OPTIONAL PURE / DIFFERENTIAL EQUATIONS & NUMERICAL METHODS

Numerical Solution of Differential Equations

Mark scheme

13 questions · 94 marks

EXAM BOARDS Edexcel FP1 9FM0/3A · AQA 7367

Questions

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	Total	**94**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last accuracy mark of the working that reaches a printed result: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{xy+2}{x+1}$ $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 = \frac{1\times3+2}{1+1} = 2.5$	**B1**	Correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$, oe. The value 2.5 is not needed for this mark.
	$y_1 = 3 + 0.1 \times$ "2.5" $\Rightarrow y_1 = \ldots$	**M1**	Uses the forward difference with $y_0 = 3$, $h = 0.1$ and their gradient at $(1, 3)$.
	$y(1.1) \approx 3.25$	**A1**	
		[3]	
1(b)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_1 = \frac{1.1\times\text{"3.25"}+2}{1.1+1}$ $\left[= \frac{223}{84}\right]$	**B1ft**	ft their $y(1.1)$ from (a), with $x = 1.1$ in their expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. Exact or awrt 2.655, soi.
	$y_2 = 3 + 2 \times 0.1 \times$ "$\frac{223}{84}$" $\Rightarrow y_2 = \ldots$	**M1**	Uses the central difference with $y_0 = 3$, $2h = 0.2$ and their gradient at $x = 1.1$. Not for their $y(1.1)$ in place of 3.
	$y(1.2) \approx 3.531$	**A1**	awrt 3.531.
		[3]	
Total for Question 1		**6**	

Question	Scheme	Marks	Notes
2(a)	$\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)_0 = 9.8 - 0.05 \times 20^2 = -10.2$	**B1**	-10.2 soi by a correct v_1.
	$v_1 = 20 + 0.2 \times$ "-10.2" $= \ldots$	**M1**	Uses $v_1 = v_0 + 0.2\times$ their gradient at $v_0 = 20$. Condone sign slips.
	$v_1 = 17.96$	**A1**	
	$\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)_1 = 9.8 - 0.05 \times$ "17.96"^{2} $v_2 =$ "17.96" $+ 0.2 \times \ldots = \ldots$	**dM1**	Finds the gradient at their v_1 and uses $v_2 = v_1 + 0.2\times$ it.
	$v_2 = 16.694\ldots$	**A1**	awrt 16.7.
	$\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)_2 = 9.8 - 0.05 \times$ "16.69"^{2} $v_3 =$ "16.69" $+ 0.2 \times \ldots = \ldots$	**dM1**	Finds the gradient at their v_2 and uses $v_3 = v_2 + 0.2\times$ it.
	$v_3 \approx 15.867 \Rightarrow 15.9\,\mathrm{m\,s^{-1}}$	**A1**	
		[7]	
2(b)	Replaces 0.05 by k, with $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$, $v = 12$: $9.8 - 144k = 0 \Rightarrow k = \ldots$	**M1**	Replaces the coefficient of v^2 by an unknown, keeping 9.8, sets the derivative to zero with $v = 12$ and solves.
	$k = 0.0681$ $\frac{\mathrm{d}v}{\mathrm{d}t} = 9.8 - 0.0681v^2$	**A1**	awrt 0.0681, with the refined equation or a statement that 0.05 becomes 0.0681.
		[2]	
	Total for Question 2	**9**	

Question	Scheme	Marks	Notes
3(a)	$\left(\frac{\mathrm{d}H}{\mathrm{d}t}\right)_0 = \frac{12-5\times0.25\times\sqrt{1}}{2(1+3)}$ $= 1.34375$	**B1**	1.34375 oe, such as $\frac{43}{32}$ or awrt 1.34; soi by a correct H_1.
	$H_1 = 1 + 0.75 \times$ "1.34375"	**M1**	Uses $H_1 = 1 + h\times$ their gradient, with their step length h (0.75 correct). Condone a slip in their gradient.
	$H_1 = 2.0078125$	**A1**	awrt 2.01, or $\frac{257}{128}$.
	$\left(\frac{\mathrm{d}H}{\mathrm{d}t}\right)_1 = \frac{12-5\times1\times\sqrt{\text{"2.0078"}}}{2(\text{"2.0078"}+3)}$ $[\approx 0.4907]$ $H_2 =$ "2.0078" $+ 0.75 \times \ldots$	**dM1**	Evaluates the gradient at $t = 0.25 + h$ with their h and their H_1, and applies the formula again with their h. Condone slips in evaluation.
	$H_2 \approx 2.38$	**A1**	awrt 2.38.
		[5]	
3(b)	Use a smaller step length h, with more applications of the formula from $t = 0.25$ to $t = 1.75$	**E1**	States reducing the step length, or using more steps over the same interval. Not for a different formula or method alone.
		[1]	
	Total for Question 3	**6**	

Question	Scheme	Marks	Notes
4(a)	$\left(\frac{\mathrm{d}P}{\mathrm{d}t}\right)_0 = 0.2 \times 50 - 0.0008 \times 50^2 = 8$	**B1**	8 soi by a correct P_1.
	$P_1 = 50 + 1 \times$ "8" $= \ldots$	**M1**	Uses $P_1 = P_0 + h\times$ their gradient at $P_0 = 50$, with $h = 1$.
	$P_1 = 58$	**A1**	
	$\left(\frac{\mathrm{d}P}{\mathrm{d}t}\right)_1 = 0.2 \times$ "58" $- 0.0008 \times$ "58"^{2} $P_2 =$ "58" $+ \ldots$	**dM1**	Evaluates the gradient at their P_1 and adds it to their P_1. Condone arithmetic slips.
	$P_2 = 66.9088$	**A1**	awrt 66.9
	$\left(\frac{\mathrm{d}P}{\mathrm{d}t}\right)_2 = 0.2 \times$ "66.9088" $- 0.0008 \times$ "66.9088"^{2} $P_3 =$ "66.9088" $+ \ldots$	**dM1**	Evaluates the gradient at their P_2 and adds it to their P_2. Condone arithmetic slips.
	$P_3 \approx 76.7$	**A1**	awrt 76.7
	"76.7" ≥ 75, so Priya can begin selling after 3 months	**A1ft**	ft their P_3. Compares it with 75 and concludes in context, consistent with their value.
		[8]	
4(b)	$\frac{\mathrm{d}P}{\mathrm{d}t}$ increases as P increases ($P < 125$) Each step uses its least gradient $\Rightarrow$ underestimate	**E1**	States $\frac{\mathrm{d}P}{\mathrm{d}t}$ increases as P increases (values from (a) suffice), or that the curve is convex, and concludes underestimate. Condone no mention of tangents. Not for "underestimate" alone.
	Model's value $> 76.7 \geq 75$ Conclusion in (a) still valid	**E1**	States the model's value exceeds their estimate, so their conclusion stands (their estimate at least 75) or is not reliable (their estimate below 75).
		[2]	
	Total for Question 4	**10**	

Question	Scheme	Marks	Notes
5(a)	$2\frac{\mathrm{d}v}{\mathrm{d}t} = (8+2t) - 0.3v^2$	**M1**	Forms an equation of motion with mass times acceleration and both forces, acceleration as a or $\frac{\mathrm{d}v}{\mathrm{d}t}$. Condone sign slips.
	$20\frac{\mathrm{d}v}{\mathrm{d}t} = 80 + 20t - 3v^2$	**A1 AG**	Correct equation with $\frac{\mathrm{d}v}{\mathrm{d}t}$, leading without error to the printed result.
		[2]	
5(b)	$\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)_0 = 4 + 0 - 0.15 \times 1^2 = 3.85$	**B1**	Or $20\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)_0 = 77$ used correctly; soi by a correct v_1.
	$v_1 = 1 + 0.2 \times$ "3.85" $\Rightarrow v_1 = \ldots$	**M1**	Uses $v_1 = v_0 + 0.2\times$ their gradient at $(0, 1)$.
	$v_1 = 1.77$	**A1**	
	$\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)_1 = 4.2 - 0.15 \times$ "1.77"^{2} $v_2 =$ "1.77" $+ 0.2 \times$ "3.730065" $= \ldots$	**dM1**	Evaluates the gradient at $t = 0.2$ with their v_1 and applies a second step with $h = 0.2$. Condone slips in the gradient.
	$v_2 = 2.516013$	**A1**	awrt 2.516
	$\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)_2 = 4.4 - 0.15 \times$ "2.516013"^{2} $v_3 =$ "2.516013" $+ 0.2 \times$ "3.450452" $= \ldots$	**dM1**	Evaluates the gradient at $t = 0.4$ with their v_2 and applies a third step with $h = 0.2$. Condone slips in the gradient.
	$v_3 = 3.206$	**A1**	awrt 3.206
		[7]	
Total for Question 5		**9**	

Question	Scheme	Marks	Notes
6(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 = \frac{0.25-2.4}{1.5+1.2} = -0.7963$	**B1**	awrt -0.796, or $-\frac{43}{54}$ oe; soi by a correct y_1.
	$y_1 = -1.2 + 0.05 \times$ "-0.7963" $= \ldots$	**M1**	Uses $y_1 = y_0 + h\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0$ with $h = 0.05$ and their gradient at $(0.5, -1.2)$.
	$y_1 = -1.2398$	**A1**	awrt -1.240.
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_1 = \frac{0.3025+2\times\text{"}-1.2398\text{"}}{1.65-\text{"}-1.2398\text{"}}$ $= \ldots$ [$= -0.7534$] $y_2 = -1.2 + 2 \times 0.05 \times$ "-0.7534" $= \ldots$	**dM1**	Evaluates the gradient at (0.55, their y_1) and uses it in the central difference with $y_0 = -1.2$ and $2h = 0.1$. Condone one slip in substitution.
	$y = -1.2753$	**A1**	awrt -1.2753.
		[5]	
6(b)	States the central difference needs y_{-1}, the value of y at $x = 0.45$, which is not known	**E1**	States the central difference needs a value of y before $x = 0.5$ (y_{-1}, or y at $x = 0.45$) that is not known. Accept "needs two previous values but only one is given".
		[1]	
	Total for Question 6	**6**	

Question	Scheme	Marks	Notes
7(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 = 1 \times \ln 1 + \frac{2\times 3}{1} = 6$	**B1**	Gradient 6 at $(1, 3)$, soi.
	$y_1 = 3 + 0.2 \times$ "6" $= \ldots$	**M1**	Uses the forward difference as $y_1 = 3 + 0.2\times$ their gradient at $(1, 3)$ from the differential equation.
	$y_1 = 4.2$	**A1**	
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_1 = 1.2 \ln 1.2 + \frac{2\times \text{"4.2"}}{1.2}$ $y_2 = 3 + 0.4 \times$ "7.2188" $= \ldots$	**dM1**	Substitutes $x = 1.2$ and their y_1 into the differential equation, and uses the central difference as $y_2 = 3 + 0.4\times$ their gradient.
	$y_2 \approx 5.8875$	**A1**	awrt 5.888
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_2 = 1.4 \ln 1.4 + \frac{2\times \text{"5.8875"}}{1.4}$ $y_3 =$ "4.2" $+ 0.4 \times$ "8.8818" $= \ldots$	**dM1**	Substitutes $x = 1.4$ and their y_2 into the differential equation, and uses the central difference as $y_3 =$ their $y_1 + 0.4\times$ their gradient.
	$y \approx 7.7527$	**A1**	awrt 7.7527
		[7]	
7(b)	$y = \frac{1}{2} \times 1.6^2 \left((\ln 1.6)^2 + 6\right) \approx 7.9628$	**B1**	awrt 7.96
	$\frac{\text{"7.7527"} - \text{"7.9628"}}{\text{"7.9628"}} \times 100 = \ldots$	**M1**	Forms the difference between their estimate from (a) and their value from the curve's equation, divided by that value, times 100. Condone either sign. Not for dividing by the estimate.
	$(-)2.64\%$	**A1**	awrt 2.64, with either sign.
		[3]	
	Total for Question 7	**10**	

Question	Scheme	Marks	Notes
8	$2 + 0.25 \times \frac{a^2+2\times 2}{2a} = 2.6$	**M1 A1**	**M1** Sets $2 + 0.25\times$ their gradient at $(a, 2)$ equal to 2.6. Gradient from substituting $x = a$, $y = 2$; condone slips in it. **A1** Correct equation in a, oe; need not be simplified.
	$5a^2 - 24a + 20 = 0 \Rightarrow a = \ldots$	**dM1**	Rearranges to a three-term quadratic in a and solves it by any method, leading to $a = \ldots$; roots correct for their quadratic imply it. Condone slips in rearranging.
	$a = \frac{12\pm 2\sqrt{11}}{5}$	**A1**	Both values, any exact equivalent such as $\frac{24\pm\sqrt{176}}{10}$, with neither rejected.
	Total for Question 8	**4**	

Question	Scheme	Marks	Notes
9(a)	$\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)_0 = -0.002(80-20)^2 = -7.2$	**B1**	-7.2 soi.
	$\theta_1 = 80 + 2 \times$ "-7.2" $= \ldots$	**M1**	Uses $\theta_1 = 80 + h\times$ their gradient at $\theta = 80$, with their step length h (2 correct).
	$\theta_1 = 65.6$	**A1**	
	$\theta_2 =$ "65.6" $+ 2 \times (-0.002)$ $\times$ ("65.6" $- 20)^2 = \ldots$	**dM1**	Evaluates the gradient at their θ_1 and applies the formula again with their h.
	$\theta_2 = 57.3$	**A1**	awrt 57.3
		[5]	
9(b)	$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -2k(\theta-20)\frac{\mathrm{d}\theta}{\mathrm{d}t}$ $= -2k(\theta-20)\left(-k(\theta-20)^2\right)$	**M1**	Differentiates by the chain rule to a multiple of $k(\theta-20)\frac{\mathrm{d}\theta}{\mathrm{d}t}$ and substitutes for $\frac{\mathrm{d}\theta}{\mathrm{d}t}$, giving an expression in θ and k. Condone coefficient and sign slips.
	$= 2k^2(\theta-20)^3 > 0$ as $\theta > 20$ Convex, tangents below curve: underestimate	**E1**	States correct $2k^2(\theta-20)^3$ oe is positive because $\theta > 20$, and concludes underestimate. Condone no mention of convexity or tangents. Not for underestimate alone.
		[2]	
	Total for Question 9	**7**	

Question	Scheme	Marks	Notes
10(a)	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)_0 = \frac{0.4\times0.3\times4}{6.4} = 0.075$	**B1**	soi by a correct x_1.
	$x_1 = 0.4 + 0.25 \times$ "0.075" $\Rightarrow x_1 = \ldots$	**M1**	Uses $x_1 = 0.4 + 0.25\times$ their gradient from substituting $t = 2$, $x = 0.4$ into the differential equation.
	$x_1 = 0.41875$	**A1**	0.41875, or awrt 0.419.
	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)_1 = \frac{\text{"0.41875"}(0.7-\text{"0.41875"})\times4.25}{6.75+\text{"0.41875"}}$ $= \ldots$ $[\approx 0.06982]$ $x_2 =$ "0.41875" $+ 0.25 \times \ldots$	**dM1**	Evaluates the differential equation at $t = 2.25$ with their x_1 and applies the formula again with $h = 0.25$. Condone slips in evaluating.
	$x_2 \approx 0.436$	**A1**	0.436, or 43.6%.
		[5]	
10(b)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 0$ at $x = 0.7$, so $x \to 0.7$ (70%)	**B1**	0.7 or 70%. No justification needed.
		[1]	
	Total for Question 10	**6**	

Question	Scheme	Marks	Notes
11(a)	$\frac{\theta_{n+1}-2\theta_n+\theta_{n-1}}{0.1^2} + 0.2 \times \frac{\theta_{n+1}-\theta_{n-1}}{2\times 0.1}$ $+ 9\sin\theta_n = 0$	**M1**	Substitutes both approximations into the differential equation with $h = 0.1$, for general n or with $n = 1$. Condone sign and coefficient slips.
	$101\theta_{n+1} - 200\theta_n + 99\theta_{n-1} + 9\sin\theta_n = 0$	**A1**	Any correct form, need not be simplified; may already have $\theta_0 = 0.5$ and $\theta_1 = 0.48$ substituted.
	$101\theta_2 = 200 \times 0.48 - 99 \times 0.5 - 9\sin 0.48$ $\Rightarrow \theta_2 = \ldots$	**dM1**	Uses $\theta_0 = 0.5$ and $\theta_1 = 0.48$ in their equation with $n = 1$ and solves for θ_2. Condone arithmetic slips.
	$\theta_2 = 0.4192$	**A1**	awrt 0.4192
		[4]	
11(b)	$101\theta_3 = 200 \times$ "0.4192" $- 99 \times 0.48$ $- 9\sin$ "0.4192" $\Rightarrow \theta_3 = \ldots$	**M1**	Uses their equation from (a) linking three successive values, with $n = 2$, $\theta_1 = 0.48$ and their θ_2, rounded or unrounded, in both places.
	$\theta_3 = 0.323$	**A1**	awrt 0.323
		[2]	
	Total for Question 11	**6**	

Question	Scheme	Marks	Notes
12(a)	$\frac{y_{n+1}-2y_n+y_{n-1}}{0.01} + x_n\frac{y_{n+1}-y_{n-1}}{0.2}$ $= 3y_n - x_n^2$	**M1**	Substitutes both approximations into the differential equation at x_n, with $h = 0.1$ used at some stage. Condone sign slips; need not be simplified.
	$20(y_{n+1} - 2y_n + y_{n-1}) + x_n(y_{n+1} - y_{n-1})$ $= 0.6y_n - 0.2x_n^2$ $\Rightarrow (20 + x_n)\, y_{n+1} = \ldots$	**dM1**	Clears both denominators and collects the y_{n+1} terms on one side. Condone arithmetic slips.
	$(20 + x_n)\, y_{n+1} = 40.6\, y_n - (20 - x_n)\, y_{n-1} - 0.2x_n^2$	**A1 AG**	Correct substituted equation leading without error to the printed result; combined steps accepted.
		[3]	
12(b)	$\frac{y_1-y_{-1}}{0.2} = -1 \Rightarrow y_{-1} = y_1 + 0.2$	**B1**	oe, such as $y_1 = y_{-1} - 0.2$.
	$21y_1 = 81.2 - 19(\text{“}y_1 + 0.2\text{”}) - 0.2$ $\Rightarrow y_1 = \ldots$	**M1**	Substitutes $n = 0$ in the recurrence, replacing y_{-1} by their expression in y_1, and solves for y_1. Condone arithmetic slips.
	$y_1 = 1.93$	**A1**	
	$21.1y_2 = 40.6 \times \text{“}1.93\text{”} - 18.9 \times 2 - 0.2 \times 1.1^2$ $\Rightarrow y_2 = \ldots$	**dM1**	Substitutes $n = 1$ with $x_1 = 1.1$, $y_0 = 2$ and their numerical y_1, and solves for y_2. Condone arithmetic slips.
	$y_2 = \frac{40.316}{21.1} \approx 1.911$	**A1**	awrt 1.911.
		[5]	
	Total for Question 12	**8**	

Question	Scheme	Marks	Notes
13	$\frac{y_1-y_{-1}}{0.4} = 0.5$ $[\Rightarrow y_{-1} = y_1 - 0.2]$	**B1**	Any correct equation in y_1 and y_{-1} from the central difference with $n = 0$; need not be rearranged.
	$\frac{y_1-2(1)+y_{-1}}{0.04} + 0 - 1^2 = 0$	**M1 A1**	**M1** Substitutes the second difference into the differential equation with $n = 0$, $x = 0$ and $y_0 = 1$. Condone one slip; the zero term may be omitted. **A1** Correct equation, unsimplified or oe such as $y_1 + y_{-1} = 2.04$; y_{-1} may already be eliminated.
	$2y_1 - 0.2 = 2.04 \Rightarrow y_1 = \ldots$	**dM1**	Uses their relation of the form $y_1 - y_{-1} = k$ to eliminate y_{-1} from their equation and solves for y_1.
	$y_1 = 1.12$	**A1**	
	$\frac{y_2-2(\text{“}1.12\text{”})+1}{0.04} + 2(0.2)\left(\frac{y_2-1}{0.4}\right) - (\text{“}1.12\text{”})^2$ $= 3(0.2) \Rightarrow y_2 = \ldots$	**dM1**	Substitutes both approximations into the differential equation with $n = 1$, $x = 0.2$, $y_0 = 1$ and their value of y_1, and solves for y_2. Condone one slip.
	$26y_2 = 33.8544 \Rightarrow y_2 = 1.30$	**A1**	awrt 1.30
	Total for Question 13	**7**	