OPTIONAL PURE / FURTHER CALCULUS

L'Hôpital's Rule

Questions by topic 12 questions · 53 marks

EXAM BOARDS Edexcel FP1 9FM0/3A · AQA 7367

Questions

Question 1	*1 mark*	2
Question 2	*1 mark*	2
Question 3	*1 mark*	2
Question 4	*5 marks*	3
Question 5	*5 marks*	4
Question 6	*5 marks*	5
Question 7	*5 marks*	6
Question 8	*6 marks*	7
Question 9	*6 marks*	8
Question 10	*3 marks*	9
Question 11	*6 marks*	10
Question 12	*9 marks*	11

1. Which of the following limits is an indeterminate form to which L'Hôpital's rule can be applied directly?

$$\lim_{x\to 0}\left(\frac{e^{2x}}{\sin x}\right) \qquad \lim_{x\to \frac{\pi}{2}}\left(\frac{\cos x}{1+\sin x}\right) \qquad \lim_{x\to \infty}\left(\frac{\ln x}{x^2}\right) \qquad \lim_{x\to 0}\left(\frac{\sinh x}{e^x+1}\right) \quad \textbf{[1]}$$

---

2. Find

$$\lim_{x\to 0}\left(\frac{\tan 6x}{\ln(1+4x)}\right)$$

$$\frac{2}{3} \qquad 1 \qquad \frac{3}{2} \qquad 6 \quad \textbf{[1]}$$

---

3. Which one of the expressions below is equal to 1?

$$\lim_{x\to \infty}\left(\frac{\ln x}{x}\right) \qquad \lim_{x\to 0}\left(\frac{e^{2x}-1}{x}\right) \qquad \lim_{x\to 0^+}\left(x^x\right) \qquad \lim_{x\to 0}\left(\frac{\ln(1+3x)}{x}\right) \quad \textbf{[1]}$$

4. Use L'Hôpital's rule to determine

$$\lim_{x \to 0} \left(\frac{\sec(2x) - 1}{x \sin(3x)} \right)$$ **[5]**

5. Use L'Hôpital's rule to prove that

$$\lim_{x\to 1}\left(\frac{x\ln x - x + 1}{\sin^2(\pi x)}\right) = \frac{1}{2\pi^2} \qquad \textbf{[5]}$$

6. Use L'Hôpital's rule to show that

$$\lim_{x\to\frac{\pi}{6}} \frac{\ln(2\sin x) + e^{\cos(3x)} - 1}{\sin(6x)} = \frac{3-\sqrt{3}}{6} \qquad \textbf{[5]}$$

7. Prove that

$$\lim_{x\to 0}\left(\frac{\ln(\cosh x)-\frac{1}{2}\tan^2 x}{x^4}\right)=-\frac{5}{12} \qquad \textbf{[5]}$$

8. Use L'Hôpital's rule to show that

$$\lim_{x \to 0^+} \left(x(\ln x)^2 \right) = 0$$

Fully justify your answer. **[6]**

9. Use L'Hôpital's rule to find the exact value of

$$\lim_{x \to 0} \left(\frac{1}{\ln(1+x)} - \frac{1}{x} \right) \qquad \textbf{[6]}$$

10. Given that k is a positive constant, use the substitution $a = \frac{1}{p}$ to find

$$\lim_{p\to\infty}\left[p\ln\left(\frac{p+k}{p-k}\right)\right] \qquad \textbf{[3]}$$

11. **(a)** Write down the expansion of $\cos 3x$ in ascending powers of x up to and including the term in x^4. **[1]**

(b) The constant a is such that the following limit exists and is finite.

$$\lim_{x \to 0} \left[\frac{ax^2 + 2\cos 3x - 2}{x^3 \ln(1+x)} \right]$$

Find the value of a and the value of the limit. **[5]**

12. **(a)** (i) Write down the expansion of $\ln(1+3x)$ in ascending powers of x up to and including the term in x^4. **[1]**

(ii) Hence, or otherwise, find the first two non-zero terms in the expansion of

$$\ln\left(\frac{1+3x}{1-3x}\right)$$

in ascending powers of x, and state the range of values of x for which the expansion is valid. **[3]**

(b) Find

$$\lim_{x\to 0}\left[\frac{\ln\left(\frac{1+3x}{1-3x}\right)-6x}{x\left(2-\sqrt{4-x^2}\right)}\right] \quad \textbf{[5]}$$