OPTIONAL PURE / DIFFERENTIAL EQUATIONS & NUMERICAL METHODS

Simpson's Rule

Questions by topic 10 questions · 68 marks

EXAM BOARDS Edexcel FP1 9FM0/3A · AQA 7367

Questions

Question 1	*5 marks*	2
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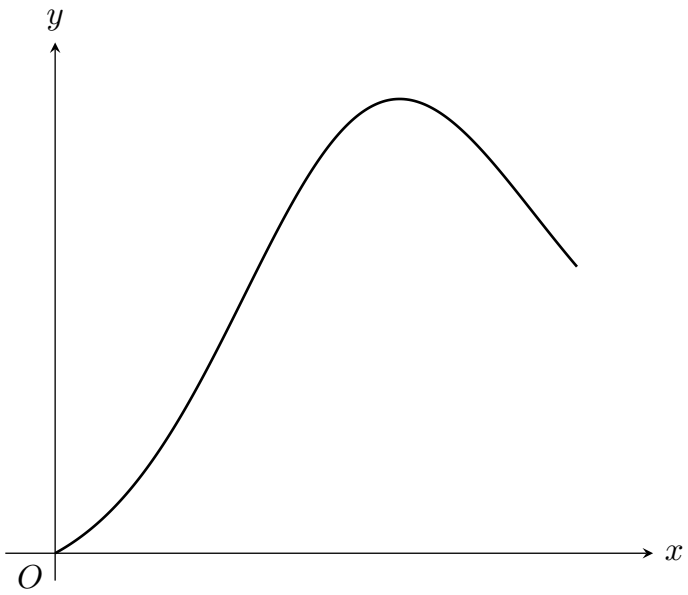
Question 10	*9 marks*	11

1. The function g is defined by

$$g(x) = x\,e^{\sin x} \qquad (0 \leq x \leq \pi)$$

The diagram below shows the graph of $y = g(x)$.



(a) Use Simpson's rule with 4 strips to estimate

$$\int_0^{\pi} g(x)\,\mathrm{d}x$$

giving your answer to 3 decimal places. **[4]**

(b) Explain how Simpson's rule could be used to find a more accurate estimate of the integral in part (a). **[1]**

2. **(a)** Use Simpson's rule with 6 strips to find an estimate for

$$\int_0^{\pi} \sqrt{\sin x}\,\mathrm{d}x$$

giving your answer to 4 significant figures. **[4]**

You are given that $\int_0^{\pi} \sqrt{\sin x}\,\mathrm{d}x = 2.3963$ to 5 significant figures.

(b) Find the percentage error in your answer to part (a), giving your answer to 2 significant figures, and comment on the accuracy of your estimate. **[2]**

3. The table below shows values of $y = \ln\left(4 + x^3\right)$ for $0 \leq x \leq 3$, with one value missing.

x	0	0.5	1	1.5	2	2.5	3
y	1.386	1.417	1.609		2.485	2.977	3.434

(a) Complete the table by finding the value of y when $x = 1.5$, giving your answer to 3 decimal places. **[1]**

(b) Use Simpson's rule with all the values of y in the completed table to estimate, to 3 significant figures, the value of

$$\int_0^3 \ln\left(4 + x^3\right)\, \mathrm{d}x$$ **[3]**

(c) Using your answer to part (b) and making your method clear, estimate the value of

$$\int_0^3 \ln\left(\frac{\left(4 + x^3\right)^2}{e^x}\right) \mathrm{d}x$$ **[2]**

4. The function f is defined by $f(x) = \sqrt{1+x^3}$.
The table below shows some values of $f(x)$, given to 4 decimal places.

x	0	0.2	0.4	0.6	0.8	1.0	1.2
$f(x)$	1.0000	1.0040		1.1027		1.4142	1.6517

(a) Complete the table, and use Simpson's rule with all the values in the table to find an estimate for

$$\int_0^{1.2} \sqrt{1+x^3}\,\mathrm{d}x$$

giving your answer to 3 decimal places. **[4]**

(b) Hence find an estimate for

$$\int_0^{1.2} \left(x^2 + 2\sqrt{1+x^3}\right)\mathrm{d}x$$ **[2]**

5. The table below shows some values of y, given to 2 decimal places, where $y = \ln(3 - \cos 2x)$.

x	0.3	0.5	0.7	0.9	1.1	1.3	1.5
y	0.78	0.90		1.17		1.35	1.38

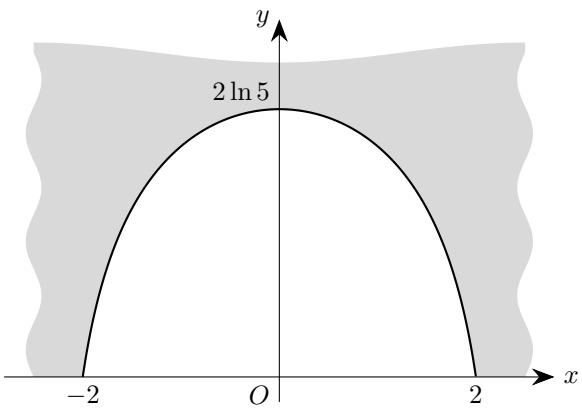
(a) Complete the table, giving the values of y corresponding to $x = 0.7$ and $x = 1.1$ to 2 decimal places. **[2]**

(b) Use Simpson's rule, with all the values of y in the completed table, to estimate, to 3 decimal places, the value of

$$\int_{0.3}^{1.5} \ln(3 - \cos 2x)\,\mathrm{d}x$$ **[3]**

(c) Using your answer to part (b), and making your method clear, estimate, to 2 decimal places, the value of

$$\int_{0.3}^{1.5} \ln\left(1 + \sin^2 x\right)\,\mathrm{d}x$$ **[2]**

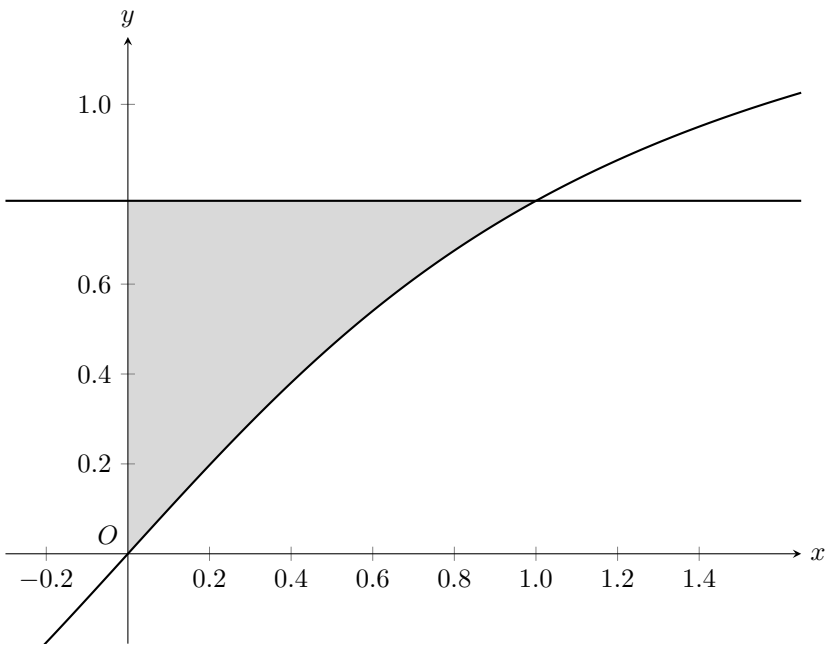


6. The diagram above shows a sketch of the vertical cross-section of the entrance to a tunnel. The entrance is 4 metres wide at its base.
The cross-section can be modelled by the region between the x-axis and the curve with equation $y = f(x)$, where x and y are measured in metres and

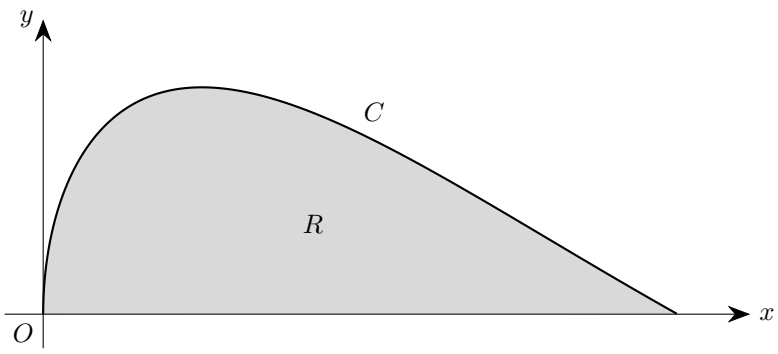
$$f(x) = 2\ln\left(5 - x^2\right), \qquad -2 \leq x \leq 2$$

(a) Use Simpson's rule with 8 strips to estimate the area of the cross-section, giving your answer in m^2 to 4 significant figures. **[4]**

(b) A wooden door of uniform thickness 75 mm is to be made to seal the tunnel entrance. The door contains a rectangular glass panel, 0.6 m wide and 1.2 m tall, which is not made of wood.
Using your answer to part (a), estimate the volume of wood required for the door, giving your answer in m^3 to 3 significant figures. **[2]**



7. The diagram above shows part of the curve $y = \arctan x$ and the line $y = \frac{\pi}{4}$, which meet at the point where $x = 1$.
The shaded region, bounded by the curve, the line $y = \frac{\pi}{4}$ and the y-axis, is rotated through 2π radians about the x-axis to form a solid.
Use Simpson's rule with 6 strips to estimate the volume of the solid. Give your answer to 4 decimal places. **[6]**



8. The diagram above shows the curve C with parametric equations

$$x = t^2, \qquad y = \sin t, \qquad 0 \leq t \leq \pi$$

The region R, shown shaded, is bounded by C and the x-axis.

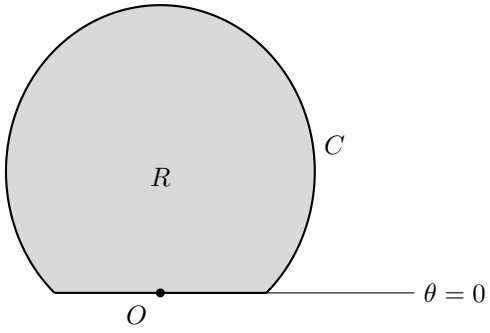
(a) Show that the area of R is given by

$$\int_0^{\pi} 2t \sin t \, \mathrm{d}t$$

[2]

(b) Use Simpson's rule with 4 strips to estimate the area of R, giving your answer to 3 decimal places. **[4]**

(c) Use integration by parts to show that the exact area of R is 2π. **[3]**



9. The diagram above shows the curve C with polar equation

$$r = e^{\sin\theta}, \qquad 0 \leq \theta \leq \pi$$

The region R, shown shaded, is enclosed by C and the line segment joining the points of C where $\theta = 0$ and $\theta = \pi$.

(a) Find the polar coordinates of the point on C that is furthest from the pole O. **[3]**

(b) Use Simpson's rule with 6 strips to estimate the area of R, giving your answer to 3 significant figures. **[5]**

10. In this question, $h > 0$ and a, b and c are constants. The curve with equation $y = ax^2 + bx + c$ passes through the points $(-h, y_0)$, $(0, y_1)$ and (h, y_2).

(a) Show that

$$\int_{-h}^{h} \left(ax^2 + bx + c\right) \mathrm{d}x = \frac{2h}{3}\left(ah^2 + 3c\right) \quad \textbf{[2]}$$

(b) By writing y_0, y_1 and y_2 in terms of a, b, c and h, show that

$$\int_{-h}^{h} \left(ax^2 + bx + c\right) \mathrm{d}x = \frac{h}{3}\left(y_0 + 4y_1 + y_2\right) \quad \textbf{[3]}$$

(c) Simpson's rule fits a curve of this form through each set of three consecutive ordinates. Explain why Simpson's rule needs an even number of strips. **[1]**

(d) Let T_1 and T_2 be the estimates of $\int_{-h}^{h} \left(ax^2 + bx + c\right) \mathrm{d}x$ given by the trapezium rule with 1 strip and with 2 strips respectively. Show that

$$\frac{4T_2 - T_1}{3} = \frac{h}{3}\left(y_0 + 4y_1 + y_2\right) \quad \textbf{[3]}$$