OPTIONAL PURE / FURTHER CALCULUS

Arc Length

Questions by topic 12 questions · 94 marks

EXAM BOARDS Edexcel FP2 9FM0/4A · OCR A Y545 · AQA 7367 · CIE 9231

Questions

Question 1	*6 marks*	2
Question 2	*6 marks*	3
Question 3	*5 marks*	4
Question 4	*7 marks*	5
Question 5	*6 marks*	6
Question 6	*5 marks*	7
Question 7	*6 marks*	8
Question 8	*10 marks*	9
Question 9	*12 marks*	10
Question 10	*8 marks*	11
Question 11	*11 marks*	12
Question 12	*12 marks*	13

1. The curve C has equation $y = \frac{1}{2}\ln(\sin 2x)$, where $0 < x < \frac{1}{2}\pi$.
Find the exact length of the arc of C from the point where $x = \frac{1}{6}\pi$ to the point where $x = \frac{1}{3}\pi$, giving your answer in the form $a \ln b$, where a is rational and b is an integer. **[6]**

2. The curve C has equation

$$y = \frac{1}{2}x\sqrt{x^2-1} - \frac{1}{2}\operatorname{arcosh} x, \qquad 2 \leq x \leq 5$$

Using calculus, find the exact length of the curve C. **[6]**

3. The curve C has polar equation $r = \theta^2$ for $0 \leq \theta \leq \sqrt{5}$.
Find the length of C, giving your answer in an exact form. **[5]**

4. The curve C has parametric equations

$$x = \frac{4}{3}t^3 - \ln t, \qquad y = kt^{\frac{3}{2}}, \qquad 1 \leq t \leq 2$$

where k is a positive constant.
It is given that, for $1 \leq t \leq 2$,

$$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = \left(4t^2 + \frac{1}{t}\right)^2$$

Find the value of k, and find the exact length of C, giving your answer in the form $p + \ln q$, where p is rational and q is an integer. **[7]**

5. A curve has equation

$$y = \ln\left(\tanh\frac{x}{2}\right), \qquad x > 0$$

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \operatorname{cosech} x$. **[2]**

(b) Hence show that the arc length of the curve from $x = a$ to $x = b$, where $0 < a < b$, is equal to

$$\ln\left(\frac{\sinh b}{\sinh a}\right)$$ **[4]**

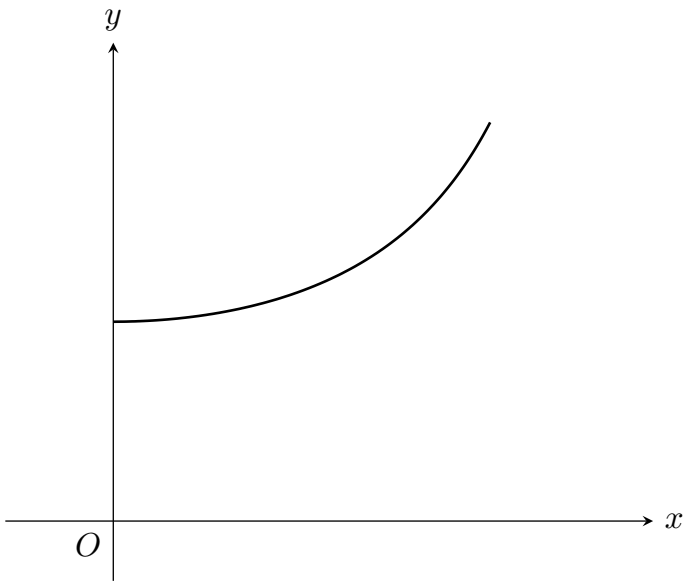
6. The curve C has parametric equations

$$x = 2\sinh t, \qquad y = \cosh 2t, \qquad 0 \leq t \leq \ln 2$$

The length of C is denoted by s.
By using the substitution $u = 2\sinh t$, show that

$$s = \int_0^{\frac{3}{2}} \sqrt{1+u^2}\,\mathrm{d}u \qquad \textbf{[5]}$$

7. The curve C has polar equation $r = 4e^{-\frac{5}{12}\theta}$ for $0 \leq \theta \leq \beta$, where $\beta > 0$.
Given that the length of C is s, find β in terms of s, and state the range of possible values of s. **[6]**



8. The diagram above shows a sketch of the curve C with equation

$$y = \frac{1}{\cos^2 x - \sin^2 x}, \qquad 0 \leq x \leq \frac{\pi}{6}$$

(a) Show that the length of C is given by

$$\int_0^{\frac{\pi}{6}} \left(2\sec^2 2x - 1\right)\, \mathrm{d}x \qquad \textbf{[6]}$$

(b) Hence find the exact length of C. **[4]**

9. The curve C has equation

$$y = 1 + 3\ln\left(x^2 - 9\right), \qquad 4 \leq x \leq 5$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and hence show that the length of C is given by

$$\int_4^5 \frac{x^2 + 9}{x^2 - 9}\,\mathrm{d}x \qquad \textbf{[4]}$$

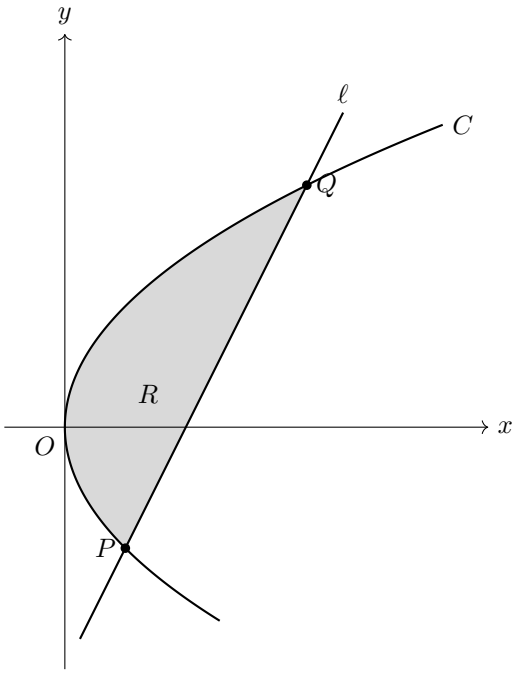
(b) Express $\frac{x^2+9}{x^2-9}$ in partial fractions. **[4]**

(c) Hence, using algebraic integration, find the exact length of C, giving your answer in the form $p + q\ln r$, where p and q are integers and r is a rational number. **[4]**

10. The curve C has parametric equations

$$x = \frac{1}{3}t^3 - t(\ln t)^2 + 2t\ln t - 2t + \frac{5}{3}, \qquad y = \frac{1}{2}t^2(2\ln t - 1) + \frac{1}{2}, \qquad 1 \leq t \leq e$$

Find the exact length of C, giving your answer in terms of e. **[8]**



11. The diagram above shows a sketch of part of the curve C with equation $y^2 = 4x$ and part of the line ℓ with equation $y = 2x - 4$.
The line ℓ meets C at the points P and Q. The region R, shown shaded in the diagram above, is bounded by C and ℓ.

(a) Show that the perimeter of R is given by

$$\alpha + \int_{\gamma}^{\beta} \sqrt{1 + \frac{y^2}{4}}\,\mathrm{d}y$$

where α, β and γ are constants to be determined. **[4]**

(b) Use the substitution $y = 2\sinh u$ and algebraic integration to determine the exact perimeter of R, giving your answer in simplest form. **[7]**

12. A curve has equation $y = 16\sqrt{x}$, where $x > 0$. The length of the arc of the curve between the points on the curve where $x = 36$ and $x = 225$ is denoted by s.

(a) Show that

$$s = \int_{36}^{225} \sqrt{\frac{x+64}{x}}\,\mathrm{d}x \qquad \textbf{[3]}$$

(b) (i) Show that $\operatorname{arsinh}\frac{15}{8} = 2\operatorname{arsinh}\frac{3}{4}$. **[3]**

(ii) Use the substitution $x = 64\sinh^2\theta$ to show that $s = p + q\ln 2$, where p and q are positive integers to be determined. **[6]**