OPTIONAL PURE / DIFFERENTIAL EQUATIONS & NUMERICAL METHODS

Simpson's Rule

Mark scheme 10 questions · 68 marks

EXAM BOARDS Edexcel FP1 9FM0/3A · AQA 7367

Questions

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	Total	**68**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last accuracy mark of the working that reaches a printed result: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [***n***] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	$h = \frac{\pi}{4}$	**B1**	
	$x = 0,\ \frac{\pi}{4},\ \frac{\pi}{2},\ \frac{3\pi}{4},\ \pi$ $y = 0,\ 1.5929,\ 4.2699,\ 4.7786,\ 3.1416$	**B1**	All five, exact or to at least 3 d.p.; implied by a bracket total of awrt 37.17.
	$\frac{1}{3} \times \frac{\pi}{4}\{(\text{“}0 + 3.1416\text{”}) + 4(\text{“}1.5929 + 4.7786\text{”}) + 2(\text{“}4.2699\text{”})\} = \ldots$	**M1**	Applies $\frac{h}{3}$ with coefficients 1, 4, 2, 4, 1 to their five equally spaced ordinates on $0 \leq x \leq \pi$, with their h.
	$\int_0^{\pi} g(x)\,\mathrm{d}x \approx 9.730$	**A1**	awrt 9.730.
		[4]	
1(b)	More strips, so a smaller h: such as 8 strips, $h = \frac{\pi}{8}$	**E1**	States more strips or ordinates, or a smaller h. A stated number must give an even number of strips. Not for more decimal places alone.
		[1]	
	Total for Question 1	**5**	

Question	Scheme	Marks	Notes
2(a)	$h = \frac{\pi}{6}$	**B1**	
	y: 0, 0.7071, 0.9306, 1, 0.9306, 0.7071, 0	**B1**	All seven ordinates, exact or to at least 3 d.p.; may be seen in the formula.
	$\frac{1}{3} \times \frac{\pi}{6} \{(0+0) + 4(0.7071 + 1 + 0.7071) + 2(0.9306 + 0.9306)\}$	**M1**	Uses $\frac{1}{3}h$ with their h and coefficients 1, 4, 2, 4, 2, 4, 1 on their seven ordinates. Condone one slip in an ordinate.
	≈ 2.335	**A1**	awrt 2.335
		[4]	
2(b)	$\frac{2.3963 - \text{“2.335”}}{2.3963} \times 100$	**M1**	Divides the difference between 2.3963 and their estimate by 2.3963 and multiplies by 100. Condone the difference in either order.
	$= 2.6\%$ Fairly accurate, within about 3% of the true value	**A1ft**	awrt 2.6%, ft their estimate, with a comment on accuracy that matches it, “accurate” or “fairly accurate” oe for an error below 5%, otherwise not accurate. “Underestimate” alone is not enough.
		[2]	
Total for Question 2		**6**	

Question	Scheme	Marks	Notes
3(a)	$y = \ln 7.375 = 1.998$	**B1**	
		[1]	
3(b)	$h = 0.5$	**B1**	soi, as $\frac{0.5}{3}$ or $\frac{1}{6}$ in the rule.
	$\frac{0.5}{3}\{(1.386 + 3.434)$ $+ 4(1.417 + \text{“}1.998\text{”} + 2.977)$ $+ 2(1.609 + 2.485)\}$	**M1**	Uses $\frac{h}{3}$ with their h, end ordinates once, odd-numbered ordinates times 4, even-numbered times 2, all seven values. Condone one slip in a y-value.
	≈ 6.43	**A1**	awrt 6.43
		[3]	
3(c)	$\ln\left(\frac{(4+x^3)^2}{e^x}\right) = 2\ln(4 + x^3) - x$ $2 \times \text{“}6.43\text{”} - \int_0^3 x \, \mathrm{d}x = \ldots$	**M1**	Uses log laws to reach $2\ln(4 + x^3) - x$ oe, then forms $2\times$ their (b) minus their $\int_0^3 x \, \mathrm{d}x$. Not for a fresh Simpson's rule estimate.
	$12.86 - 4.5 \approx 8.36$	**A1ft**	awrt 8.36, or ft $2\times$ their (b) $- 4.5$ to 3 s.f.
		[2]	
	Total for Question 3	**6**	

Question	Scheme	Marks	Notes
4(a)	$f(0.4) = 1.0315$	**B1**	
	$f(0.8) = 1.2296$	**B1**	
	$\frac{0.2}{3}\{(1.0000 + 1.6517) + 4(1.0040 + 1.1027 + 1.4142) + 2(\text{“}1.0315\text{”} + \text{“}1.2296\text{”})\}$	**M1**	Uses $\frac{0.2}{3}$ with the end ordinates once, 4 times the odd ordinates and 2 times the even, with their missing values. Condone one slip copying an ordinate.
	$= \frac{0.2}{3} \times 21.2575 = 1.417$	**A1**	
		[4]	
4(b)	$\left[\frac{x^3}{3}\right]_0^{1.2} + 2 \times \text{“}1.417\text{”}$	**M1**	Evaluates $\int_0^{1.2} x^2 \, \mathrm{d}x$, or 0.576 seen, and adds twice their (a). Condone a slip in evaluating $\frac{x^3}{3}$.
	$= 0.576 + 2.834 = 3.410$	**A1ft**	ft $0.576 + 2\times$ their (a); awrt 3.41 from a correct (a).
		[2]	
	Total for Question 4	**6**	

Question	Scheme	Marks	Notes
5(a)	$x = 0.7: \ y = 1.04$	**B1**	
	$x = 1.1: \ y = 1.28$	**B1**	
		[2]	
5(b)	$h = 0.2$	**B1**	
	$\frac{0.2}{3}\{(0.78 + 1.38) + 4(0.90 + 1.17 + 1.35) + 2(\text{“}1.04\text{”} + \text{“}1.28\text{”})\}$	**M1**	Uses $\frac{1}{3}$ their h with coefficients 1, 4, 2, 4, 2, 4, 1 on all seven ordinates, including their (a) values. Condone one copying slip.
	≈ 1.365	**A1**	awrt 1.365
		[3]	
5(c)	$3 - \cos 2x = 2\left(1 + \sin^2 x\right)$ $\Rightarrow \int_{0.3}^{1.5} \ln\left(1 + \sin^2 x\right) \mathrm{d}x$ $\approx \text{“}1.365\text{”} - 1.2\ln 2$	**M1**	Uses a double angle formula to write the integrand as $\ln(3 - \cos 2x) - \ln 2$ and subtracts a multiple of $\ln 2$ from their (b). Condone a slip in the interval length.
	≈ 0.53	**A1ft**	awrt 0.53, ft their (b) $- 1.2\ln 2$.
		[2]	
	Total for Question 5	**7**	

Question	Scheme	Marks	Notes
6(a)	$h = \frac{2-(-2)}{8} = 0.5$	**B1**	
	y: 0, 2.0232, 2.7726, 3.1163, 3.2189, 3.1163, 2.7726, 2.0232, 0	**B1**	All nine ordinates correct, exact or to at least 4 s.f.; implied by correct sums 10.279 and 8.764.
	$\frac{1}{3}(0.5)\{0 + 4(\text{“}10.279\text{”}) + 2(\text{“}8.764\text{”})\}$	**M1**	Uses $\frac{h}{3}$ with their h and nine ordinates, coefficients $1, 4, 2, 4, 2, 4, 2, 4, 1$. Condone one ordinate wrong or missing.
	Area ≈ 9.774	**A1**	awrt 9.774. Units not required.
		[4]	
6(b)	$(\text{“}9.774\text{”} - 0.6 \times 1.2) \times 0.075$	**M1**	Subtracts the panel's area from their (a), or the panel's volume from the door's, with thickness 0.075. Condone arithmetic slips.
	$= 0.679$	**A1ft**	awrt 0.679, or ft 0.075(their (a) $- 0.72$) to 3 s.f. Units not required.
		[2]	
Total for Question 6		**6**	

Question	Scheme	Marks	Notes
7	$V = \pi\int_0^1 \left(\frac{\pi}{4}\right)^2 \mathrm{d}x - \pi\int_0^1 (\arctan x)^2\,\mathrm{d}x$	**M1**	Subtracts the curve's $\pi\int y^2\,\mathrm{d}x$ from the line's, which may be the cylinder $\frac{\pi^3}{16}$. Not for $\pi\int\left(\frac{\pi}{4} - \arctan x\right)^2\mathrm{d}x$.
	$V = \pi\int_0^1 \left(\frac{\pi^2}{16} - (\arctan x)^2\right)\mathrm{d}x$	**A1**	Correct, with limits 0 and 1, oe as two integrals or the cylinder minus an integral.
	$h = \frac{1}{6}$	**B1**	
	0.616850, 0.589576, 0.513327, 0.401881, 0.271103, 0.134189, 0	**B1**	All seven correct to at least 4 d.p., for $\frac{\pi^2}{16} - (\arctan x)^2$ or for $(\arctan x)^2$; correct sums imply them.
	$\frac{1}{3}\times\frac{1}{6}\{\text{“0.616850”} + 4(\text{“1.125646”}) + 2(\text{“0.784430”})\}$	**M1**	Uses $\frac{h}{3}$ with their h and coefficients 1, 4, 2, 4, 2, 4, 1 on their seven ordinates. Condone one slip in an ordinate.
	$V \approx \pi \times 0.371572 \approx 1.1673$	**A1**	awrt 1.1673.
	Total for Question 7	**6**	

Question	Scheme	Marks	Notes
8(a)	$\int y \frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t,\ \frac{\mathrm{d}x}{\mathrm{d}t} = 2t$	**M1**	Uses $\int y\frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t$, or $\int y\,\mathrm{d}x$ with $\mathrm{d}x = kt\,\mathrm{d}t$, where $\frac{\mathrm{d}x}{\mathrm{d}t}$ is of the form kt. Not for $\int y\,\mathrm{d}t$.
	$x = 0 \Rightarrow t = 0,\ x = \pi^2 \Rightarrow t = \pi$ $\int_0^{\pi} 2t\sin t\,\mathrm{d}t$	**A1 AG**	Limits 0 and π found from the ends of C, by x or by $y = 0$, leading to the printed integral without error.
		[2]	
8(b)	$h = \frac{\pi}{4}$	**B1**	
	t: $0,\ \frac{\pi}{4},\ \frac{\pi}{2},\ \frac{3\pi}{4},\ \pi$ $2t\sin t$: $0,\ 1.1107,\ 3.1416,\ 3.3322,\ 0$	**B1**	All five ordinates, exact or awrt 3 s.f., seen or implied by the substitution.
	$\frac{1}{3}\times\frac{\pi}{4}\{(0+0)$ $+4(\text{“}1.1107\text{”}+\text{“}3.3322\text{”})+2\times\text{“}3.1416\text{”}\}$	**M1**	Uses Simpson's rule with their h and five ordinates, end ordinates once, coefficients 4, 2, 4 inside, and the factor $\frac{h}{3}$. Condone one slip in an ordinate.
	≈ 6.298	**A1**	awrt 6.298. An exact form alone does not score.
		[4]	
8(c)	$\int 2t\sin t\,\mathrm{d}t = -2t\cos t + \int 2\cos t\,\mathrm{d}t$	**M1**	Uses integration by parts with $u = 2t$ or $u = t$, reaching the form $\alpha t\cos t + \beta\int\cos t\,\mathrm{d}t$. Condone sign and coefficient slips.
	$= -2t\cos t + 2\sin t\ (+c)$	**A1**	
	$[-2t\cos t + 2\sin t]_0^{\pi}$ $= (-2\pi\cos\pi + 2\sin\pi) - (0 + 2\sin 0)$ $= 2\pi$	**A1 AG**	Both limits substituted into a correct integral, with $\cos\pi = -1$ used, reaching 2π without error.
		[3]	
	Total for Question 8	**9**	

Question	Scheme	Marks	Notes
9(a)	$\frac{\mathrm{d}r}{\mathrm{d}\theta} = \cos\theta\, e^{\sin\theta} = 0 \Rightarrow \theta = \ldots$	**M1**	Differentiates r to the form $k\cos\theta\, e^{\sin\theta}$ and sets it to zero, or uses $\sin\theta$ greatest, or takes $\theta = \frac{\pi}{2}$ from the diagram. Condone a coefficient slip.
	$\theta = \frac{\pi}{2}$	**A1**	
	$\left(e, \frac{\pi}{2}\right)$	**A1**	Or $r = e$, $\theta = \frac{\pi}{2}$ stated, or awrt $(2.72, 1.57)$. No justification that this is the furthest point is needed.
		[3]	
9(b)	$A = \frac{1}{2}\int_0^{\pi} e^{2\sin\theta}\,\mathrm{d}\theta$	**B1**	Integrand $e^{2\sin\theta}$ or $\left(e^{\sin\theta}\right)^2$, limits 0 and π, and the factor $\frac{1}{2}$; soi by later working.
	$h = \frac{\pi}{6}$	**B1**	
	$y = 1,\ e,\ e^{\sqrt{3}},\ e^2,\ e^{\sqrt{3}},\ e,\ 1$	**B1**	All seven ordinates of $e^{2\sin\theta}$, exact or awrt $1, 2.72, 5.65, 7.39, 5.65, 2.72, 1.$
	$\int_0^{\pi} e^{2\sin\theta}\,\mathrm{d}\theta$ $\approx \frac{1}{3}\times\frac{\pi}{6}\left\{(1+1)+4\left(e+e^2+e\right)\right.$ $\left.+2\left(e^{\sqrt{3}}+e^{\sqrt{3}}\right)\right\} = \ldots$ $[\approx 13.249]$	**M1**	Applies Simpson's rule to their seven ordinates with coefficients 1, 4, 2, 4, 2, 4, 1 and $\frac{h}{3}$. Condone a slip in one ordinate or in h.
	$A \approx \frac{1}{2}\times$ "13.249" ≈ 6.62	**A1**	awrt 6.62. Needs the factor $\frac{1}{2}$. Ordinates rounded to 3 s.f. give 6.63, A0. An exact form alone does not score.
		[5]	
	Total for Question 9	**8**	

Question	Scheme	Marks	Notes
10(a)	$\left[\frac{ax^3}{3}+\frac{bx^2}{2}+cx\right]_{-h}^{h}$ $=\left(\frac{ah^3}{3}+\frac{bh^2}{2}+ch\right)$ $\quad-\left(-\frac{ah^3}{3}+\frac{bh^2}{2}-ch\right)$	**M1**	Integrates, with the power of at least one term increased by 1, and substitutes h and $-h$ and subtracts, or doubles the integral from 0 to h of the even terms.
	$=\frac{2ah^3}{3}+2ch=\frac{2h}{3}\left(ah^2+3c\right)$	**A1 AG**	Correct integral and substitution leading without error to the printed result.
		[2]	
10(b)	$y_0=ah^2-bh+c,\quad y_1=c,\quad y_2=ah^2+bh+c$	**B1**	All three correct.
	$y_0+4y_1+y_2=$ "ah^2-bh+c" $+4$"c" $+$ "ah^2+bh+c"	**M1**	Substitutes their three ordinates, each found from ax^2+bx+c, into $y_0+4y_1+y_2$ or $\frac{h}{3}(y_0+4y_1+y_2)$ and collects terms. Condone slips.
	$y_0+4y_1+y_2=2ah^2+6c$ $\Rightarrow\frac{h}{3}\left(y_0+4y_1+y_2\right)=\frac{2h}{3}\left(ah^2+3c\right)$ $\quad=\int_{-h}^{h}\left(ax^2+bx+c\right)\mathrm{d}x$ by (a)	**A1 AG**	Correct ordinates leading without error to $\frac{2h}{3}(ah^2+3c)$, equated to the integral or with a reference to (a).
		[3]	
10(c)	Each quadratic passes through three consecutive ordinates, so spans two strips Strips are used in pairs $\Rightarrow$ even number	**E1**	States that each quadratic covers two adjacent strips, or three ordinates, so the strips are taken in pairs. Not for restating that the number is even.
		[1]	
10(d)	$T_1=\frac{1}{2}\times 2h\left(y_0+y_2\right)=h\left(y_0+y_2\right)$ $T_2=\frac{h}{2}\left(y_0+2y_1+y_2\right)$	**M1 A1**	**M1** Applies the trapezium rule with width $2h$ for T_1 and h for T_2, using the ordinates at the strip ends. Condone a coefficient slip. **A1** Both correct, oe.
	$4T_2-T_1=2h\left(y_0+2y_1+y_2\right)-h\left(y_0+y_2\right)$ $\quad=h\left(y_0+4y_1+y_2\right)$ $\Rightarrow\frac{4T_2-T_1}{3}=\frac{h}{3}\left(y_0+4y_1+y_2\right)$	**A1 AG**	Correct T_1 and T_2 substituted into $4T_2-T_1$, leading without error to the printed result.
		[3]	
	Total for Question 10	**9**	