[4]	
	Total for Question 7	**9**	

Question	Scheme	Marks	Notes
8(a)	$n(A) = 50,\ n(B) = 30,\ n(C) = 21$ $n(A \cap B) = 10,\ n(A \cap C) = 7,\ n(B \cap C) = 4$ $n(A \cap B \cap C) = 1$	**B1**	All seven counts, for the multiples of 3, 5, 7 and their intersections, with any labels; soi.
	$n(A \cup B \cup C) = 50 + 30 + 21 - 10 - 7 - 4 + 1\ [= 81]$ $150 -$ "81" $= \ldots$	**M1**	Adds their three single counts, subtracts their three pairwise counts, adds their triple count, and subtracts from 150, in either order.
	69	**A1**	
		[3]	
8(b)	1-digit: 9, 2-digit: 90, 3-digit: 51	**B1**	
	Splits by card lengths: $3+1$, $2+2$, $2+1+1$, $1+1+1+1$	**M1**	Splits into cases by the cards' numbers of digits, with at least three of the four correct cases, listed ordered or unordered.
	$51 \times 9 \times 2 + 90 \times 89$ $+ 90 \times \binom{9}{2} \times 3! + {}^9P_4 = \ldots$	**M1**	Finds the number of ordered arrangements of distinct cards in at least three of their cases, with an ordering factor where needed, and adds. Condone one wrong factor.
	$918 + 8010 + 19\,440 + 3024 = 31\,392$	**A1**	
		[4]	
	Total for Question 8	**7**	

Question	Scheme	Marks	Notes
9(a)(i)	$3 \times 3 \times 2 = 18$	**M1 A1**	**M1** Finds the number without repetition, excluding a leading 0, as $3 \times 3 \times 2$, $4 \times 3 \times 2 - 3 \times 2$ or a full listing.
		[2]	
9(a)(ii)	$3 \times 4 \times 4 = 48$	**M1 A1**	**M1** Uses 3 choices for the first digit and 4 for each other, or $4^3 - 4^2$.
		[2]	
9(a)(iii)	"48" − "18" = 30	**M1 A1ft**	**M1** Subtracts their (a)(i) from their (a)(ii), or counts the numbers with a repeated digit directly. **A1ft** ft their (a)(ii) minus their (a)(i).
		[2]	
9(b)	$3 \times 4 \times 4 \times 2 = 96$	**M1 A1**	**M1** Finds the even numbers with unlimited repetition, 3 choices first and 2 last, or one direct pattern count, such as 10 all distinct. Condone one slip in a middle factor. **A1** 96, or one correct pattern count on the direct route.
	Counts numbers using a digit three or more times, digit by digit: digit 0: 3, digit 1: 2, digit 3: 2	**M1 A1**	**M1** Finds the number of even numbers using some digit three or more times, for at least two digits, to subtract from their unrestricted count, or at least two further pattern counts on the direct route. **A1** All three counts correct, or two pattern counts correct on the direct route.
	digit 2: $1 + 2 + 3 + 3 + 1 = 10$	**A1**	Correct count, including 2222, or all three pattern counts correct on the direct route.
	$96 - (3 + 2 + 2 + 10) = 79$	**A1**	
		[6]	
	Total for Question 9	**12**	

Question	Scheme	Marks	Notes
10(a)(i)	$8! = 40\,320$	**B1**	
		[1]	
10(a)(ii)	$5! \times {}^6P_3 = \ldots$ $[= 14\,400]$	**M1**	Puts the geometry questions in different gaps between the arranged algebra questions, as $5! \times {}^6P_3$, or chooses positions as $\binom{6}{3}$. Condone one wrong factor.
	$\dfrac{\text{“}14\,400\text{”}}{40\,320} = \ldots$	**dM1**	Divides their count by 8!, or their position count by $\binom{8}{3}$.
	$\dfrac{5}{14}$	**A1**	$\frac{5}{14}$ oe, or awrt 0.357.
		[3]	
10(b)(i)	$2 \times 5! \times 3! = \ldots$	**M1**	Multiplies the arrangements within each section by the number of section orders. Not for $5! \times 3!$ alone.
	1440	**A1**	
		[2]	
10(b)(ii)	$\dfrac{4! \times 2}{5!} = \ldots$	**M1**	Finds the number of orders with A and B adjacent over the matching total, within the algebra section or over all arrangements, or adjacent position pairs over $\binom{5}{2}$.
	$\dfrac{2}{5}$	**A1**	$\frac{2}{5}$ oe.
		[2]	
10(b)(iii)	Algebra first, C the mth of 5, H the nth of 3: $(5 - m) + (n - 1) \geq 3$ $\Rightarrow m \leq n + 1$	**M1**	Finds the number of positions of C and H with at least three questions between, for one section order, by inequality or listing. Condone one miscounted case.
	$2 + 3 + 4 = 9$ of the $5 \times 3 = 15$ pairs	**A1**	9 of 15, or an equivalent count such as 432 of 720.
	$\dfrac{1}{2} \times \dfrac{\text{“}9\text{”}}{15} + \dfrac{1}{2} \times \dfrac{\text{“}9\text{”}}{15} = \ldots$	**M1**	Combines both section orders, each weighted by $\frac{1}{2}$, or adds both favourable counts over 1440; one order alone earns it, as the orders are equal by symmetry.

Question 10 continued

Question	Scheme	Marks	Notes
	$\frac{3}{5}$	**A1**	$\frac{3}{5}$ oe.
		[4]	
	Total for Question 10	**12**	