OPTIONAL PURE / MISCELLANEOUS OPTIONAL PURE

Algebraic and Modulus Inequalities

Mark scheme 15 questions · 107 marks

EXAM BOARDS Edexcel FP1 9FM0/3A · AQA 7367 · CIE 9231

Questions

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	Total	**107**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last accuracy mark of the working that reaches a printed result: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

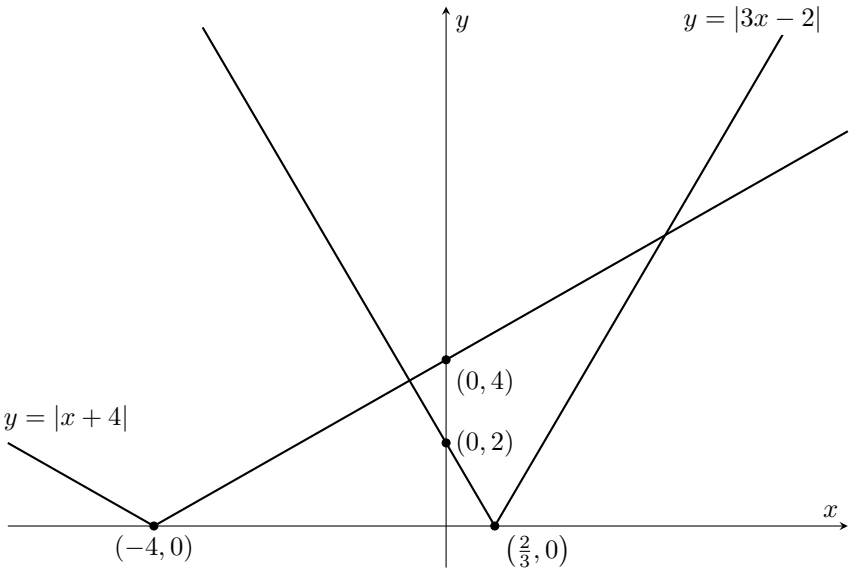
Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1	$-1 < 2\cos x - 1 < 1$	**M1**	Uses a double inequality, two cases or squaring to remove the modulus, or sets $2\cos x - 1$ equal to 1 and -1. Condone $\leq$ for $<$.
	$0 < \cos x < 1$	**A1**	Or the critical equations $\cos x = 0$ and $\cos x = 1$.
	$\cos x = 0 \Rightarrow x = \ldots$	**dM1**	Solves at least one of their critical equations in $\cos x$ for a value of x in $0 \leq x \leq 2\pi$.
	$x = \frac{\pi}{2},\ \frac{3\pi}{2}$ $[\cos x = 1 \Rightarrow x = 0,\ 2\pi]$	**A1**	Both, or awrt 1.57 and 4.71.
	$\left\{x \in \mathbb{R} \mid 0 < x < \frac{\pi}{2}\right\} \cup$ $\left\{x \in \mathbb{R} \mid \frac{3\pi}{2} < x < 2\pi\right\}$	**A1**	Both pieces with strict signs, excluding 0 and 2π, in any set notation, such as a union of open intervals. Not for inequalities alone.
	Total for Question 1	**5**	

Question	Scheme	Marks	Notes
2(a)			
	Shape	**B1**	Two V shapes on the same axes, each with its vertex on the x-axis, $y = \lvert 3x - 2 \rvert$ the steeper.
	Intercepts	**B1**	$\left(\frac{2}{3}, 0\right)$ and $(0, 2)$ for $y = \lvert 3x - 2 \rvert$, stated or marked on the axes.
	Intercepts	**B1**	$(-4, 0)$ and $(0, 4)$ for $y = \lvert x + 4 \rvert$, stated or marked on the axes.
		[3]	
2(b)	$3x - 2 = x + 4 \Rightarrow x = \ldots$	**M1**	Solves $3x - 2 = \pm(x + 4)$ for one critical value, or squares both sides and solves their three-term quadratic. Condone sign slips.
	$x = 3, \quad x = -\frac{1}{2}$	**A1**	Both critical values, by any valid method.
	$x < -\frac{1}{2}$ or $x > 3$	**A1**	Both ranges with strict inequalities and no others, oe including set notation.
		[3]	
	Total for Question 2	**6**	

Question	Scheme	Marks	Notes
3(a)	$(4x+1)(2x+3) \leq -3(2x+3)^2$ $\Rightarrow (4x+1)(2x+3)+3(2x+3)^2 \leq 0$	**M1**	Multiplies by $(2x+3)^2$, or uses another sign-safe method such as one fraction on one side, and collects terms on one side. Condone sign slips.
	$10(2x+3)(x+1) \leq 0 \Rightarrow x = -\frac{3}{2},\ x = -1$	**A1**	Both critical values.
	$-\frac{3}{2} \ldots x \ldots -1$	**dM1**	Chooses the region between their two critical values, with any inequality signs.
	$-\frac{3}{2} < x \leq -1$	**A1**	Both ends with correct signs, oe such as $x > -\frac{3}{2}$ and $x \leq -1$.
		[4]	
3(b)	Replaces x by $\frac{1}{y}$: "$-\frac{3}{2}$" $< \frac{1}{y} \leq$ "-1" $\Rightarrow y \ldots$	**M1**	Replaces x by $\frac{1}{y}$ in their two-ended range from (a) and inverts at least one end to reach a bound on y. Condone signs not reversed.
	$-1 \leq y < -\frac{2}{3}$	**A1**	Both ends with correct signs, oe such as $y \geq -1$ and $y < -\frac{2}{3}$.
		[2]	
	Total for Question 3	**6**	

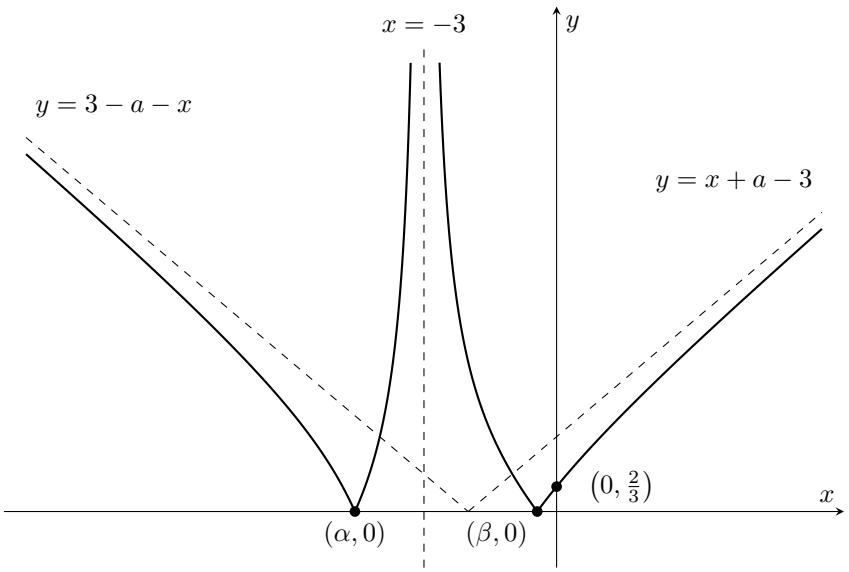
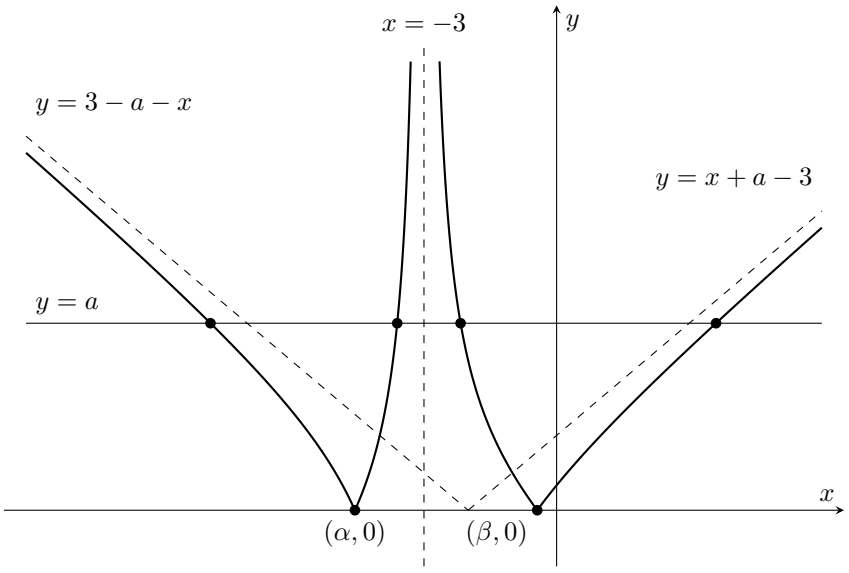
Question	Scheme	Marks	Notes
4(a)(i)	$f(-4) = 0$ $f(x) = (x+4)\left(9x^2 + \ldots\right)$	**M1**	Uses the factor theorem to find a linear factor, then divides or compares coefficients to obtain a quadratic factor. Condone slips in the division.
	$f(x) = (3x-2)^2(x+4)$	**A1**	oe, such as $(3x-2)(3x-2)(x+4)$ or $(2-3x)^2(x+4)$.
		[2]	
4(a)(ii)	$(3x-2)^2 \geq 0 \Rightarrow f(x) > 0$ for $x+4>0$, $x \neq \frac{2}{3}$	**M1**	Uses their factorisation from (a)(i) and its critical values to decide where $f(x) > 0$, by sign analysis or a sketch, attending to any repeated factor.
	$-4 < x < \frac{2}{3}$ or $x > \frac{2}{3}$	**A1**	Both pieces with strict signs, oe such as $x > -4$, $x \neq \frac{2}{3}$, or set notation.
		[2]	
4(b)	$g(x) = f(2-x)$ $= (3(2-x)-2)^2\left((2-x)+4\right)$	**M1**	Replaces x by $-x$ and x by $x-2$ in f or their factorised form. Condone $f(-x-2)$ from the wrong order.
	$g(x) = (3x-4)^2(6-x)$	**A1**	Linear factors, oe such as $(3x-4)(3x-4)(6-x)$ or $-(3x-4)^2(x-6)$. Not an expanded cubic.
	$(3x-4)^2 \geq 0 \Rightarrow g(x) \leq 0$ for $6-x \leq 0$ or $x = \frac{4}{3}$	**M1**	Uses their factorised g, considering the sign of each factor and any repeated root, by sign analysis or a sketch.
	$x = \frac{4}{3}$ or $x \geq 6$	**A1**	Both, with $\geq$, oe in set notation. A0 if $x = \frac{4}{3}$ is omitted.
		[4]	
	Total for Question 4	**8**	

Question	Scheme	Marks	Notes
5(a)	$a = 4$	**B1**	
	$\frac{3\times\text{“4”}+b}{3-2} = 7 \Rightarrow b = -5$	**B1ft**	ft their a, as $b = 7 - 3a$.
		[2]	
5(b)	$(4x-5)(x-2) \geq (2-x)(x-2)^2$ $\Rightarrow (4x-5)(x-2) + (x-2)^3 \geq 0$	**M1**	Multiplies by $(x-2)^2$, or forms one fraction compared with zero, and collects terms on one side. Condone slips. Not for multiplying by $x-2$ alone.
	$(x-2)(x^2-1) \geq 0$	**A1**	Correct inequality oe, expanded or as $\frac{x^2-1}{x-2} \geq 0$.
	$(x+1)(x-1)(x-2) \geq 0 \Rightarrow x = \ldots$	**dM1**	Factorises their cubic, or numerator and denominator, to find the critical values. Condone sign slips.
	$x = -1,\ 1,\ 2$	**A1**	
	Selects where the cubic is non-negative: $-1 \ldots x \ldots 1$ or $x \ldots 2$	**M1**	Uses the sign of their cubic or fraction, by sketch, sign table or test values, to choose the regions between and beyond their critical values.
	$-1 \leq x \leq 1$ or $x > 2$	**A1**	cso. Both regions, with -1 and 1 included and 2 excluded. Not for a calculator's solution without working.
		[6]	
	Total for Question 5	**8**	

Question	Scheme	Marks	Notes
6	$7 = 5\cosh x - \sinh x$ $= \frac{5}{2}\left(e^x + e^{-x}\right) - \frac{1}{2}\left(e^x - e^{-x}\right)$	**M1**	Clears $\sinh x$ from the denominators and substitutes the exponential forms of $\cosh x$ and $\sinh x$, in either order. Condone sign slips.
	$2e^{2x} - 7e^x + 3 = 0$	**A1**	Correct three-term quadratic in e^x, oe, as an equation or an inequality.
	$(2e^x - 1)(e^x - 3) = 0 \Rightarrow x = \ln \ldots$	**M1**	Solves their three-term quadratic in e^x by any means and takes logarithms of at least one positive root.
	$x = -\ln 2, \quad x = \ln 3$	**A1**	Both values, oe such as $\ln \frac{1}{2}$.
	$-\ln 2 < x < 0$ or $x > \ln 3$	**A1**	Both ranges with strict inequalities and 0 excluded, in logarithmic form, oe such as $\ln \frac{1}{2}$ or set notation.
	Total for Question 6	**5**	

Question	Scheme	Marks	Notes
7	Roots of the quartic: $-2,\ -1,\ 3,\ c$	**B1**	soi, as by the factors $(x+2)$, $(x+1)$, $(x-3)$, $(x-c)$.
	Pairs roots with product -12 for $x^2+ax-12$: $-2c=-12$ or $-c=-12 \Rightarrow c=\ldots$	**M1**	Uses product of roots -12, or substitutes a known root into $x^2+ax-12=0$, to pair two roots and find a value of c.
	$c=6$, roots -2 and 6	**A1**	
	Other roots -1, 3: sum 2 as needed $x^2-2x+b=(x+1)(x-3) \Rightarrow b=\ldots$	**dM1**	Uses the remaining two roots in x^2-2x+b, by sum and product or by expanding their factors.
	$a=-4$, $b=-3$	**A1**	Both values, cso.
	Total for Question 7	**5**	

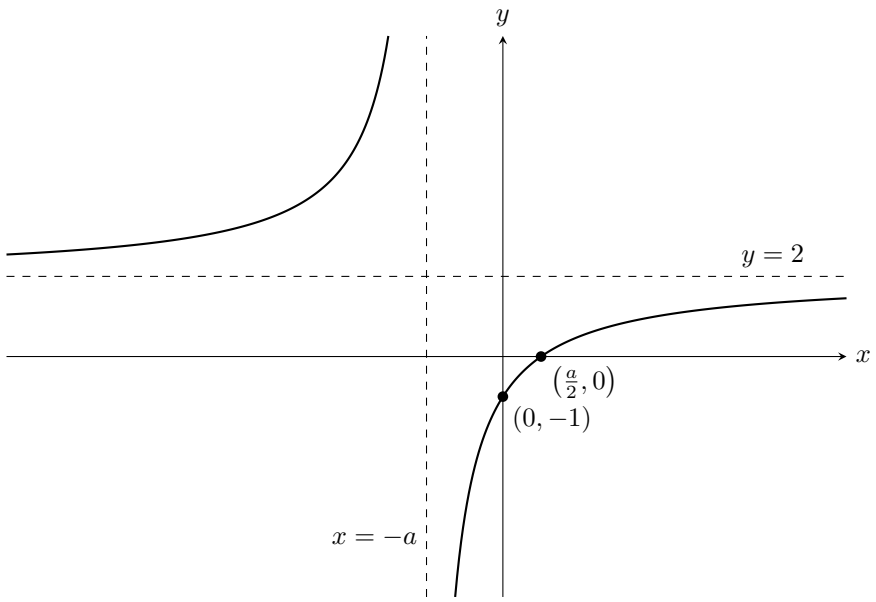
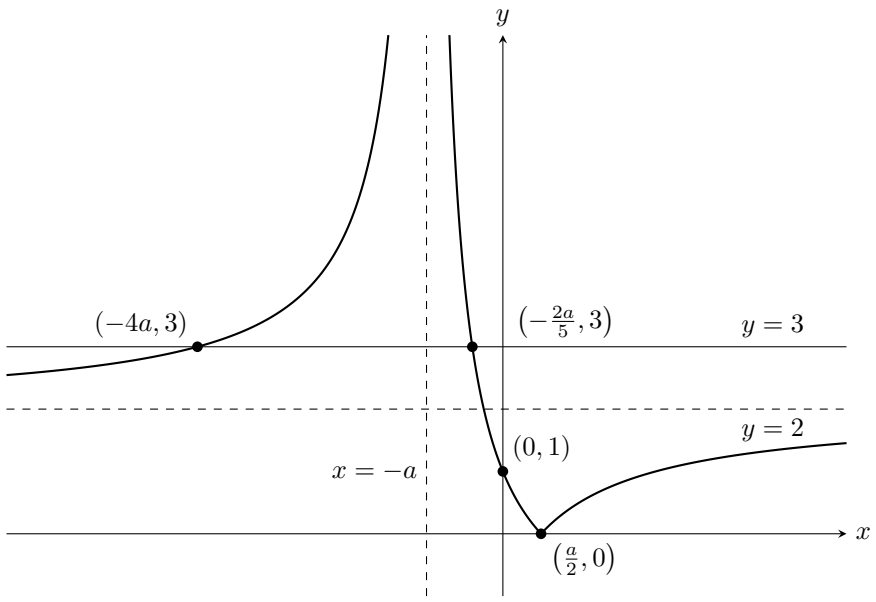
Question	Scheme	Marks	Notes
8(a)	$\left(2t^2-15t+3\right)^2 > (3t-25)^2$	**M1**	Squares both sides to form a polynomial inequality or equation, or sets $2t^2-15t+3$ equal to $3t-25$ or $-(3t-25)$. Condone $=$ or $\geq$ for $>$.
	$\left(2t^2-15t+3-3t+25\right)\left(2t^2-15t+3+3t-25\right) > 0$	**M1**	Factorises as a difference of two squares, expands and factorises the quartic, or forms both $\pm$ cases as quadratics. Condone sign slips.
	$\left(2t^2-18t+28\right)\left(2t^2-12t-22\right) > 0$	**A1**	Both quadratic factors correct, oe such as $\left(t^2-9t+14\right)\left(t^2-6t-11\right)$, or the quartic correctly factorised.
	$t^2-9t+14=0 \Rightarrow t=\ldots$ $t^2-6t-11=0 \Rightarrow t=\ldots$	**dM1**	Solves their quadratic factor(s), or both $\pm$ quadratics, by any method, leading to $t=\ldots$; a calculator is allowed.
	$t=2,\ t=7$	**A1**	
	Rejects $3-2\sqrt{5}<0$: $t=3+2\sqrt{5}$	**A1**	$3+2\sqrt{5}$ or awrt 7.47, with $3-2\sqrt{5}$ rejected or not used in the intervals.
	Uses the signs of $(t-2)(t-7)\left(t-3-2\sqrt{5}\right)$ $\Rightarrow 2<t<7,\ t>3+2\sqrt{5}$	**M1**	Chooses intervals between their critical values by a sign table, a sketch or test values, with at least one interval correct for their values.
	$2<t<7$ or $3+2\sqrt{5}<t\leq 10$	**A1**	Both intervals, strict at 2, 7 and $3+2\sqrt{5}$ (or awrt 7.47), with $t\leq 10$; set notation accepted.
		[8]	
8(b)	$(7-2)+\left(10-\text{“}3+2\sqrt{5}\text{”}\right)$	**M1**	Adds the lengths of their two intervals from (a), using 10 as the upper end of the second.
	$12-2\sqrt{5}$ [≈ 7.53]	**A1**	Any exact equivalent.
	$7.53>6$: model overestimates, so needs refining	**E1**	Compares a correct total, exact or awrt 7.5, with 6 and concludes the model overestimates the time, so is not accurate or needs refining.
		[3]	
	Total for Question 8	**11**	

Question	Scheme	Marks	Notes
9(a)			
	Shape	**B1**	Graph on or above the x-axis, one branch each side of $x=-3$, each touching the x-axis at a sharp point and rising to infinity both ways.
	Asymptotes	**B1**	Vertical asymptote $x=-3$ shown or labelled, with both outer branches tending to straight lines; the oblique asymptotes' equations are not required.
		[2]	
9(b)			
	Intercepts	**B1**	Horizontal line meeting the graph at exactly four points, two on each side of $x=-3$.
		[1]	

Question 9 continued

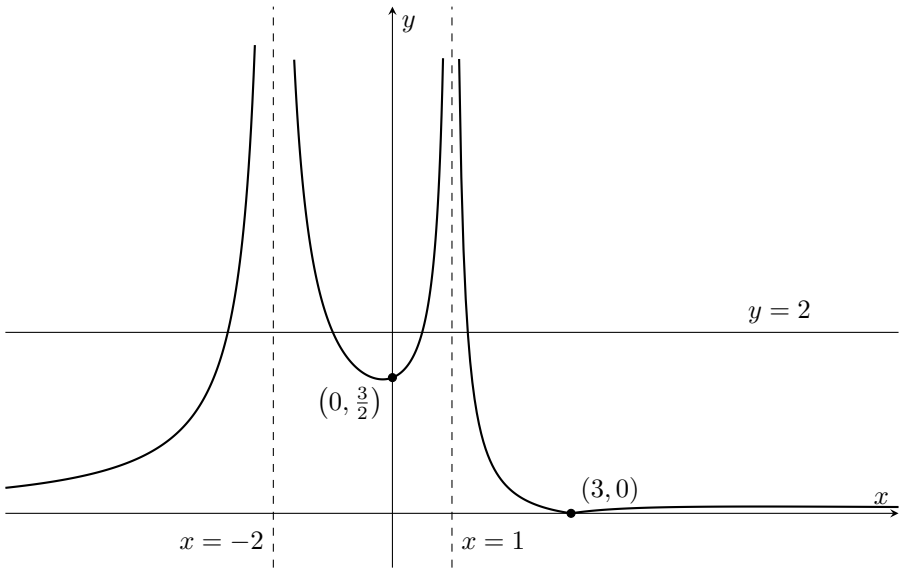
Question	Scheme	Marks	Notes
9(c)	$\frac{16+4a+2}{7} = a \Rightarrow a = \ldots$	**M1**	Substitutes $x = 4$ into $f(x)$, sets it equal to a and solves for a.
	$a = 6$	**A1**	
	$x^2 + 6x + 2 = 6(x + 3) \Rightarrow x = \ldots$ $x^2 + 6x + 2 = -6(x + 3) \Rightarrow x = \ldots$	**M1**	Solves both $f(x) = a$ and $f(x) = -a$ with their a, or squares both sides and solves. Quadratics solved by any means.
	$x = -10,\ -4,\ -2,\ 4$	**A1**	
	$-10 < x < -4$ or $-2 < x < 4$	**A1**	cao. Both ranges with strict inequalities, oe in set notation.
		[5]	
	Total for Question 9	**8**	

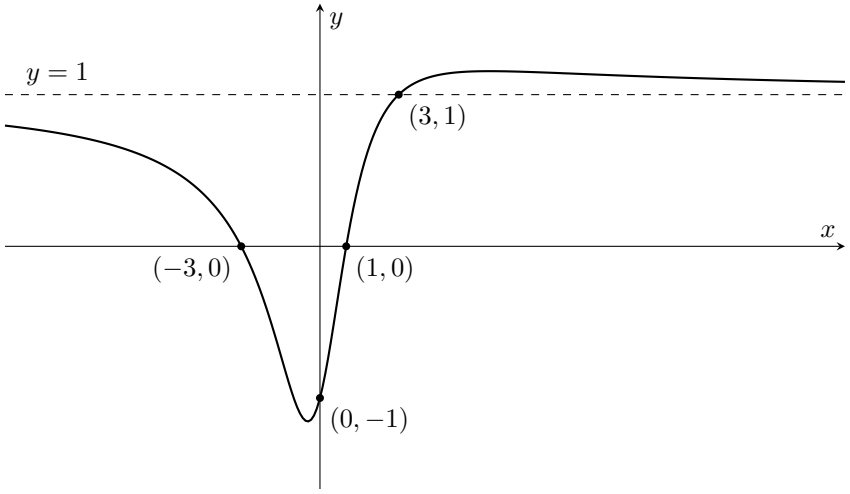
Question	Scheme	Marks	Notes
10(a)	$kx(x+3)(x-2)^2 \leq (x-1)(x-2)(x+3)^2$	**M1**	Multiplies both sides by $(x+3)^2(x-2)^2$, or compares a single combined fraction with zero. Condone slips.
	$0 \leq (x-2)(x+3)\left[(x-1)(x+3) - kx(x-2)\right]$	**dM1**	Collects terms on one side and takes out the factor $(x-2)(x+3)$, leaving a quadratic factor. Condone sign slips.
	$(x-2)(x+3)\left((k-1)x^2 - (2k+2)x + 3\right) \leq 0$ $p = k-1, \quad q = -2k-2, \quad r = 3$	**A1**	cso. Correct p, q and r, or any positive multiple of all three, in the printed form with $\leq$.
		[3]	
10(b)	$9(k-1) + 3(-2k-2) + 3 = 0 \Rightarrow k = \ldots$	**M1**	Substitutes $x = 3$ into their quadratic factor in terms of k, sets it equal to zero and solves for k.
	$k = 4$	**A1**	
	$3x^2 - 10x + 3 = (3x-1)(x-3) = 0 \Rightarrow x = \ldots$	**M1**	Solves their quadratic factor with their k by any means, or otherwise finds the critical values of the inequality with their k.
	Critical values $x = -3,\ \frac{1}{3},\ 2,\ 3$; regions chosen by the sign of the quartic	**dM1**	Uses the sign of their quartic, by sketch, sign table or test values, to choose the regions between and beyond their critical values. Condone errors at the endpoints.
	$-3 < x \leq \frac{1}{3}$ or $2 < x \leq 3$	**A1**	cso. Both ranges, strict at -3 and 2, non-strict at $\frac{1}{3}$ and 3, and no others. Accept set notation.
		[5]	
	Total for Question 10	**8**	

Question	Scheme	Marks	Notes
11(a)			
	Shape	**B1**	Two branches approaching both asymptotes, above $y = 2$ for $x < -a$ and below $y = 2$ for $x > -a$.
	Asymptotes	**B1**	Both asymptotes drawn, with equations $x = -a$ and $y = 2$ stated.
	Intercepts	**B1**	$(0, -1)$ and $\left(\frac{a}{2}, 0\right)$ marked or stated; condone $y = -1$ and $x = \frac{a}{2}$.
		[3]	
11(b)			
	Shape	**B1ft**	ft their sketch in (a). The part below the x-axis reflected, the curve wholly on or above it and touching it at their x-intercept.

Question 11 continued

Question	Scheme	Marks	Notes
	$\frac{2x-a}{x+a} = 3 \Rightarrow x = \ldots$ or $\frac{2x-a}{x+a} = -3 \Rightarrow x = \ldots$	**M1**	Solves either equation, or squares both sides and solves the quadratic, for at least one critical value in terms of a. Condone sign slips.
	$x = -4a$, $x = -\frac{2a}{5}$	**A1**	Both values, oe.
	$-4a < x < -a$ or $-a < x < -\frac{2a}{5}$	**A1**	Both ranges, strict inequalities, with $-a$ excluded; oe such as $-4a < x < -\frac{2a}{5}$, $x \neq -a$, or set notation.
		[4]	
Total for Question 11		**7**	

Question	Scheme	Marks	Notes
12			
	Shape	**B1**	Three branches, all on or above the x-axis, the middle one wholly above it, the right-hand one touching the axis at $x = 3$.
	Asymptotes	**B1**	Vertical asymptotes $x = -2$ and $x = 1$ labelled, and the outer branches tending to the x-axis as $x \to \pm\infty$.
	$x - 3 = \pm 2\left(2 - x - x^2\right)$ $\Rightarrow 2x^2 + 3x - 7 = 0$ or $2x^2 + x - 1 = 0$ $\Rightarrow x = \ldots$	**M1**	Forms an equation for the critical values, with either sign, or squares both sides, and solves a resulting quadratic or the quartic by any means. Condone sign slips.
	$x = \frac{-3 \pm \sqrt{65}}{4}$	**A1**	Both values, exact, oe.
	$x = -1,\ x = \frac{1}{2}$	**A1**	
	$\frac{-3-\sqrt{65}}{4} < x < -1$ or $\frac{1}{2} < x < \frac{-3+\sqrt{65}}{4}$	**A1**	Both ranges, exact, with strict signs and $x = -2$, $x = 1$ included; oe in set notation or as separate ranges. Withhold if $x = -2$ or $x = 1$ is excluded.
	Total for Question 12	**6**	

Question	Scheme	Marks	Notes
13(a)			
	Shape	**B1**	Single continuous curve, no vertical asymptote, below the x-axis between its x-intercepts, crossing $y = 1$ and approaching it from below as $x \to -\infty$, from above as $x \to \infty$.
	Intercepts	**B1**	$(-3, 0)$, $(1, 0)$ and $(0, -1)$, on the sketch or stated.
	Asymptotes	**B1**	$y = 1$ drawn and its equation stated.
		[3]	
13(b)	$\left(\frac{1}{f(x)}\right)^3 \geq \frac{1}{f(x)}$ $\Rightarrow \frac{1}{f(x)}\left(\frac{1}{f(x)} - 1\right)\left(\frac{1}{f(x)} + 1\right) \geq 0$	**M1**	Multiplies by a positive square to form a cubic inequality in $\frac{1}{f(x)}$, or an equivalent inequality in x, or uses the critical values $\frac{1}{f(x)} = \pm 1$. Condone sign slips.
	$-1 \leq \frac{1}{f(x)} < 0$ or $\frac{1}{f(x)} \geq 1$	**A1**	Both conditions, oe. Condone $\frac{1}{f(x)} \leq 0$.
	$x^2 + 2x - 3 = -\left(x^2 + 3\right) \Rightarrow 2x(x + 1) = 0$ $\Rightarrow x = \ldots\ [-1,\ 0]$	**M1**	Sets $\frac{1}{f(x)} = -1$ and solves the resulting quadratic by any means.
	$-3 < x \leq -1$ or $0 \leq x < 1$ or $x \geq 3$	**A1**	All three ranges with correct inequality signs, -3 and 1 excluded, and no others. Set notation accepted.
		[4]	
	Total for Question 13	**7**	

Question	Scheme	Marks	Notes
14	$\lvert x\rvert - 1 = 0 \Rightarrow x = \pm 1$	**B1**	Both $x = -1$ and $x = 1$, seen as critical values or as excluded ends of intervals.
	$x \geq 0$: $\frac{2x}{x-1} = x + 3$ $\Rightarrow 2x = (x+3)(x-1) \Rightarrow x = \ldots$	**M1**	Replaces $\lvert x\rvert$ by x, clears the fraction in the equation or inequality and solves. Condone sign slips.
	$x^2 = 3 \Rightarrow x = \sqrt{3}$	**A1**	$\sqrt{3}$ or awrt 1.73 only, with $-\sqrt{3}$ rejected or not used.
	$x < 0$: $\frac{2x}{-x-1} = x + 3$ $\Rightarrow x^2 + 6x + 3 = 0 \Rightarrow x = \ldots$	**M1**	Replaces $\lvert x\rvert$ by $-x$, forms a three-term quadratic and solves it by any means. Condone sign slips.
	$x = -3 \pm \sqrt{6}$	**A1**	Both values, or awrt -0.551 and -5.45, with neither rejected.
	Selects where the curve is on or above the line: $x \leq \ldots$, $-1 < x \leq \ldots$, $1 < x \leq \ldots$	**M1**	Uses the sketch or test values to choose at least two intervals bounded by their critical values and ± 1.
	$x \leq -3 - \sqrt{6}$ or $-1 < x \leq \sqrt{6} - 3$ or $1 < x \leq \sqrt{3}$	**A1 A1**	**A1** Any two of the three intervals correct, with both ends and their inequality signs. **A1** All three and no others, ± 1 excluded and the other ends included. Accept set notation oe.
	Total for Question 14	**8**	

Question	Scheme	Marks	Notes
15(a)	$5+4x-x^2=mx+c$ $\Rightarrow x^2+(m-4)x+(c-5)=0$ $(m-4)^2-4(c-5)=0$	**M1**	Equates the line to $-(x^2-4x-5)$, forms a three-term quadratic and sets its discriminant to zero, or equates values and gradients at the contact point. Condone sign slips.
	$m^2-8m+16-4c+20=0$ $\Rightarrow m^2-8m-4c+36=0$	**A1 AG**	Correct discriminant set to zero, leading without error to the printed equation.
		[2]	
15(b)	$2=-2m+c \Rightarrow c=2m+2$ $m^2-8m-4(2m+2)+36=0$ $\Rightarrow m^2-16m+28=0 \Rightarrow m=\ldots$	**M1**	Substitutes $(-2,2)$ into the line and eliminates c (or m) from the equation in (a), then solves the quadratic by any means.
	$m=2$ or $m=14$	**A1**	
	Finds the point of contact, $x=\frac{4-m}{2}$ $m=14 \Rightarrow x=-5$, $m=2 \Rightarrow x=1$	**M1**	Finds the point of contact for at least one value of m, or rejects a value for another valid reason, such as the contact point lying below the x-axis.
	$m=14$ rejected, as -5 is not in $-1<x<5$ $m=2,\ c=6$	**A1**	$m=2$ and $c=6$ only, with $m=14$ rejected for a correct reason.
		[4]	
15(c)	$x^2-4x-5=2x+6$ $\Rightarrow x^2-6x-11=0 \Rightarrow x=\ldots$	**M1**	Equates x^2-4x-5 to their $mx+c$ from (b) and solves the three-term quadratic by any means.
	$x=3\pm 2\sqrt{5}$	**A1**	Both values, oe, or awrt -1.47 and 7.47.
	$x<3-2\sqrt{5}$ or $x>3+2\sqrt{5}$	**A1**	Both ranges with strict inequalities, oe or awrt as above. A0 if any interval inside $-1<x<5$ is included.
		[3]	
	Total for Question 15	**9**	