OPTIONAL PURE / DIFFERENTIAL EQUATIONS & NUMERICAL METHODS

Series Solutions of Differential Equations

Mark scheme

12 questions · 105 marks

EXAM BOARDS Edexcel FP1 9FM0/3A

Questions

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Question 12	(a) 2 (b) 4 (c) 4	10	16
	Total	**105**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last accuracy mark of the working that reaches a printed result: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1	$x = 0$: $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \cos 0 - 0 - 2 = -1$	**B1**	
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = -2\sin 2x - 3\frac{\mathrm{d}y}{\mathrm{d}x} - 2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	**M1**	Differentiates the equation, with the product rule on $2x\frac{\mathrm{d}y}{\mathrm{d}x}$ giving two terms. Condone sign and coefficient slips.
	$x = 0$: $\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 0 - 3(-1) - 0 = 3$	**A1**	
	$\frac{\mathrm{d}^4y}{\mathrm{d}x^4} = -4\cos 2x - 5\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2x\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ $x = 0$: $\frac{\mathrm{d}^4y}{\mathrm{d}x^4} = -4 - 5\,(\text{“}-1\text{”}) = \ldots\ [= 1]$	**dM1**	Differentiates their third-derivative equation, with the product rule, and substitutes $x = 0$ values to find the fourth derivative. Condone sign slips.
	$y = 2 - x + \frac{\text{“}-1\text{”}}{2!}x^2 + \frac{\text{“}3\text{”}}{3!}x^3 + \frac{\text{“}1\text{”}}{4!}x^4 + \cdots$	**M1**	Uses Maclaurin's series with their values at $x = 0$ and the factorials 2!, 3!, 4!, or 2, 6, 24.
	$y = 2 - x - \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{24}x^4 + \cdots$	**A1**	Correct series, oe.
	Alternative: Substituting a power series		
	$y = \sum_{n=0}^{\infty} a_n x^n$ $\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + 2x\sum_{n=1}^{\infty} n a_n x^{n-1}$ $\quad + \sum_{n=0}^{\infty} a_n x^n = \cos 2x$	**M1**	Substitutes a power series for y and its first two derivatives into the equation, in summation notation or written out term by term. Condone index and coefficient slips.
	$\sum_{n=0}^{\infty}\left[(n+2)(n+1)a_{n+2} + (2n+1)a_n\right]x^n$ $\quad = \cos 2x$	**A1**	Correct coefficient of x^n, oe, or correct coefficients of x^0, x^1 and x^2 when written out term by term.
	$\cos 2x = 1 - \frac{(2x)^2}{2!} + \cdots = 1 - 2x^2 + \cdots$	**B1**	To at least the term in x^2.
	Equates coefficients, with $a_0 = 2$, $a_1 = -1$: $2a_2 + 2 = 1 \Rightarrow a_2 = \ldots$ $6a_3 - 3 = 0 \Rightarrow a_3 = \ldots$ $12a_4 + 5a_2 = -2 \Rightarrow a_4 = \ldots$	**dM1**	Equates coefficients of at least x^0 and x^1, with a_0 and a_1 from the initial conditions, and solves for a_2 and a_3.
	$a_2 = -\frac{1}{2}$, $a_3 = \frac{1}{2}$	**A1**	
	$y = 2 - x - \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{24}x^4 + \cdots$	**A1**	Correct series, oe.
	Total for Question 1	**6**	

Question	Scheme	Marks	Notes
2(a)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + (2+x)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ $= 2y\frac{\mathrm{d}y}{\mathrm{d}x} - 3\frac{\mathrm{d}y}{\mathrm{d}x} - 3x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	**M1 A1**	**M1** Differentiates both sides, using the product rule on at least one product and $y^2 \to ky\frac{\mathrm{d}y}{\mathrm{d}x}$. Condone sign slips. **A1** Correct differentiated equation, unsimplified.
	$(2+x)\frac{\mathrm{d}^3y}{\mathrm{d}x^3} + (3x+1)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ $= (2y-3)\frac{\mathrm{d}y}{\mathrm{d}x}$	**A1 AG**	Printed result reached from a correct differentiated equation without error; collected steps accepted.
		[3]	
2(b)	$2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2^2 \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2$ $2\frac{\mathrm{d}^3y}{\mathrm{d}x^3} + 2 = (4-3)(-1)$ $\Rightarrow \frac{\mathrm{d}^3y}{\mathrm{d}x^3} = -\frac{3}{2}$	**M1 A1**	**M1** Substitutes the conditions into the equation and into part (a) to find both higher derivatives. Condone arithmetic slips. **A1** $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2$ and $\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = -\frac{3}{2}$, soi.
	$y \approx 2 - x + \frac{\text{“2”}}{2!}x^2 + \frac{\text{“}-\frac{3}{2}\text{”}}{3!}x^3$ $y \approx 2 - x + x^2 - \frac{1}{4}x^3$	**A1**	Correct series, oe, using 2! and 3!; coefficients evaluated.
		[3]	
2(c)	$y \approx 2 - 0.2 + 0.2^2 - \frac{1}{4}(0.2)^3$	**M1**	Substitutes $x = 0.2$ into their series from part (b) with at least three terms.
	$= 1.838$	**A1**	cao
		[2]	
	Total for Question 2	**8**	

Question	Scheme	Marks	Notes
3(a)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2 \times 1 \times 2 - 1^2 = e^0$ $\Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -2$	**B1**	
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + x\frac{\mathrm{d}^3y}{\mathrm{d}x^3} + 2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ $+ 2y\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 2e^{2x-2}$	**M1**	Differentiates every term of the equation with respect to x, using the product rule on at least one product. Condone one slip.
	$-2 + \frac{\mathrm{d}^3y}{\mathrm{d}x^3} + 8 - 4 - 4 = 2$ $\Rightarrow \frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 4$	**A1**	cso, from a correct differentiated equation.
	$y \approx 1 + 2(x-1) + \frac{\text{“}-2\text{”}}{2!}(x-1)^2$ $+ \frac{\text{“}4\text{”}}{3!}(x-1)^3$	**M1**	Uses $y = 1$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 2$ and their two higher derivatives in a series in powers of $(x-1)$, dividing by 2! and 3!.
	$y \approx 1 + 2(x-1) - (x-1)^2 + \frac{2}{3}(x-1)^3$	**A1**	Correct series, oe; coefficients need not be simplified.
		[5]	
3(b)	$y \approx 1 + 2(0.1) - (0.1)^2 + \frac{2}{3}(0.1)^3$	**M1**	Substitutes $x - 1 = 0.1$ into their series from (a) in powers of $(x-1)$.
	$y \approx 1.19$	**A1**	awrt 1.19 from a correct series, with no incorrect values seen.
		[2]	
	Total for Question 3	**7**	

Question	Scheme	Marks	Notes
4(a)	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + y\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2$	**M1 A1**	**M1** Differentiates both sides, with the product rule on $y\frac{\mathrm{d}y}{\mathrm{d}x}$ giving two terms. Condone sign slips. **A1** Correct differentiated equation, in any arrangement.
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 2 - \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 - y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	**A1 AG**	Printed result from a correct differentiated equation with no errors seen; differentiating and rearranging may be combined.
		[3]	
4(b)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 4, \qquad \frac{\mathrm{d}^3y}{\mathrm{d}x^3} = -7$	**B1**	Both values, soi in the series.
	$\frac{\mathrm{d}^4y}{\mathrm{d}x^4} = -3\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - y\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$	**M1**	Differentiates the part (a) result, or the equation twice, with the product rule on at least one product. Condone sign and coefficient slips.
	$\frac{\mathrm{d}^4y}{\mathrm{d}x^4} = -3(-1)(4) - 2(-7) = 26$	**A1**	
	$y = 2 - (x-1) + \frac{\text{“4”}}{2!}(x-1)^2 + \frac{\text{“}-7\text{”}}{3!}(x-1)^3 + \frac{\text{“26”}}{4!}(x-1)^4 + \cdots$	**M1**	Substitutes their derivative values into a Taylor series in powers of $(x-1)$ with factorial denominators, at least as far as the $(x-1)^3$ term.
	$y = 2-(x-1)+2(x-1)^2-\frac{7}{6}(x-1)^3+\frac{13}{12}(x-1)^4+\cdots$	**A1**	Every coefficient in simplest form; $y \approx$ accepted. Decimals, or an expansion in powers of x, score A0.
		[5]	
	Total for Question 4	**8**	

Question	Scheme	Marks	Notes
5	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -0 \times 3 - 2\cos 0 = -2$	**B1**	
	$\frac{\mathrm{d}^3x}{\mathrm{d}t^3} = -\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 - x\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\sin x\frac{\mathrm{d}x}{\mathrm{d}t}$	**M1**	Differentiates the equation with respect to t, using the product rule on $x\frac{\mathrm{d}x}{\mathrm{d}t}$ and the chain rule on $\cos x$. Condone sign and coefficient slips.
	$t = 0$: $\frac{\mathrm{d}^3x}{\mathrm{d}t^3} = -9$	**A1**	cso, from a correct expression for the third derivative.
	$\frac{\mathrm{d}^4x}{\mathrm{d}t^4} = -3\frac{\mathrm{d}x}{\mathrm{d}t}\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - x\frac{\mathrm{d}^3x}{\mathrm{d}t^3} + 2\cos x\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + 2\sin x\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ $t = 0$: $\frac{\mathrm{d}^4x}{\mathrm{d}t^4} = \ldots$ $[= 36]$	**dM1**	Differentiates their third derivative, with product and chain rules where needed, and substitutes their values at $t = 0$. Condone slips.
	$x = 3t + \frac{\text{“}-2\text{”}}{2!}t^2 + \frac{\text{“}-9\text{”}}{3!}t^3 + \frac{\text{“}36\text{”}}{4!}t^4 + \cdots$	**M1**	Substitutes their values at $t = 0$ into the series up to t^4, dividing by 2!, 3!, 4! or 2, 6, 24.
	$x = 3t - t^2 - \frac{3}{2}t^3 + \frac{3}{2}t^4 + \cdots$	**A1**	Correct series, oe.
	Total for Question 5	**6**	

Question	Scheme	Marks	Notes
6(a)	$2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + (-1)^2 - 0 = 4 \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{3}{2}$	**B1**	
	$\frac{\mathrm{d}}{\mathrm{d}x}\left[(x+y)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right]$ $= \left(1+\frac{\mathrm{d}y}{\mathrm{d}x}\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + (x+y)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$	**M1 A1**	**M1** Differentiates the equation with respect to x, applying the product rule to at least one product. Condone sign and coefficient slips. **A1** Correct derivative of $(x+y)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$, oe such as from the expanded product.
	$\left(1+\frac{\mathrm{d}y}{\mathrm{d}x}\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + (x+y)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ $+ 2\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2y - 2x\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	**A1**	Fully correct differentiated equation, oe.
	$0 + 2\frac{\mathrm{d}^3y}{\mathrm{d}x^3} + 2(-1)\left(\text{“}\frac{3}{2}\text{”}\right) - 4 - 0 = 0$ $\Rightarrow \frac{\mathrm{d}^3y}{\mathrm{d}x^3} = \ldots$	**dM1**	Substitutes $x = 0$, $y = 2$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -1$ and their $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ into their differentiated equation containing a third-derivative term, and solves for it.
	$2\frac{\mathrm{d}^3y}{\mathrm{d}x^3} - 7 = 0 \Rightarrow \frac{\mathrm{d}^3y}{\mathrm{d}x^3} = \frac{7}{2}$	**A1 AG**	Correct differentiated equation with $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{3}{2}$ substituted, leading without error to $\frac{7}{2}$.
		[6]	
6(b)	$y \approx 2 - x + \frac{\text{“}3/2\text{”}}{2!}x^2 + \frac{7/2}{3!}x^3$	**M1**	Uses Maclaurin's series with their $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $\frac{7}{2}$, with 2! and 3! denominators. Need not be simplified.
	$y \approx 2 - x + \frac{3}{4}x^2 + \frac{7}{12}x^3$	**A1**	Correct series, oe.
	$y \approx 2 - 0.2 + \frac{3}{4}(0.2)^2 + \frac{7}{12}(0.2)^3$	**M1**	Substitutes $x = 0.2$ into their series of at least three terms in ascending powers of x.
	$y \approx 1.835$	**A1**	1.835 to 3 d.p.
		[4]	
	Total for Question 6	**10**	

Question	Scheme	Marks	Notes
7(a)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2\times 2\times(-1)^2 + 2 = 0$ $\Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2$	**B1**	
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(2y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)$ $= 2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3 + 4y\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	**M1 A1**	**M1** Differentiates the equation, or a rearrangement of it, term by term, with the product rule on at least one product. Condone sign and coefficient slips. **A1** Correct derivative of this term, oe.
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + x\frac{\mathrm{d}^3y}{\mathrm{d}x^3} - 2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3$ $- 4y\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2xy + x^2\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	**A1**	Fully correct differentiated equation, oe.
	"2" $+ \frac{\mathrm{d}^3y}{\mathrm{d}x^3} - 2(-1)^3 - 4(2)(-1)\,(\text{“}2\text{”}) + 4 - 1 = 0$ $\Rightarrow \frac{\mathrm{d}^3y}{\mathrm{d}x^3} = \ldots$	**dM1**	Substitutes $x = 1$, $y = 2$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -1$ and their $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ into their differentiated equation and solves for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$.
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = -23$	**A1**	
		[6]	
7(b)	$y \approx 2 + (x-1)(-1) + \frac{(x-1)^2}{2!}\times$ "2" $+ \frac{(x-1)^3}{3!}\times$ "-23"	**M1**	Substitutes their values into a Taylor series in powers of $(x-1)$, with 2! and 3!, as far as the term in $(x-1)^3$.
	$y \approx 2 - (x-1) + (x-1)^2 - \frac{23}{6}(x-1)^3$	**A1**	Correct series, each coefficient in simplest form.
	$y \approx 2 - 0.1 + 0.1^2 - \text{“}\frac{23}{6}\text{”}\times 0.1^3$	**M1**	Substitutes $x - 1 = 0.1$ into their series of at least three terms in powers of $(x-1)$.
	$y \approx 1.906$	**A1**	
		[4]	
	Total for Question 7	**10**	

Question	Scheme	Marks	Notes
8(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} + (1+x)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 1 + 2y\frac{\mathrm{d}y}{\mathrm{d}x}$	**M1 A1**	**M1** Differentiates both sides, product rule on the left and $y^2 \to ky\frac{\mathrm{d}y}{\mathrm{d}x}$ on the right. Condone one slip. **A1** Correct equation, oe.
	$(1+x)\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + (2y-2)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ $\frac{\mathrm{d}^3y}{\mathrm{d}x^3} + (1+x)\frac{\mathrm{d}^4y}{\mathrm{d}x^4}$ $= 6\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + (2y-2)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$	**dM1**	Differentiates twice more, with the product rule on at least one product each time, reaching an equation in $\frac{\mathrm{d}^4y}{\mathrm{d}x^4}$. Condone sign and coefficient slips.
	$(1+x)\frac{\mathrm{d}^4y}{\mathrm{d}x^4} = (2y-3)\frac{\mathrm{d}^3y}{\mathrm{d}x^3} + 6\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ $a = 2,\ b = -3,\ c = 6$	**A1**	cso. All three values, stated or seen in the correct equation.
		[4]	
8(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 0 + 1^2 = 1,\ \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 1 + (2-1)\times 1 = \ldots$	**M1**	Substitutes $x = 0$, $y = 1$ into $(*)$ and their derivative equations in turn, using their part (a) for the fourth derivative. Condone slips.
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 1,\ \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2$ $\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 2,\ \frac{\mathrm{d}^4y}{\mathrm{d}x^4} = 10$	**A1**	All four values correct.
	$y \approx 1 + x + \frac{\text{“2”}}{2!}x^2 + \frac{\text{“2”}}{3!}x^3 + \frac{\text{“10”}}{4!}x^4$	**M1**	Uses Maclaurin's series with their values and factorials 2!, 3!, 4!, to the term in x^4.
	$y \approx 1 + x + x^2 + \frac{1}{3}x^3 + \frac{5}{12}x^4$	**A1**	Correct series, oe; coefficients need not be simplified.
	$y \approx 1 + 0.2 + 0.2^2 + \frac{1}{3}(0.2)^3 + \frac{5}{12}(0.2)^4$ ≈ 1.243	**A1**	awrt 1.243, from a correct series.
		[5]	
	Total for Question 8	**9**	

Question	Scheme	Marks	Notes
9(a)	$x\frac{\mathrm{d}^3y}{\mathrm{d}x^3} + \frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + x\frac{\mathrm{d}y}{\mathrm{d}x} + y = 0$ $\Rightarrow x\frac{\mathrm{d}^3y}{\mathrm{d}x^3} + 3\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + x\frac{\mathrm{d}y}{\mathrm{d}x} + y = 0$	**M1 A1**	**M1** Differentiates the equation, applying the product rule to $x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and xy. Condone one slip. **A1** Correct third-order equation, oe uncollected.
	$x\frac{\mathrm{d}^4y}{\mathrm{d}x^4} + 4\frac{\mathrm{d}^3y}{\mathrm{d}x^3} + x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	**dM1**	Differentiates their third-order equation, with the product rule on its x terms, and collects like derivatives. Condone one slip.
	$a = 4,\ b = 2$	**A1**	Both values, cso.
		[4]	
9(b)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2 + 2 = 2 \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \ldots$ $\frac{\mathrm{d}^3y}{\mathrm{d}x^3} + 3\text{“2”} - 1 + 2 = 0 \Rightarrow \frac{\mathrm{d}^3y}{\mathrm{d}x^3} = \ldots$ $\frac{\mathrm{d}^4y}{\mathrm{d}x^4} + 4\text{“}-7\text{”} + \text{“2”} - 2 = 0 \Rightarrow \frac{\mathrm{d}^4y}{\mathrm{d}x^4} = \ldots$	**M1**	Substitutes the values at $x = 1$ into the equation and their equations from (a) to find the three higher derivatives. Condone arithmetic slips.
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2,\ \frac{\mathrm{d}^3y}{\mathrm{d}x^3} = -7,\ \frac{\mathrm{d}^4y}{\mathrm{d}x^4} = 28$	**A1**	All three, soi in the series.
	$y \approx 2 - (x-1) + \frac{\text{“2”}}{2!}(x-1)^2$ $\quad + \frac{\text{“}-7\text{”}}{3!}(x-1)^3 + \frac{\text{“28”}}{4!}(x-1)^4$ $y \approx 2 - (x-1) + (x-1)^2 - \frac{7}{6}(x-1)^3 + \frac{7}{6}(x-1)^4$	**A1**	Correct series in powers of $(x-1)$, using 2!, 3! and 4!, each term simplified. Not for a polynomial expanded in x.
		[3]	
9(c)	$\frac{\mathrm{d}y}{\mathrm{d}x} \approx -1 + 2(x-1) - \frac{7}{2}(x-1)^2 + \frac{14}{3}(x-1)^3$	**M1**	Differentiates their series from (b) term by term, the power of at least one term decreased by 1.
	$\frac{\mathrm{d}y}{\mathrm{d}x} \approx -1 + 2(-0.1) - \frac{7}{2}(-0.1)^2 + \frac{14}{3}(-0.1)^3$	**dM1**	Substitutes $x - 1 = -0.1$ into their differentiated series, or $x = 0.9$ if it is in powers of x.
	$\frac{\mathrm{d}y}{\mathrm{d}x} \approx -1.240$	**A1**	awrt -1.240
		[3]	
	Total for Question 9	**10**	

Question	Scheme	Marks	Notes
10(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right) = \frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + y\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$	**M1 A1**	**M1** Differentiates the equation, or a rearrangement of it, with respect to x, attempting the product or quotient rule and the chain rule. Condone sign and coefficient slips. **A1** Correct derivative of $y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$, or of $\frac{2}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ if the equation is rearranged first.
	$\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + y\frac{\mathrm{d}^3y}{\mathrm{d}x^3} + 4\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 6$	**A1**	Fully correct differentiated equation, oe.
	$y\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 6 - 5\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ $= 6 - \frac{5}{y}\frac{\mathrm{d}y}{\mathrm{d}x}\left(6x - 2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)$	**M1**	Substitutes for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ from the original equation, in terms of x, y and $\frac{\mathrm{d}y}{\mathrm{d}x}$, into their differentiated equation. Condone sign slips.
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = \frac{6}{y} + \frac{10}{y^2}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3 - \frac{30x}{y^2}\frac{\mathrm{d}y}{\mathrm{d}x}$	**A1 AG**	Correct substituted equation leading without error to the printed result; combined steps accepted.
		[5]	
10(b)	$2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2(-1)^2 = 6 \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2$ $\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = \frac{6}{2} + \frac{10}{4}(-1)^3 - \frac{30}{4}(-1) = 8$	**M1 A1**	**M1** Substitutes the conditions at $x = 1$ into the equation and into the result of (a) to find both derivatives. Condone arithmetic slips. **A1** Both values correct, soi by correct coefficients.
	$y \approx 2 - (x-1) + \frac{\text{“2”}}{2!}(x-1)^2 + \frac{\text{“8”}}{3!}(x-1)^3$ $y \approx 2 - (x-1) + (x-1)^2 + \frac{4}{3}(x-1)^3$	**A1**	Correct series, oe, using 2! and 3!; coefficients simplified.
		[3]	
10(c)	$y \approx 2 - 0.2 + 0.2^2 + \text{“}\frac{4}{3}\text{”} \times 0.2^3$	**M1**	Substitutes $x - 1 = 0.2$ into their series from (b).
	≈ 1.851	**A1**	1.851; $\frac{694}{375}$ alone does not score.
		[2]	
	Total for Question 10	**10**	

Question	Scheme	Marks	Notes
11(a)	$2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+(1+x^2)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ $+\frac{\mathrm{d}y}{\mathrm{d}x}+x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-6\frac{\mathrm{d}y}{\mathrm{d}x}=0$	**M1**	Differentiates the equation, using the product rule on at least one of the first two terms and obtaining a third-derivative term. Condone sign and coefficient slips.
	$(1+x^2)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+3x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-5\frac{\mathrm{d}y}{\mathrm{d}x}=0$	**A1**	Correct third-order equation, oe; need not be collected.
	$2x\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+(1+x^2)\frac{\mathrm{d}^4y}{\mathrm{d}x^4}$ $+3\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+3x\frac{\mathrm{d}^3y}{\mathrm{d}x^3}-5\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$	**dM1**	Differentiates their third-order equation, product rule on both products, and collects terms into the printed form. Condone sign and coefficient slips.
	$(1+x^2)\frac{\mathrm{d}^4y}{\mathrm{d}x^4}+5x\frac{\mathrm{d}^3y}{\mathrm{d}x^3}-2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$ $a=5,\ b=-2$	**A1**	Both values, or the equation with them, cso.
		[4]	
11(b)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=12$ $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=-5$ $\frac{\mathrm{d}^4y}{\mathrm{d}x^4}=24$	**B1**	All three values at $x=0$ correct.
	$(1+x^2)\frac{\mathrm{d}^5y}{\mathrm{d}x^5}+7x\frac{\mathrm{d}^4y}{\mathrm{d}x^4}+3\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=0$	**M1**	Differentiates their part (a) equation, product rule on at least one product, obtaining a fifth-derivative term. Need not be collected; condone sign and coefficient slips.
	$\frac{\mathrm{d}^5y}{\mathrm{d}x^5}=15$	**A1**	
	$y=2-x+\frac{\text{“12”}}{2!}x^2+\frac{\text{“}-5\text{”}}{3!}x^3$ $+\frac{\text{“24”}}{4!}x^4+\frac{\text{“15”}}{5!}x^5+\cdots$	**M1**	Substitutes their derivative values into Maclaurin's series with the correct factorials, at least as far as the term in x^4.
	$y=2-x+6x^2-\frac{5}{6}x^3+x^4+\frac{1}{8}x^5+\cdots$	**A1**	Correct series, oe, as far as the term in x^5; coefficients need not be simplified.
	Alternative: Substituting a power series		
	$y=\sum_{n=0}^{\infty}a_nx^n$ $\sum_{n=2}^{\infty}n(n-1)a_nx^{n-2}+\sum_{n=2}^{\infty}n(n-1)a_nx^n$ $+\sum_{n=1}^{\infty}na_nx^n-6\sum_{n=0}^{\infty}a_nx^n=0$	**M1**	Substitutes a power series for y, in summation notation or term by term, and its first two derivatives into the equation. Condone slips in limits and coefficients.
	$(n+2)(n+1)a_{n+2}+(n^2-6)a_n=0$ $\Rightarrow a_{n+2}=\frac{(6-n^2)a_n}{(n+1)(n+2)}$	**A1**	Correct recurrence, oe, or correct equations from the coefficients of x^0 to x^3.

Question 11 continued

Question	Scheme	Marks	Notes
	$a_0 = 2,\ a_1 = -1$ $a_2 = \frac{6\times 2}{2} = \ldots,\ a_3 = \frac{5\times(-1)}{6} = \ldots$	**dM1**	Uses their recurrence or coefficient equations with $a_0 = 2$ and $a_1 = -1$ to find at least a_2 and a_3.
	$a_2 = 6,\ a_3 = -\frac{5}{6}$	**A1**	
	$a_4 = 1,\ a_5 = \frac{1}{8}$ $y = 2 - x + 6x^2 - \frac{5}{6}x^3 + x^4 + \frac{1}{8}x^5 + \cdots$	**A1**	Correct series, oe, as far as the term in x^5; coefficients need not be simplified.
		[5]	
11(c)	$y \approx 2 - 0.1 + 6(0.1)^2 - \frac{5}{6}(0.1)^3$ $+ (0.1)^4 + \frac{1}{8}(0.1)^5$	**M1**	Substitutes $x = 0.1$ into their series from (b), with at least four nonzero terms.
	$y \approx 1.9593$	**A1**	
		[2]	
	Total for Question 11	**11**	

Question	Scheme	Marks	Notes
12(a)	$\frac{1}{2}\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2(1)(0) = 1^2 \times \frac{\sqrt{3}}{2}$ $\Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \ldots$	**M1**	Substitutes $x = \frac{\pi}{6}$, $y = 1$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ into the differential equation and solves for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. Condone a slip in a trigonometric value.
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \sqrt{3} > 0$, so minimum	**A1**	$\sqrt{3}$, or awrt 1.73, with its positive sign used to conclude "minimum".
		[2]	
12(b)	$\cos x \frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \sin x \frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ $- 2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 - 2y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ $= 2y\frac{\mathrm{d}y}{\mathrm{d}x}\cos x - y^2 \sin x$	**M1 A1**	**M1** Differentiates both sides, applying the product rule to at least two of the three products. Condone sign and coefficient slips. **A1** Correct differentiated equation, oe.
	$\frac{\sqrt{3}}{2} \times$ "$\sqrt{3}$" $+ \frac{1}{2}\frac{\mathrm{d}^3y}{\mathrm{d}x^3} - 2$"$\sqrt{3}$" $= -\frac{1}{2} \Rightarrow \frac{\mathrm{d}^3y}{\mathrm{d}x^3} = \ldots$	**dM1**	Substitutes $x = \frac{\pi}{6}$, $y = 1$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and their $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ into their differentiated equation and solves for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$.
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 4\sqrt{3} - 4$	**A1**	Any exact equivalent.
		[4]	
12(c)	$y \approx 1 + \frac{\text{"}\sqrt{3}\text{"}}{2!}\left(x - \frac{\pi}{6}\right)^2$ $+ \frac{\text{"}4\sqrt{3}-4\text{"}}{3!}\left(x - \frac{\pi}{6}\right)^3$	**M1**	Uses $y = 1$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and their second and third derivatives in the Taylor series in powers of $\left(x - \frac{\pi}{6}\right)$, with 2! and 3!.
	$y \approx 1 + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)^2$ $+ \frac{2\sqrt{3}-2}{3}\left(x - \frac{\pi}{6}\right)^3$	**A1**	Exact coefficients, need not be simplified.
	$y \approx 1 + \frac{\sqrt{3}}{2}\left(0.6 - \frac{\pi}{6}\right)^2 + \ldots$	**M1**	Substitutes $x = 0.6$ into their series in powers of $\left(x - \frac{\pi}{6}\right)$ and evaluates.
	$y \approx 1.0053$	**A1**	
		[4]	
	Total for Question 12	**10**	