OPTIONAL PURE / FURTHER CALCULUS

Leibnitz's Theorem

Mark scheme 10 questions · 78 marks

EXAM BOARDS Edexcel FP1 9FM0/3A

Questions

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	Total	**78**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last accuracy mark of the working that reaches a printed result: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1	$u' = 4x^3,\ u'' = 12x^2,\ u''' = 24x,\ u^{(4)} = 24$ $v' = -k\sin kx,\ v'' = -k^2\cos kx$ $v''' = k^3\sin kx,\ v^{(4)} = k^4\cos kx$	**M1**	Differentiates x^4 and $\cos kx$ repeatedly, up to the fourth derivative of each; may be seen within the substitution. Condone sign slips and wrong powers of k.
	$\frac{\mathrm{d}^4y}{\mathrm{d}x^4} = x^4(k^4\cos kx) + 4(4x^3)(k^3\sin kx)$ $+ 6(12x^2)(-k^2\cos kx)$ $+ 4(24x)(-k\sin kx) + 24\cos kx$	**M1 A1**	**M1** Uses Leibnitz's theorem with $n = 4$: at least three of the five terms, each a binomial coefficient from $1, 4, 6, 4, 1$ times their derivatives. Condone slips in their derivatives. **A1** Correct unsimplified expression, oe.
	$\frac{\mathrm{d}^4y}{\mathrm{d}x^4} = (k^4x^4 - 72k^2x^2 + 24)\cos kx$ $+ 16kx(k^2x^2 - 6)\sin kx$	**dM1**	Collects the $\cos kx$ and $\sin kx$ terms and takes a factor of kx from the $\sin kx$ coefficient. Condone arithmetic slips.
	$A = -72,\ B = 24,\ C = 16,\ D = -6$	**A1**	All four values, or the correct expression in the given form.
	Total for Question 1	**5**	

Question	Scheme	Marks	Notes
2	$u' = 3x^2,\ u'' = 6x,\ u''' = 6,\ u^{(4)} = 0$ $v'' = 4\sinh 2x,\ v''' = 8\cosh 2x$ $v^{(4)} = 16\sinh 2x,\ v^{(5)} = 32\cosh 2x$	**M1**	Obtains successive derivatives of both x^3 and $\sinh 2x$, the latter alternating between multiples of $\cosh 2x$ and $\sinh 2x$. Condone coefficient slips.
	$\frac{\mathrm{d}^5y}{\mathrm{d}x^5} = x^3 \times 32\cosh 2x + 5 \times 3x^2 \times 16\sinh 2x$ $+ 10 \times 6x \times 8\cosh 2x + 10 \times 6 \times 4\sinh 2x$	**M1 A1**	**M1** Applies Leibnitz's theorem with $n = 5$: at least three of the four non-zero terms, binomial coefficients from 1, 5, 10, 10, with their derivatives. **A1** Correct expression, oe; need not be simplified.
	$= 32x\left(x^2 + 15\right)\cosh 2x + 240\left(x^2 + 1\right)\sinh 2x$	**dM1**	Collects the $\cosh 2x$ and $\sinh 2x$ terms and factorises each towards the given form. Condone coefficient slips.
	$P = 32,\ Q = 15,\ R = 240$	**A1**	cao. Accept the correct expression in the given form with the values not listed separately.
	Total for Question 2	**5**	

Question	Scheme	Marks	Notes
3(a)	$u' = 3,\ u'' = 0$ $v^{(k)} = 2^k e^{2x}$	**B1**	General derivative $2^k e^{2x}$, any letter for the order, seen or used.
	$\frac{\mathrm{d}^n y}{\mathrm{d}x^n} = (3x-1) \times 2^n e^{2x}$ $\quad + n \times 3 \times 2^{n-1} e^{2x}$	**M1 A1**	**M1** Uses Leibnitz's theorem with $u = 3x-1$, $v = e^{2x}$, either way round, keeping two non-zero terms with binomial coefficients. Condone slips in powers of 2.
	$= 2^{n-1} e^{2x} \left(2(3x-1) + 3n\right)$	**dM1**	Takes out the common factor $2^{n-1} e^{2x}$, or otherwise collects both terms over one power of 2.
	$= 2^{n-1} \left(6x + 3n - 2\right) e^{2x}$	**A1 AG**	Correct two-term expression leading without error to the printed result; combined steps accepted.
		[5]	
3(b)	$2^9 \left(0 + 30 - 2\right) e^0 = \ldots$	**M1**	Substitutes $n = 10$ and $x = 0$ into the result of (a).
	$14\,336$	**A1**	Accept 7×2^{11} oe.
		[2]	
Total for Question 3		**7**	

Question	Scheme	Marks	Notes
4	$u' = 3x^2,\ u'' = 6x,\ u''' = 6,\ u^{(4)} = 0$ $v^{(4)} = 16\cos 2x,\ v^{(5)} = -32\sin 2x$ $v^{(6)} = -64\cos 2x,\ v^{(7)} = 128\sin 2x$	**M1 A1**	**M1** Differentiates both: x^3 to its constant third derivative, and $\cos 2x$ to terms of the form $\pm 2^r \sin 2x$ or $\pm 2^r \cos 2x$. Condone sign and coefficient slips. **A1** $u''' = 6$ and $v^{(4)}$ to $v^{(7)}$ correct, soi in their expansion.
	$f^{(7)}(x) = x^3(128\sin 2x) + 7(3x^2)(-64\cos 2x)$ $+ 21(6x)(-32\sin 2x) + 35(6)(16\cos 2x)$	**M1**	Uses Leibnitz's theorem with $n = 7$: at least three of the four non-zero terms, each with a binomial coefficient and their derivatives. Condone coefficient slips.
	$f^{(7)}(x) = 128x^3\sin 2x - 1344x^2\cos 2x$ $- 4032x\sin 2x + 3360\cos 2x$	**A1**	Correct expression, need not be simplified.
	$\sin\pi = 0,\ \cos\pi = -1$ $f^{(7)}\left(\frac{\pi}{2}\right) = -1344\left(\frac{\pi^2}{4}\right)(-1) + 3360(-1)$	**M1**	Substitutes $x = \frac{\pi}{2}$ into their seventh derivative, using $\sin\pi = 0$ or $\cos\pi = -1$.
	$f^{(7)}\left(\frac{\pi}{2}\right) = 336\pi^2 - 3360$	**A1**	
	$\frac{\text{“}336\pi^2 - 3360\text{”}}{5040} = \ldots$	**dM1**	Divides their value of $f^{(7)}\left(\frac{\pi}{2}\right)$ by 7! or 5040.
	$\frac{1}{15}\pi^2 - \frac{2}{3}$ $p = \frac{1}{15},\quad q = -\frac{2}{3}$	**A1**	cao. Both values as exact fractions, or the coefficient $\frac{1}{15}\pi^2 - \frac{2}{3}$ oe.
	Total for Question 4	**8**	

Question	Scheme	Marks	Notes
5(a)	$v = \ln x \Rightarrow v' = \frac{1}{x},\ v'' = -\frac{1}{x^2},\ \ldots$	**M1**	Differentiates $\ln x$ repeatedly, reaching at least three derivatives of the form $\pm\frac{k}{x^n}$. Condone sign and coefficient slips.
	$v''' = \frac{2}{x^3},\ v^{(4)} = -\frac{6}{x^4}$ $v^{(5)} = \frac{24}{x^5},\ v^{(6)} = -\frac{120}{x^6}$	**A1**	v''' to $v^{(6)}$ all correct; may be seen within the expansion.
	$f^{(6)}(x) = 1 \times x^3 \times \left(-\frac{120}{x^6}\right) + 6 \times 3x^2 \times \frac{24}{x^5}$ $+ 15 \times 6x \times \left(-\frac{6}{x^4}\right) + 20 \times 6 \times \frac{2}{x^3}$	**M1 A1**	**M1** Applies Leibnitz's theorem with $n = 6$: at least three non-zero terms, each a $\binom{6}{r}$ coefficient times their derivatives of x^3 and $\ln x$. Condone sign slips. **A1** Correct four non-zero terms, unsimplified.
	$= \frac{-120+432-540+240}{x^3} = \frac{12}{x^3}$	**A1 AG**	Correct substituted expansion leading without error to $\frac{12}{x^3}$; combined simplification accepted.
		[5]	
5(b)	$\frac{f^{(6)}(1)}{6!} = \frac{12}{720}$	**M1**	Uses $\frac{f^{(r)}(1)}{r!}$ with part (a) for either coefficient. Not for omitting the $r!$.
	$= \frac{1}{60}$	**A1**	Simplest form only.
	$f^{(7)}(x) = -\frac{36}{x^4} \Rightarrow -\frac{36}{7!} = -\frac{1}{140}$	**A1**	$f^{(7)}$ from differentiating part (a); $-\frac{1}{140}$ in simplest form.
		[3]	
Total for Question 5		**8**	

Question	Scheme	Marks	Notes
6(a)	$u' = -e^{-x},\ u'' = e^{-x},\ u''' = -e^{-x},\ u^{(4)} = e^{-x}$ $v' = -\sin x,\ v'' = -\cos x$ $v''' = \sin x,\ v^{(4)} = \cos x$	**M1 A1**	**M1** Differentiates both e^{-x} and $\cos x$ repeatedly, up to the fourth derivative or by an evident pattern. Condone sign slips. **A1** All derivatives correct, soi in the expansion.
	$\frac{\mathrm{d}^4y}{\mathrm{d}x^4} = uv^{(4)} + 4u'v''' + 6u''v'' + 4u'''v' + u^{(4)}v$ $\frac{\mathrm{d}^4y}{\mathrm{d}x^4} = e^{-x}\cos x - 4e^{-x}\sin x - 6e^{-x}\cos x$ $\quad + 4e^{-x}\sin x + e^{-x}\cos x$	**M1 A1**	**M1** Uses Leibnitz's theorem with $n = 4$: at least three of the five terms, each a binomial coefficient from $1, 4, 6, 4, 1$ times their derivatives. Condone sign slips. **A1** Correct unsimplified expansion.
	$= -4e^{-x}\cos x = -4y$	**A1 AG**	Correct expansion collected without error to $-4e^{-x}\cos x$ and identified as $-4y$.
		[5]	
6(b)	$\frac{\mathrm{d}^8y}{\mathrm{d}x^8} = -4\frac{\mathrm{d}^4y}{\mathrm{d}x^4} = (-4)^2y$	**M1**	Uses (a) repeatedly, each four further differentiations multiplying by -4; soi by $(-4)^k y$ or a power of ± 4 times y.
	$\frac{\mathrm{d}^{20}y}{\mathrm{d}x^{20}} = (-4)^5y = -1024y$	**A1**	oe such as -4^5y.
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = uv'' + 2u'v' + u''v = \ldots\ [= 2e^{-x}\sin x]$ $\frac{\mathrm{d}^{22}y}{\mathrm{d}x^{22}} =$ "-1024" $\times \frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	**M1**	Finds $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ by any method and multiplies it by their coefficient of y in the twentieth derivative. Condone sign slips.
	$\frac{\mathrm{d}^{22}y}{\mathrm{d}x^{22}} = -1024 \times 2e^{-\pi/2} = -2048e^{-\pi/2}$	**A1**	Any exact form, such as $-2^{11}e^{-\frac{\pi}{2}}$.
		[4]	
	Total for Question 6	**9**	

Question	Scheme	Marks	Notes
7(a)	$u = e^{2x}$: $u^{(k)} = 2^k e^{2x}$ $v = \sin x$: $v' = \cos x,\ v'' = -\sin x,\ v''' = -\cos x$ $v^{(4)} = \sin x,\ v^{(5)} = \cos x$	**M1 A1**	**M1** Differentiates both e^{2x} and $\sin x$ repeatedly, to the fifth derivative or a general form. Condone sign slips. **A1** All derivatives correct, soi in the expansion.
	$\frac{\mathrm{d}^5 y}{\mathrm{d}x^5} = e^{2x}\big(\cos x + 5 \times 2\sin x$ $- 10 \times 4\cos x - 10 \times 8\sin x$ $+ 5 \times 16\cos x + 32\sin x\big)$	**M1 A1**	**M1** Uses Leibnitz's theorem with $n = 5$: at least three of the six terms, each a binomial coefficient times their derivatives of total order 5. Condone sign slips and one wrong coefficient. **A1** Correct unsimplified expansion, oe.
	$= e^{2x}(41\cos x - 38\sin x)$	**A1 AG**	Correct expansion collected to the printed result without error; collecting terms in one step accepted.
		[5]	
7(b)	$R\cos\alpha\cos x - R\sin\alpha\sin x$ $\Rightarrow R\cos\alpha = 41,\ R\sin\alpha = 38$	**M1**	Expands $R\cos(x+\alpha)$ and compares coefficients with their part (a) result, reaching $R\cos\alpha = \pm 41$ and $R\sin\alpha = \pm 38$, or uses $R = \sqrt{41^2 + 38^2}$ or $\tan\alpha = \pm\frac{38}{41}$ directly. Condone the ratio inverted.
	$R = \sqrt{41^2 + 38^2} = 25\sqrt{5}$	**A1**	Simplest surd form only; $\sqrt{3125}$ or a decimal scores A0.
	$\tan\alpha = \frac{38}{41} \Rightarrow \alpha = 0.747$	**A1**	awrt 0.747, in radians.
		[3]	
	Total for Question 7	**8**	

Question	Scheme	Marks	Notes
8(a)	$v = e^{-2x}$: $v^{(m)} = (-2)^m e^{-2x}$	**B1**	soi within the Leibnitz terms; any letter for the order.
	$\frac{\mathrm{d}^k y}{\mathrm{d}x^k} = \binom{k}{0}(-2)^k e^{-2x}\left(x^2+3\right)$ $+ \binom{k}{1}(-2)^{k-1}e^{-2x}(2x)$ $+ \binom{k}{2}(-2)^{k-2}e^{-2x}(2)$	**M1**	Applies Leibnitz's theorem with x^2+3 differentiated at most twice, giving three terms with binomial coefficients. Condone sign and power slips.
	$= (-2)^k e^{-2x}\left(x^2+3\right) + 2kx(-2)^{k-1}e^{-2x}$ $+ k(k-1)(-2)^{k-2}e^{-2x}$	**A1**	Correct three terms; binomial coefficients need not be evaluated.
	$= (-2)^{k-2}e^{-2x}\left(4\left(x^2+3\right) - 4kx + k(k-1)\right)$	**dM1**	Takes out $(-2)^{k-2}e^{-2x}$ from all three terms, adjusting the other powers of -2. Condone sign slips.
	$A = 4,\ B = -4,\ C = 1,\ D = 12$	**A1**	cso. All four values, or the correct expression in the printed form.
		[5]	
8(b)	$(-4k)^2 - 4 \times 4 \times \left(\text{“}k^2 - k + 12\text{”}\right) \geq 0$ $\Rightarrow k \geq \ldots$	**M1**	Uses the discriminant of, or completes the square on, their quadratic in x from (a) with coefficients in k, reaching a condition on k. Condone sign slips.
	$16(k-12) \geq 0 \Rightarrow k = 12$	**A1**	cso.
	$4x^2 - 48x + 144 = 0 \Rightarrow x = 6$	**A1**	
		[3]	
	Total for Question 8	**8**	

Question	Scheme	Marks	Notes
9(a)	LHS: $y^{(1)} = 2e^{2x}u + e^{2x}u^{(1)}$ RHS: $e^{2x}\left(\binom{1}{0}2^1u^{(0)} + \binom{1}{1}2^0u^{(1)}\right)$ $= e^{2x}\left(2u + u^{(1)}\right)$	**B1**	Product-rule derivative and the formula at $n = 1$ both evaluated and shown equal.
	$y^{(k+1)} = 2e^{2x}\sum_{r=0}^{k}\binom{k}{r}2^{k-r}u^{(r)}$ $+ e^{2x}\sum_{r=0}^{k}\binom{k}{r}2^{k-r}u^{(r+1)}$	**M1 A1**	**M1** Assumes the result for $n = k$ and differentiates it by the product rule, giving two sums. Condone slips in powers of 2. **A1** Correct, oe.
	$\sum_{r=0}^{k}\binom{k}{r}2^{k-r}u^{(r+1)}$ $= \sum_{r=1}^{k+1}\binom{k}{r-1}2^{k+1-r}u^{(r)}$	**M1**	Rewrites either sum with its index shifted so both sums have the same order of derivative of u, changing the limits accordingly. Condone a slip in the power of 2.
	$y^{(k+1)} = e^{2x}\Big[2^{k+1}u + u^{(k+1)}$ $+ \sum_{r=1}^{k}\left(\binom{k}{r} + \binom{k}{r-1}\right)2^{k+1-r}u^{(r)}\Big]$ $= e^{2x}\Big[2^{k+1}u + u^{(k+1)}$ $+ \sum_{r=1}^{k}\binom{k+1}{r}2^{k+1-r}u^{(r)}\Big]$	**dM1**	Separates the end terms, combines the sums over common r and uses the given result. Condone slips in the end terms.
	$y^{(k+1)} = e^{2x}\sum_{r=0}^{k+1}\binom{k+1}{r}2^{(k+1)-r}u^{(r)}$	**A1**	Correct, with the end terms identified as the $r = 0$ and $r = k + 1$ terms of the sum.
	True for $n = 1$; true for $n = k \Rightarrow$ true for $n = k + 1$ $\Rightarrow$ true for all positive integers n	**E1**	Concludes, naming the base case, the inductive step and the result for all positive integers n.
		[7]	
9(b)	$u = x^3,\ u^{(1)} = 3x^2,\ u^{(2)} = 6x,\ u^{(3)} = 6$ $e^{2x}\big[2^4x^3 + 4 \times 2^3(3x^2)$ $+ 6 \times 2^2(6x) + 4 \times 2(6)\big]$	**M1**	Uses part (a) with $n = 4$, $u = x^3$: at least three terms with binomial coefficients, powers of 2 and their derivatives of x^3. Condone one slip.
	$e^{2x}\left(16x^3 + 96x^2 + 144x + 48\right)$	**A1**	cao, in the form $e^{2x}p(x)$.
		[2]	
	Total for Question 9	**9**	

Question	Scheme	Marks	Notes
10(a)	$u = x^2,\ u' = 2x,\ u'' = 2,\ u''' = 0$ $v = \ln(1+x),\ v' = \frac{1}{1+x}$ $v'' = -\frac{1}{(1+x)^2},\ v''' = \frac{2}{(1+x)^3}$	**M1**	Attempts the derivatives of $\ln(1+x)$ up to the third, each of the form $\frac{k}{(1+x)^r}$, and those of x^2. Condone sign and coefficient slips.
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = uv''' + 3u'v'' + 3u''v'\ [+u'''v]$ $= \frac{2x^2}{(1+x)^3} - \frac{6x}{(1+x)^2} + \frac{6}{1+x}$	**M1 A1**	**M1** Uses Leibnitz's theorem with $n = 3$: at least three non-zero terms with coefficients 1, 3, 3 and their derivatives. Condone slips in the derivatives. **A1** Correct, unsimplified, oe.
	$= \frac{2x^2-6x(1+x)+6(1+x)^2}{(1+x)^3}$	**dM1**	Combines their three terms over $(1+x)^3$, multiplying each numerator by the matching power of $(1+x)$. Condone sign slips.
	$= \frac{2x^2+6x+6}{(1+x)^3} = \frac{2\left(x^2+3x+3\right)}{(1+x)^3}$	**A1 AG**	Correct unsimplified derivative combined and expanded without error to the printed result; collected steps accepted.
		[5]	
10(b)	$n = 3$: $(-1)^2\frac{0!\left(2x^2+6x+6\right)}{(1+x)^3} = \frac{2\left(x^2+3x+3\right)}{(1+x)^3}$ which is $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ by (a)	**B1**	Formula at $n = 3$ simplified to the result of (a), with a reference to (a) or the third derivative.
	Assumes true for $n = k$: $\frac{\mathrm{d}^ky}{\mathrm{d}x^k} = (-1)^{k-1}\frac{(k-3)!\left(2x^2+2kx+k(k-1)\right)}{(1+x)^k}$ $\frac{\mathrm{d}^{k+1}y}{\mathrm{d}x^{k+1}} = \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}^ky}{\mathrm{d}x^k}\right)$	**M1**	States the assumption for $n = k$ and sets out to differentiate it for the $(k+1)$th derivative.
	$\frac{\mathrm{d}^{k+1}y}{\mathrm{d}x^{k+1}} = (-1)^{k-1}(k-3)!\big[(4x+2k)(1+x)^{-k}$ $-k\left(2x^2+2kx+k(k-1)\right)(1+x)^{-k-1}\big]$	**M1 A1**	**M1** Applies the product or quotient rule to the $n = k$ expression: one term from differentiating the quadratic, one from the power of $(1+x)$. Condone sign and coefficient slips. **A1** Correct, unsimplified, oe.
	$= (-1)^{k-1}(k-3)!\,\frac{-(k-2)\left[2x^2+2(k+1)x+(k+1)k\right]}{(1+x)^{k+1}}$ $= (-1)^{(k+1)-1}\frac{((k+1)-3)!\left(2x^2+2(k+1)x+(k+1)k\right)}{(1+x)^{k+1}}$	**A1**	Numerator factorised and $(k-2)(k-3)! = (k-2)!$ used, reaching the $n = k+1$ form without error; $(-1)^k$, $(k-2)!$ and $(k+1)k$ accepted.

Question 10 continued

Question	Scheme	Marks	Notes
	True for $n = k$ implies true for $n = k + 1$ True for $n = 3$, so true for every integer $n \geq 3$	**E1**	States all three: true for $n = 3$; true for $n = k$ implies true for $n = k + 1$; so true for every integer $n \geq 3$.
		[6]	
	Total for Question 10	**11**	