OPTIONAL PURE / FURTHER CALCULUS

The t-Formulae

Mark scheme 10 questions · 93 marks

EXAM BOARDS Edexcel FP1 9FM0/3A

Questions

Question	Marks by part	Marks	Page
Question 1	(a) 1 (b)(i) 2 (b)(ii) 2 (c) 5	10	4
Question 2	(a) 3 (b) 5	8	5
Question 3	(a) 3 (b) 6	9	6
Question 4	(a) 2 (b) 3 (c) 2	7	7
Question 5	(a) 1 (b) 3 (c) 5	9	8
Question 6	(a) 1 (b) 3 (c) 5 (d) 1	10	9
Question 7	(a) 4 (b) 3	7	10
Question 8	(a) 3 (b) 4 (c) 6	13	11
Question 9	(a) 3 (b) 4 (c) 2	9	13
Question 10		11	14
	Total	**93**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last accuracy mark of the working that reaches a printed result: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [***n***] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	$\tan x = \frac{2t}{1-t^2}$	**B1**	
		[1]	
1(b)(i)	$\sin x = \tan x \cos x = \frac{2t}{1-t^2} \times \frac{1-t^2}{1+t^2}$	**M1**	Uses $\sin x = \tan x \cos x$ with their $\tan x$ of the form $\frac{2t}{1\pm t^2}$ and the given $\cos x$.
	$= \frac{2t}{1+t^2}$	**A1 AG**	Correct product from $\tan x = \frac{2t}{1-t^2}$, cancelling to the printed result without error.
		[2]	
1(b)(ii)	$\tan x = \frac{2/\sqrt{5}}{1-\frac{1}{5}} = \ldots$ $\sin x = \frac{2/\sqrt{5}}{1+\frac{1}{5}} = \ldots$	**M1**	Substitutes $t = \frac{1}{\sqrt{5}}$ into at least one of the t-formulae for $\tan x$ and $\sin x$.
	$\tan x = \frac{\sqrt{5}}{2}, \quad \sin x = \frac{\sqrt{5}}{3}$	**A1**	Both, any exact equivalents such as $\frac{5}{2\sqrt{5}}$ and $\frac{5}{3\sqrt{5}}$.
		[2]	
1(c)	$\frac{8t}{1+t^2} + \frac{3(1-t^2)}{1+t^2} + 3 = 0$ $\Rightarrow 8t + 3 - 3t^2 + 3 + 3t^2 = 0$	**M1**	Substitutes both t-formulae and multiplies every term by $1 + t^2$. Condone sign slips.
	$8t + 6 = 0 \Rightarrow t = -\frac{3}{4}$	**A1**	
	$\tan\left(\frac{\theta}{2}\right) =$ "$-\frac{3}{4}$" $\Rightarrow \theta = 2 \times \ldots$	**dM1**	Solves $\tan\left(\frac{\theta}{2}\right) =$ their numerical t for $\frac{\theta}{2}$ and doubles.
	$\theta = -73.7°$	**A1**	awrt $-73.7°$.
	Checks $\theta = 180°$, where $t = \tan 90°$ is undefined: $4\sin 180° + 3\cos 180° + 3 = 0$, so $\theta = 180°$	**E1**	States t is undefined at $\theta = 180°$ and shows $\theta = 180°$ satisfies the equation, giving it as a solution. Both points needed.
		[5]	
Total for Question 1		**10**	

Question	Scheme	Marks	Notes
2(a)	$7\left(\frac{2t}{1+t^2}\right)+\frac{1-t^2}{1+t^2}=5$	**M1**	Substitutes t-formulae of the correct form for both $\sin x$ and $\cos x$. Condone one sign slip in a formula.
	$14t+1-t^2=5+5t^2$ $\Rightarrow 6t^2-14t+4=0$	**dM1**	Multiplies through by $1+t^2$ and collects terms into a three-term quadratic. Condone sign slips.
	$3t^2-7t+2=0$	**A1 AG**	Correct substituted equation leading without error to the printed quadratic; combined steps accepted.
		[3]	
2(b)	$(3t-1)(t-2)=0\Rightarrow t=\ldots$	**M1**	Solves the quadratic by any method; calculator accepted.
	$t=\frac{1}{3},\ t=2$	**A1**	
	$\frac{x}{2}=\arctan$ "$\frac{1}{3}$" $[\approx 18.43^\circ]$ or "18.43°" $+180^\circ$ $\Rightarrow x=\ldots$	**M1**	Uses arctan of their t for $\frac{x}{2}$, adds 180° and doubles, for at least one value of t.
	$x=36.9^\circ,\ 126.9^\circ,\ 396.9^\circ,\ 486.9^\circ$	**A1**	All four, to 1 d.p., and no others in $0^\circ\le x\le 720^\circ$.
	$\tan\left(\frac{x}{2}\right)$ undefined at $x=180^\circ,\ 540^\circ$ $7\sin x+\cos x=7(0)+(-1)=-1\neq 5$	**E1**	States t is undefined only at $x=180^\circ$ and 540°, and shows neither satisfies the equation.
		[5]	
	Total for Question 2	**8**	

Question	Scheme	Marks	Notes
3(a)	$\sin 2\theta + 3\cos 2\theta + 2$ $= \frac{2t}{1+t^2} + \frac{3(1-t^2)}{1+t^2} + 2$	**M1**	Substitutes expressions in t of the form $\frac{\ldots}{1+t^2}$ for both $\sin 2\theta$ and $\cos 2\theta$. Condone a sign slip in one.
	$= \frac{2t+3-3t^2+2+2t^2}{1+t^2}$	**dM1**	Writes all three terms over $1+t^2$, including the 2, and combines the numerators. Condone sign slips.
	$= \frac{5+2t-t^2}{1+t^2}$	**A1 AG**	Correct substituted expression leading without error to the printed result; combined steps accepted.
		[3]	
3(b)	$\frac{5+2t-t^2}{1+t^2} = 3 \Rightarrow 5+2t-t^2 = 3+3t^2$	**M1**	Sets the expression from (a) equal to 3 and clears the denominator to reach a quadratic in t. Condone sign slips.
	$2t^2 - t - 1 = 0$	**A1**	oe, such as $4t^2 - 2t - 2 = 0$.
	$(2t+1)(t-1) = 0 \Rightarrow t = \ldots$	**M1**	Solves their three-term quadratic: factors whose constants multiply to its constant term, the formula, or completing the square. Calculator allowed.
	$t = 1$ or $t = -\frac{1}{2}$	**A1**	
	$\theta = n\pi + \arctan$ "1" $\theta = n\pi + \arctan\left(\text{"}-\frac{1}{2}\text{"}\right)$	**M1**	Uses arctan with period π (or $180°$) for at least one of their values of t, as $\theta = n\pi + \arctan t$ oe.
	$\theta = n\pi + \frac{\pi}{4}$ or $\theta = n\pi - \arctan\frac{1}{2}$	**A1**	Both families, exact, oe such as $n\pi + \arctan\left(-\frac{1}{2}\right)$ or degrees; $n \in \mathbb{Z}$ need not be stated. A0 for a decimal angle or an extra family.
		[6]	
Total for Question 3		**9**	

Question	Scheme	Marks	Notes
4(a)	$\operatorname{cosec} x + \cot x = \frac{1+t^2}{2t} + \frac{1-t^2}{2t}$	**M1**	Substitutes the t-formulae for $\sin x$ and $\cos x$ into both terms. Condone sign slips.
	$= \frac{2}{2t} = \frac{1}{t}$	**A1 AG**	Correct substituted expression combined without error to $\frac{1}{t}$; combined steps accepted.
		[2]	
4(b)	$\operatorname{cosec} x - \cot x = \frac{1+t^2}{2t} - \frac{1-t^2}{2t} = \ldots$	**M1**	Writes $\operatorname{cosec} x - \cot x$, or $1 \pm \cos x$, in terms of t using the t-formulae. Condone sign slips.
	$\operatorname{cosec} x - \cot x = t$ $\Rightarrow \frac{\operatorname{cosec} x + \cot x}{\operatorname{cosec} x - \cot x} = \frac{1/t}{t} = \frac{1}{t^2}$	**A1**	LHS $= \frac{1}{t^2}$ oe, by any correct route.
	$\frac{1+\cos x}{1-\cos x} = \frac{2}{1+t^2} \times \frac{1+t^2}{2t^2}$ $= \frac{1}{t^2}$, so LHS = RHS	**A1 AG**	RHS reached as $\frac{1}{t^2}$ without error, with a conclusion such as LHS = RHS.
		[3]	
4(c)	$\frac{1+\cos x}{1-\cos x} = 3 \Rightarrow \cos x = \ldots$	**M1**	Uses part (b) to set $\frac{1+\cos x}{1-\cos x} = 3$, or $\frac{1}{t^2} = 3$, and solves for $\cos x$ or t. Condone sign slips.
	$\cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3},\ \frac{5\pi}{3}$	**A1**	Both values, or awrt 1.05 and 5.24, with no others in $0 < x < 2\pi$.
		[2]	
	Total for Question 4	**7**	

Question	Scheme	Marks	Notes
5(a)	$x = 59 \Rightarrow T \approx 8.67$	**B1**	awrt 8.67, from $x = 59$.
		[1]	
5(b)	$T = 1.2\left(\frac{2t}{1+t^2}\right) - 6\left(\frac{1-t^2}{1+t^2}\right) + 11$	**M1**	Replaces both $\sin\left(\frac{x}{60}\right)$ and $\cos\left(\frac{x}{60}\right)$ by t-formulae in $t = \tan\left(\frac{x}{120}\right)$. Condone one slip in a formula.
	$= \frac{2.4t-6+6t^2+11+11t^2}{1+t^2}$	**dM1**	Combines all three terms, including the constant 11, over the single denominator $1 + t^2$. Condone sign slips.
	$T = \frac{17t^2+2.4t+5}{1+t^2}$	**A1**	Correct expression, or $a = 17$, $b = 2.4$, $c = 5$ stated; $b = \frac{12}{5}$ accepted.
		[3]	
5(c)	$\frac{\text{“}17t^2+2.4t+5\text{”}}{1+t^2} \geq 15$ $\Rightarrow 2t^2 + 2.4t - 10 \geq 0$	**M1**	Sets their expression of the form $\frac{at^2+bt+c}{1+t^2}$ equal to, or at least, 15 and multiplies by $1 + t^2$ to reach a three-term quadratic in t.
	$t^2 + 1.2t - 5 = 0 \Rightarrow t = \frac{-1.2\pm\sqrt{21.44}}{2}$ $t \approx 1.715, \quad t \approx -2.915$	**A1**	Correct quadratic oe and both critical values, exact or awrt 1.72 and -2.92.
	$x = 120\arctan(\text{“}1.715\text{”}) = \ldots$ $x = 120\left(\pi - \arctan(\text{“}2.915\text{”})\right) = \ldots$	**dM1**	Uses $x = 120\arctan t$, or $x = 120(\pi + \arctan t)$ for a negative t, for at least one of their values.
	$x \approx 125.15, \quad x \approx 228.15$	**A1**	Both, awrt 125.2 and 228.2.
	$x = 126$: 7th May 2031 $x = 228$: 17th August 2031	**A1**	Both dates, cao. The check at $x = 60\pi$ is not needed.
		[5]	
Total for Question 5		**9**	

Question	Scheme	Marks	Notes
6(a)	$x = 6 \Rightarrow D = 6 + 0 - \frac{16}{5} = 2.8\,\text{m}$	**B1**	2.8 or $\frac{14}{5}$.
		[1]	
6(b)	$D = 6 + \frac{12}{5} \times \frac{2t}{1+t^2} + \frac{16}{5} \times \frac{1-t^2}{1+t^2}$	**M1**	Substitutes t-formulae for both $\sin(30x)^\circ$ and $\cos(30x)^\circ$, with $t = \tan(15x)^\circ$. Condone sign slips.
	$D = \dfrac{30\left(1+t^2\right) + 24t + 16\left(1-t^2\right)}{5\left(1+t^2\right)}$	**dM1**	Combines all three terms over a single denominator $5\left(1+t^2\right)$ or a multiple of it. Condone slips in the numerator.
	$D = \dfrac{14t^2 + 24t + 46}{5\left(1+t^2\right)}$ $A = 14,\ B = 24,\ C = 46$	**A1**	cso. Correct expression, or A, B, C stated.
		[3]	
6(c)	$\dfrac{\text{“}14t^2 + 24t + 46\text{”}}{5\left(1+t^2\right)} = 4$ $\Rightarrow 3t^2 - 12t - 13 = 0 \Rightarrow t = \ldots$	**M1**	Sets their expression from (b) equal to 4 and rearranges to a three-term quadratic in t, solved by any means.
	$t = 2 \pm \frac{5\sqrt{3}}{3}$ $[\approx 4.887,\ -0.887]$	**A1**	Both roots, exact or awrt 4.89 and -0.887.
	$15x = \arctan$ “4.887” $\Rightarrow x = \ldots$ $15x = 180 - \arctan$ “0.887” $\Rightarrow x = \ldots$	**dM1**	Finds x from $15x = \arctan t$ for their roots, taking $0 \leq 15x < 180$, so adding 180 for a negative root.
	$x \approx 5.229$: 05:14	**A1**	One time correct to the nearest minute, oe.
	$x \approx 9.229$: 09:14	**A1**	Both 05:14 and 09:14, oe, and no others in $0 \leq x < 12$.
		[5]	
6(d)	$x = 6 \Rightarrow t = \tan 90^\circ$, undefined	**E1**	States that at $x = 6$, $t = \tan 90^\circ$, which is undefined, oe.
		[1]	
	Total for Question 6	**10**	

Question	Scheme	Marks	Notes
7(a)	$\dfrac{13+\frac{12(1-t^2)}{1+t^2}+\frac{10t}{1+t^2}}{13-\frac{12(1-t^2)}{1+t^2}-\frac{10t}{1+t^2}}$	**M1**	Substitutes t-formulae for both $\sin\theta$ and $\cos\theta$ in numerator and denominator. Condone sign slips.
	$=\dfrac{13(1+t^2)+12(1-t^2)+10t}{13(1+t^2)-12(1-t^2)-10t}$	**dM1**	Clears the fractions, by multiplying through by $1+t^2$ or combining over a common denominator. Condone sign slips.
	$=\dfrac{t^2+10t+25}{25t^2-10t+1}$	**A1**	
	$=\dfrac{(t+5)^2}{(5t-1)^2}=\left(\dfrac{t+5}{5t-1}\right)^2$	**A1 AG**	Correct expression in t without error, matched to the printed result by factorising or expanding the right-hand side, with a conclusion such as LHS $\equiv$ RHS.
		[4]	
7(b)	$\tan\left(\frac{\theta}{2}-\alpha\right)=\dfrac{t-\frac{1}{5}}{1+\frac{1}{5}t}$	**M1**	Uses the addition formula for tangent with $t=\tan\left(\frac{\theta}{2}\right)$ and $\tan\alpha=\frac{1}{5}$. Condone sign slips; not for quoting the formula alone.
	$=\dfrac{5t-1}{t+5}$	**A1**	oe, such as $\cot\left(\frac{\theta}{2}-\alpha\right)=\frac{t+5}{5t-1}$ directly.
	$\cot\left(\frac{\theta}{2}-\alpha\right)=\dfrac{t+5}{5t-1}$ $\Rightarrow \text{LHS}\equiv\left(\dfrac{t+5}{5t-1}\right)^2\equiv\cot^2\left(\frac{\theta}{2}-\alpha\right)$	**A1 AG**	Reciprocal squared and equated to the result of (a), reaching the printed identity with no errors seen.
		[3]	
	Total for Question 7	**7**	

Question	Scheme	Marks	Notes
8(a)	$\sin x = 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)$	**M1**	Uses the double angle formula to write $\sin x$ in terms of $\sin\left(\frac{x}{2}\right)$ and $\cos\left(\frac{x}{2}\right)$.
	$= \frac{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)+\sin^2\left(\frac{x}{2}\right)}$ $= \frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$	**dM1**	Divides by $\cos^2\left(\frac{x}{2}\right)+\sin^2\left(\frac{x}{2}\right)$ then through by $\cos^2\left(\frac{x}{2}\right)$, or uses $\cos^2\left(\frac{x}{2}\right) = \frac{1}{1+\tan^2\left(\frac{x}{2}\right)}$ oe, to reach an expression in $\tan\left(\frac{x}{2}\right)$. A right-angled triangle for $\frac{x}{2}$ accepted.
	$= \frac{2t}{1+t^2}$	**A1 AG**	Printed result reached without error, in either direction; combined steps accepted. Not for a right-angled triangle.
		[3]	
8(b)	$\frac{8t}{1-t^2} + \frac{6t}{1+t^2} + \frac{6(1-t^2)}{1+t^2} + 6 = 0$	**M1 A1**	**M1** Substitutes t-formulae for all three of $\tan x$, $\sin x$ and $\cos x$. Condone one slip in a formula. **A1** Correct equation in t, oe.
	$8t(1+t^2) + (6t+12)(1-t^2) = 0$	**dM1**	Multiplies through by $(1-t^2)(1+t^2)$, or combines over a common denominator, and expands to a polynomial equation. Condone sign slips.
	$2t^3 - 12t^2 + 14t + 12 = 0$ $\Rightarrow t^3 - 6t^2 + 7t + 6 = 0$	**A1 AG**	Correct substituted equation leading without error to the printed cubic; combined steps accepted.
		[4]	
8(c)	$t = 3:\ 27 - 54 + 21 + 6 = 0$ $\Rightarrow (t-3)(t^2 + \ldots) = 0$	**M1**	Finds a root by the factor theorem and attempts the quadratic factor. Calculator roots earn M1 A1 M1 A1 when all three are correct.
	$(t-3)(t^2 - 3t - 2) = 0$	**A1**	
	$t^2 - 3t - 2 = 0 \Rightarrow t = \ldots$	**M1**	Solves their three-term quadratic factor by formula, completing the square or calculator.
	$t = 3,\ \frac{3+\sqrt{17}}{2},\ \frac{3-\sqrt{17}}{2}$	**A1**	All three values, exact or awrt 3.56 and -0.562.
	$x = 2\arctan t \Rightarrow x = \ldots$	**M1**	Uses $x = 2\arctan t$ for at least one of their values of t. Not for $x = \arctan t$.

Question 8 continued

Question	Scheme	Marks	Notes
	$x = -58.6°,\ 143.1°,\ 148.6°$	**A1**	awrt $-58.6°$, $143.1°$ and $148.6°$, and no others in the range.
		[6]	
	Total for Question 8	**13**	

Question	Scheme	Marks	Notes
9(a)	$1+\cos x = 1+\frac{1-t^2}{1+t^2}$ $3+2\sin x+\cos x$ $\quad = 3+\frac{4t}{1+t^2}+\frac{1-t^2}{1+t^2}$	**M1**	Substitutes t-formulae, or equivalent half-angle identities in t, into both numerator and denominator. Condone one sign or coefficient slip.
	$1+\cos x = \frac{2}{1+t^2}$ $3+2\sin x+\cos x = \frac{2t^2+4t+4}{1+t^2}$	**A1**	Both correct, oe, as single fractions or after multiplying through by $1+t^2$.
	$f(x) = \frac{2}{2t^2+4t+4} = \frac{1}{t^2+2t+2}$ $\quad = \frac{1}{(t+1)^2+1}$	**A1 AG**	Cancels $1+t^2$ and completes the square, without error, to the printed form; combined steps accepted.
		[3]	
9(b)	$\frac{1}{(t+1)^2+1} = \frac{1}{5} \Rightarrow (t+1)^2 = 4$ $\Rightarrow t = \ldots$	**M1**	Sets the form from (a) equal to $\frac{1}{5}$ and solves for t by any method. Condone sign slips.
	$t = 1, \quad t = -3$	**A1**	
	$\tan\left(\frac{x}{2}\right) =$ "1" $\Rightarrow \frac{x}{2} = \ldots$ $\tan\left(\frac{x}{2}\right) =$ "-3" $\Rightarrow \frac{x}{2} = \pi - \arctan 3$ $\Rightarrow x = \ldots$	**M1**	Solves $\tan\left(\frac{x}{2}\right) =$ their t with $0 < \frac{x}{2} < \pi$ and doubles; a negative t needs $\frac{x}{2}$ in the second quadrant.
	$x = \frac{\pi}{2}, \quad x = 2\pi - 2\arctan 3 \approx 3.79$	**A1**	$\frac{\pi}{2}$ exact, and awrt 3.79 or $2\pi - 2\arctan 3$, with no others in $0 < x < 2\pi$.
		[4]	
9(c)	$(t+1)^2 = 0 \Rightarrow \tan\left(\frac{x}{2}\right) = -1 \Rightarrow x = \ldots$	**M1**	Uses $(t+1)^2 = 0$, or $t = -1$ by any method, for the greatest value and solves $\tan\left(\frac{x}{2}\right) = -1$ for x.
	Greatest value 1, at $x = \frac{3\pi}{2}$	**A1**	Both, with x exact.
		[2]	
	Total for Question 9	**9**	

Question	Scheme	Marks	Notes
10	$4\left(\frac{2t}{1+t^2}\right)+\frac{2t}{1-t^2}=\frac{3}{t}$	**M1 A1**	**M1** Substitutes t-formulae for all three of $\sin x$, $\tan x$ and $\cot\left(\frac{x}{2}\right)$. Condone sign slips in the formulae. **A1** Correct equation in t only, oe.
	$8t^2(1-t^2)+2t^2(1+t^2)=3(1-t^4)$	**dM1**	Multiplies through by a common denominator to an equation with no fractions. Condone expansion slips.
	$3t^4-10t^2+3=0$	**A1**	Any equivalent polynomial form.
	$(3t^2-1)(t^2-3)=0\Rightarrow t^2=\ldots$	**M1**	Solves their three-term quadratic in t^2 by factorising, formula or completing the square, to $t^2=\ldots$; a calculator is allowed.
	$t=\pm\frac{1}{\sqrt{3}},\quad t=\pm\sqrt{3}$	**A1**	All four values, oe.
	$\tan\left(\frac{x}{2}\right)=\frac{1}{\sqrt{3}}\Rightarrow\frac{x}{2}=\frac{\pi}{6}+n\pi$ $\Rightarrow x=\ldots$	**M1**	Solves $\tan\left(\frac{x}{2}\right)=$ one of their values, adding a multiple of π for $\frac{x}{2}$, then doubles.
	$x=\pm\frac{\pi}{3}+2n\pi,\quad x=\pm\frac{2\pi}{3}+2n\pi$	**A1**	All four families, oe; implied by a correct combined form.
	Checks where t is undefined, $x=(2n+1)\pi$: $4\sin x+\tan x=\ldots,\quad 3\cot\left(\frac{x}{2}\right)=\ldots$	**M1**	Identifies $x=(2n+1)\pi$, or $x=\pi$, as where the substitution fails and substitutes into the original equation.
	LHS $=0=$ RHS $x=(2n+1)\pi$ are lost solutions	**A1**	Both sides shown to be 0, concluding $x=(2n+1)\pi$, oe, are solutions.
	$x=n\pi\pm\frac{\pi}{3}$ or $x=(2n+1)\pi,\ n\in\mathbb{Z}$	**A1**	Families combined to $x=n\pi\pm\frac{\pi}{3}$ oe, with $x=(2n+1)\pi$ oe. Not for four separate families.
	Total for Question 10	**11**	