OPTIONAL PURE / FURTHER CALCULUS

The Weierstrass Substitution

Mark scheme 10 questions · 82 marks

EXAM BOARDS Edexcel FP1 9FM0/3A · CIE 9231

Questions

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	Total	**82**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last accuracy mark of the working that reaches a printed result: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1	$\int \frac{1}{5+4\left(\frac{1-t^2}{1+t^2}\right)} \times \frac{2}{1+t^2}\,\mathrm{d}t$	**M1**	Substitutes for both $\cos x$ and $\mathrm{d}x$ in terms of t. Condone sign or coefficient slips; quoting the identities alone is not enough.
	$= \int \frac{2}{t^2+9}\,\mathrm{d}t$	**A1**	Correct integrand in t, oe. Limits need not be shown here.
	$\int \frac{2}{t^2+9}\,\mathrm{d}t = k\arctan\left(\frac{t}{3}\right)$	**M1**	Integrates their $\frac{p}{t^2+q}$, $q>0$, to the form $k\arctan\left(\frac{t}{\sqrt{q}}\right)$.
	$= \frac{2}{3}\arctan\left(\frac{t}{3}\right)$	**A1**	
	$\left[\frac{2}{3}\arctan\left(\frac{t}{3}\right)\right]_0^{\sqrt{3}}$ $= \frac{2}{3}\arctan\left(\frac{\sqrt{3}}{3}\right) - \frac{2}{3}\arctan 0$	**dM1**	Applies limits 0 and $\sqrt{3}$ to their arctan form in t, or returns to x and uses 0 and $\frac{2}{3}\pi$.
	$= \frac{2}{3} \times \frac{\pi}{6} = \frac{\pi}{9}$	**A1 AG**	cso. Correct limits substituted and evaluated exactly, without error, to $\frac{\pi}{9}$. A decimal check alone scores nothing.
	Total for Question 1	**6**	

Question	Scheme	Marks	Notes
2	$1+\sin x+2\cos x = 1+\frac{2t}{1+t^2}+\frac{2(1-t^2)}{1+t^2}$ $\mathrm{d}x = \frac{2}{1+t^2}\,\mathrm{d}t$	**M1**	Replaces $\sin x$, $\cos x$ and $\mathrm{d}x$ by expressions in t of the forms $\frac{2t}{1+t^2}$, $\frac{1-t^2}{1+t^2}$, $\frac{k}{1+t^2}\,\mathrm{d}t$. Condone one sign or coefficient slip.
	$I = \int \frac{2}{3+2t-t^2}\,\mathrm{d}t$	**A1**	oe, such as $\frac{2}{(3-t)(1+t)}$.
	$\frac{2}{(3-t)(1+t)} = \frac{A}{3-t}+\frac{B}{1+t} \Rightarrow A=\ldots,\ B=\ldots$ $\Rightarrow \ldots\ln(3-t)+\ldots\ln(1+t)$	**M1**	Uses partial fractions on their quadratic denominator, or completes the square, and integrates to ln or artanh terms. Condone sign and coefficient slips.
	$\frac{1}{2}\ln(1+t)-\frac{1}{2}\ln(3-t)$	**A1**	oe, such as $\frac{1}{2}\ln\frac{1+t}{3-t}$.
	$I = \frac{1}{2}\left[\ln(1+t)-\ln(3-t)\right]_0^1$ $= \frac{1}{2}\left((\ln 2-\ln 2)-(\ln 1-\ln 3)\right)$	**dM1**	Substitutes $t=0$ and $t=1$, or $x=0$ and $x=\frac{\pi}{2}$ after returning to x, into their integral and subtracts.
	$I = \frac{1}{2}\ln 3$	**A1**	Any equivalent $p\ln q$ form, such as $\frac{1}{4}\ln 9$.
	Total for Question 2	**6**	

Question	Scheme	Marks	Notes
3(a)	$\int \frac{1+t^2}{3(1+t^2)-4t-(1-t^2)} \times \frac{2}{1+t^2}\,\mathrm{d}t$	**M1**	Replaces $\sin\theta$, $\cos\theta$ and $\mathrm{d}\theta$ by expressions in t. Condone one sign or coefficient slip.
	$\int \frac{2}{4t^2-4t+2}\,\mathrm{d}t$, $4t^2-4t+2=(2t-1)^2+1$	**M1**	Completes the square on their three-term quadratic denominator, to $(bt+c)^2+r$ or $p(t+q)^2+r$. Condone arithmetic slips.
	$\int \frac{2}{(2t-1)^2+1}\,\mathrm{d}t \Rightarrow a=2,\ b=2,\ c=-1$	**A1**	All three, stated or seen in the integral; cso.
		[3]	
3(b)	$\int_0^1 \frac{2}{(2t-1)^2+1}\,\mathrm{d}t = [k\arctan(2t-1)]_0^1$	**M1**	Integrates their part (a) form to $k\arctan(bt+c)$, any nonzero k.
	$[\arctan(2t-1)]_0^1$	**A1**	Correct, oe, such as $\frac{1}{2}\times 2\arctan\left(\frac{t-\frac{1}{2}}{\frac{1}{2}}\right)$.
	$\arctan 1-\arctan(-1)$	**dM1**	Substitutes $t=1$ and $t=0$ and subtracts, or returns to θ and uses $\frac{\pi}{2}$ and 0.
	$\frac{\pi}{2}$	**A1**	
		[4]	
	Total for Question 3	**7**	

Question	Scheme	Marks	Notes
4	$\int_0^1 \frac{1}{3+5\left(\frac{1-t^2}{1+t^2}\right)} \times \frac{2}{1+t^2}\,\mathrm{d}t$	**M1**	Replaces both $\cos\theta$ and $\mathrm{d}\theta$ by expressions in t of the correct form. Condone sign slips; not if $\mathrm{d}\theta$ is left unconverted.
	$\int_0^1 \frac{1}{4-t^2}\,\mathrm{d}t$	**A1**	oe, such as $\frac{2}{8-2t^2}$. Limits not needed for this mark.
	$\frac{1}{4-t^2} = \frac{A}{2-t} + \frac{B}{2+t} \Rightarrow A = \ldots,\ B = \ldots$ $\int \frac{1}{4-t^2}\,\mathrm{d}t = p\ln(2-t) + q\ln(2+t)$	**M1**	Uses partial fractions with linear denominators, or the standard result for $\int \frac{1}{a^2-t^2}\,\mathrm{d}t$, reaching log or artanh terms. Condone sign and coefficient slips.
	$\frac{1}{4}\left[\ln(2+t) - \ln(2-t)\right]_0^1$	**A1**	oe, such as $\frac{1}{4}\ln\left(\frac{2+t}{2-t}\right)$ or $\frac{1}{2}\operatorname{artanh}\frac{1}{2}t$. Modulus signs condoned.
	$\frac{1}{4}\left[(\ln 3 - \ln 1) - (\ln 2 - \ln 2)\right]$	**dM1**	Substitutes $t = 1$ and $t = 0$, or returns to θ and uses $\frac{1}{2}\pi$ and 0, into their integral and subtracts. Not for θ-limits in a function of t.
	$\frac{1}{4}\ln 3$	**A1**	Any exact equivalent, such as $\frac{1}{2}\ln\sqrt{3}$ or $\frac{1}{2}\operatorname{artanh}\frac{1}{2}$; cso.
	Total for Question 4	**6**	

Question	Scheme	Marks	Notes
5	$\int \frac{1+t^2}{5(1+t^2)+4(1-t^2)+6t} \times \frac{2}{1+t^2}\,\mathrm{d}t$	**M1**	Replaces $\cos x$, $\sin x$ and $\mathrm{d}x$, all three, by expressions in t of the form $\frac{\ldots}{1+t^2}$. Condone sign and coefficient slips.
	$= \int \frac{2}{(t+3)^2}\,\mathrm{d}t$	**A1**	oe, such as $\frac{2}{t^2+6t+9}$.
	$-\frac{2}{t+3}$	**M1 A1**	**M1** Integrates their $\frac{k}{(t+a)^2}$ to the form $\frac{c}{t+a}$.
	$\left(-\frac{2}{3+\sqrt{3}}\right) - \left(-\frac{2}{3}\right)$	**dM1**	Substitutes limits $\sqrt{3}$ and 0 in t, or reverts to x and uses $\frac{2\pi}{3}$ and 0, and subtracts. Not for x-limits in a function of t.
	$= \frac{\sqrt{3}-1}{3}$	**A1**	Any exact equivalent, such as $\frac{2}{3} - \frac{2}{3+\sqrt{3}}$.
	Total for Question 5	**6**	

Question	Scheme	Marks	Notes
6	$\sin x = \frac{2t}{1+t^2},\ \mathrm{d}x = \frac{2}{1+t^2}\,\mathrm{d}t$ $\Rightarrow \int \frac{1+t^2}{t^2+4t+1} \times \frac{2}{1+t^2}\,\mathrm{d}t$	**M1**	Substitutes for both $\sin x$ and $\mathrm{d}x$ in terms of t. Condone a slip in one of them.
	$\int_0^{\frac{1}{\sqrt{3}}} \frac{2}{t^2+4t+1}\,\mathrm{d}t$	**A1**	Limits need not be seen here.
	$t^2+4t+1 = (t+2)^2 - 3$	**M1**	Completes the square on their quadratic to $(t+\alpha)^2 \pm \beta$, or factorises it for partial fractions. Condone sign slips.
	$\int \frac{2}{(t+2)^2-3}\,\mathrm{d}t$ $= \frac{1}{\sqrt{3}} \ln\left(\frac{t+2-\sqrt{3}}{t+2+\sqrt{3}}\right)$	**M1 A1**	**M1** Integrates their $\frac{2}{(t+\alpha)^2 \pm \beta}$ to $k\ln\left(\frac{t+\alpha-\sqrt{\beta}}{t+\alpha+\sqrt{\beta}}\right)$ or $k\arctan\left(\frac{t+\alpha}{\sqrt{\beta}}\right)$, as their sign requires, or their partial fractions to a sum of logarithms. Condone a wrong constant k. **A1** Correct, oe, with or without modulus signs.
	$\frac{1}{\sqrt{3}} \ln\left(\frac{\frac{1}{\sqrt{3}}+2-\sqrt{3}}{\frac{1}{\sqrt{3}}+2+\sqrt{3}}\right)$ $- \frac{1}{\sqrt{3}} \ln\left(\frac{2-\sqrt{3}}{2+\sqrt{3}}\right)$	**dM1 A1**	**dM1** Substitutes $t = \frac{1}{\sqrt{3}}$ and $t = 0$ into their logarithmic integral and subtracts. Not for the x-limits used in t. **A1** Correct exact expression; need not be simplified.
	$\frac{1}{\sqrt{3}} \ln\left(\frac{3\sqrt{3}-5}{7-4\sqrt{3}}\right)$ $= \frac{1}{\sqrt{3}} \ln\left((3\sqrt{3}-5)(7+4\sqrt{3})\right)$	**M1**	Combines their logarithms into one and rationalises at least one surd denominator using its conjugate. Condone arithmetic slips.
	$\frac{1}{3}\sqrt{3}\ln\left(1+\sqrt{3}\right),\ p = \frac{1}{3},\ q = 1$	**A1**	cso. Both values, or the answer in the required form; $\frac{1}{\sqrt{3}}\ln\left(1+\sqrt{3}\right)$ alone scores nothing.
	Total for Question 6	**9**	

Question	Scheme	Marks	Notes
7(a)	$\frac{\cos x}{2-\cos x} = -1 + \frac{2}{2-\cos x}$ $\Rightarrow \int_0^{\frac{1}{2}\pi} -1 \, \mathrm{d}x = -\frac{\pi}{2}$	**B1**	$A = -1$, $B = 2$ and the first integral evaluated as $-\frac{\pi}{2}$.
	$\int \frac{2}{2-\frac{1-t^2}{1+t^2}} \times \frac{2}{1+t^2} \, \mathrm{d}t$	**M1**	Replaces both $\cos x$ and $\mathrm{d}x$ by expressions in t, with $\mathrm{d}x$ of the form $\frac{k}{1+t^2} \, \mathrm{d}t$. Condone sign or coefficient slips.
	$= \int \frac{4}{1+3t^2} \, \mathrm{d}t$	**A1**	Correct simplified integrand in t.
	$x = 0 \Rightarrow t = 0, \quad x = \frac{1}{2}\pi \Rightarrow t = 1$	**B1**	
	$I = -\frac{\pi}{2} + \int_0^1 \frac{4}{1+3t^2} \, \mathrm{d}t$	**A1 AG**	Correct substitutions, limits and $-\frac{\pi}{2}$ leading without error to the printed result; combined steps accepted.
		[5]	
7(b)	$\frac{4}{3} \int_0^1 \frac{1}{t^2+\frac{1}{3}} \, \mathrm{d}t = \left[k \arctan\left(\sqrt{3}\, t\right)\right]_0^1$	**M1**	Integrates to the form $k \arctan\left(\sqrt{3}\, t\right)$. Condone an incorrect k.
	$= \left[\frac{4\sqrt{3}}{3} \arctan\left(\sqrt{3}\, t\right)\right]_0^1$	**A1**	Correct integral, oe.
	$I = -\frac{\pi}{2} + \frac{4\sqrt{3}}{3}\left(\arctan\sqrt{3} - \arctan 0\right)$	**dM1**	Substitutes both limits into their $k \arctan\left(\sqrt{3}\, t\right)$ and adds $-\frac{\pi}{2}$.
	$I = -\frac{1}{2}\pi + \frac{4}{9}\pi\sqrt{3}$	**A1**	In the required form, or $p = -\frac{1}{2}$ and $q = \frac{4}{9}$ stated.
		[4]	
	Total for Question 7	**9**	

Question	Scheme	Marks	Notes
8(a)	$\mathrm{d}x = \frac{2}{1+t^2}\,\mathrm{d}t$ $\sin x = \frac{2t}{1+t^2},\ \cos x = \frac{1-t^2}{1+t^2}$	**M1**	Replaces $\sin x$, $\cos x$ and $\mathrm{d}x$ by expressions in t of the correct form. The t-formulae may be quoted. Condone one slip.
	$\frac{\sin x}{1+\sin x+\cos x} = \frac{2t}{(1+t^2)+2t+(1-t^2)}$ $= \frac{t}{1+t}$	**A1**	$\frac{t}{1+t}$ oe; may be implied by a correct integrand combined with the $\mathrm{d}x$ factor.
	$x = 0 \Rightarrow t = 0,\ x = \frac{\pi}{2} \Rightarrow t = 1$	**B1**	
	$I = \int_0^1 \frac{t}{1+t} \times \frac{2}{1+t^2}\,\mathrm{d}t$ $= \int_0^1 \frac{2t}{(1+t)(1+t^2)}\,\mathrm{d}t$	**A1 AG**	Correct substituted integral with t limits, leading without error to the printed result; combined steps accepted.
		[4]	
8(b)	$\frac{2t}{(1+t)(1+t^2)} = \frac{A}{1+t} + \frac{Bt+C}{1+t^2}$	**M1**	Uses the correct form, with a linear numerator over $1+t^2$.
	$2t = A(1+t^2) + (Bt+C)(1+t)$ $\Rightarrow A = \ldots,\ B = \ldots,\ C = \ldots$	**dM1**	Multiplies through and finds all three constants by substitution or equating coefficients. Condone slips.
	$-\frac{1}{1+t} + \frac{t+1}{1+t^2}$	**A1**	Correct partial fractions, or $A = -1$, $B = 1$, $C = 1$.
	$-\ln(1+t) + \frac{1}{2}\ln(1+t^2) + \arctan t$	**M1 A1**	**M1** Integrates their partial fractions of the form $\frac{A}{1+t} + \frac{Bt+C}{1+t^2}$, reaching a $\ln(1+t)$ term and an $\ln(1+t^2)$ or $\arctan t$ term. Condone coefficient slips. **A1** oe; modulus signs accepted.
	$\left(-\ln 2 + \frac{1}{2}\ln 2 + \frac{\pi}{4}\right) - (0)$	**dM1**	Substitutes 1 and 0 into their integral and subtracts. Zero lower-limit values need not be shown.
	$I = \frac{\pi}{4} - \frac{1}{2}\ln 2$	**A1**	Or $a = \frac{1}{4}$, $b = -\frac{1}{2}$. Must be in the form $a\pi + b\ln 2$: $\frac{\pi}{4} - \ln\sqrt{2}$ scores nothing.
		[7]	
	Total for Question 8	**11**	

Question	Scheme	Marks	Notes
9(a)	$\frac{1-\frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2}} \times \frac{2}{1+t^2}\,\mathrm{d}t$	**M1**	Replaces $\cos\theta$, $\sin\theta$ and $\mathrm{d}\theta$ by expressions in t. Condone a sign or coefficient slip in one of them.
	$\frac{1-\cos\theta}{1+\sin\theta} = \frac{2t^2}{(1+t)^2}$	**A1**	oe, such as $\frac{2t^2}{1+2t+t^2}$.
	$\theta = 0 \Rightarrow t = 0, \quad \theta = \frac{1}{2}\pi \Rightarrow t = 1$	**B1**	
	$I = \int_0^1 \frac{2t^2}{(1+t)^2} \times \frac{2}{1+t^2}\,\mathrm{d}t$ $= \int_0^1 \frac{4t^2}{(1+t)^2(1+t^2)}\,\mathrm{d}t$	**A1 AG**	Correct integrand with the $\frac{2}{1+t^2}$ factor and limits 0 and 1, leading without error to the printed integral; combined steps accepted.
		[4]	
9(b)	$\frac{4t^2}{(1+t)^2(1+t^2)} = \frac{A}{1+t} + \frac{B}{(1+t)^2}$ $+ \frac{Ct+D}{1+t^2}$	**M1**	Uses a form with terms over $1+t$ and $(1+t)^2$ and a linear numerator over $1+t^2$.
	$4t^2 = A(1+t)(1+t^2) + B(1+t^2)$ $+ (Ct+D)(1+t)^2 \Rightarrow A, B, C, D = \ldots$	**dM1**	Multiplies through and finds at least two constants by substituting values or equating coefficients. Condone slips.
	$A = -2,\ B = 2,\ C = 2,\ D = 0$	**A1**	All four, or the correct partial fractions.
	$\left[-2\ln(1+t) - \frac{2}{1+t} + \ln(1+t^2)\right]_0^1$	**M1 A1**	**M1** Integrates at least two of their terms to the forms $p\ln(1+t)$, $\frac{q}{1+t}$, $r\ln(1+t^2)$. Condone coefficient slips. **A1** Correct integral, oe.
	$(-2\ln 2 - 1 + \ln 2) - (0 - 2 + 0)$	**dM1**	Substitutes 1 and 0 into their integral and subtracts.
	$I = 1 - \ln 2$	**A1**	Or $a = 1$, $b = -1$ stated.
		[7]	
	Total for Question 9	**11**	

Question	Scheme	Marks	Notes
10(a)	$\sin x = \frac{2t}{1+t^2}, \quad \mathrm{d}x = \frac{2}{1+t^2}\,\mathrm{d}t$	**M1**	Replaces $\sin x$ by $\frac{2t}{1+t^2}$ and $\mathrm{d}x$ by $\frac{k}{1+t^2}\,\mathrm{d}t$ in the integral. Condone a slip in k.
	$\frac{\sin x}{1+\sin x} = \frac{2t}{(1+t)^2}$	**A1**	oe, such as $\frac{2t}{1+2t+t^2}$.
	$x = 0 \Rightarrow t = 0, \quad x = \frac{1}{3}\pi \Rightarrow t = \frac{1}{\sqrt{3}}$	**B1**	Both limits in t; $\frac{\sqrt{3}}{3}$ accepted.
	$I = \int_0^{\frac{1}{\sqrt{3}}} \frac{2t}{(1+t)^2} \times \frac{2}{1+t^2}\,\mathrm{d}t$ $= \int_0^{\frac{1}{\sqrt{3}}} \frac{4t}{(1+t)^2(1+t^2)}\,\mathrm{d}t$	**A1 AG**	Correct integrand and $\mathrm{d}t$ factor with limits in t, combined without error to the printed integral; combined steps accepted.
		[4]	
10(b)	$\frac{4t}{(1+t)^2(1+t^2)} = \frac{A}{1+t} + \frac{B}{(1+t)^2} + \frac{Ct+D}{1+t^2}$	**M1**	Uses the correct form, with terms over $1+t$ and $(1+t)^2$ and a linear numerator over $1+t^2$.
	$4t = A(1+t)(1+t^2) + B(1+t^2) + (Ct+D)(1+t)^2$ $\Rightarrow A = \ldots,\ B = \ldots,\ C = \ldots,\ D = \ldots$	**dM1**	Multiplies through and finds at least two constants, by substituting values of t or equating coefficients. Condone slips.
	$\frac{4t}{(1+t)^2(1+t^2)} = \frac{2}{1+t^2} - \frac{2}{(1+t)^2}$	**A1**	Or $A = 0$, $B = -2$, $C = 0$, $D = 2$ with the correct form.
		[3]	
10(c)	$I = \left[2\arctan t + \frac{2}{1+t}\right]_0^{\frac{1}{\sqrt{3}}}$	**M1 A1**	**M1** Integrates their partial fractions from (b), at least two terms to the forms $p\ln(1+t)$, $\frac{q}{1+t}$, $r\arctan t$ or $s\ln\left(1+t^2\right)$. Condone sign and coefficient slips. **A1** Correct integral; limits not needed.
	$= 2\arctan\frac{1}{\sqrt{3}} + \frac{2}{1+\frac{1}{\sqrt{3}}} - (0+2)$	**dM1**	Substitutes both limits into their integral and subtracts.
	$= \frac{1}{3}\pi + \frac{2\sqrt{3}}{\sqrt{3}+1} - 2$ $= \frac{1}{3}\pi + 1 - \sqrt{3}$	**A1 AG**	Exact values from correct substituted limits, leading without error to the printed result; combined steps accepted.
		[4]	
	Total for Question 10	**11**	