OPTIONAL PURE / DIFFERENTIAL EQUATIONS & NUMERICAL METHODS

2nd Order Differential Equations by Substitution

Mark scheme

12 questions · 180 marks

EXAM BOARDS Edexcel FP1 9FM0/3A · CIE 9231

Questions

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Question 3	(a) 4 (b) 2 (c) 6 (d) 1	13	6
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Question 8	(a) 5 (b) 7 (c) 6	18	14
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Question 11	(a) 5 (b) 11	16	20
Question 12	(a) 4 (b) 2 (c) 6 (d) 5	17	22
	Total	**180**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last accuracy mark of the working that reaches a printed result: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	$\frac{\mathrm{d}u}{\mathrm{d}x} = 2xy + x^2\frac{\mathrm{d}y}{\mathrm{d}x}$	**B1**	Correct first derivative, oe such as $y = ux^{-2} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = x^{-2}\frac{\mathrm{d}u}{\mathrm{d}x} - 2x^{-3}u$.
	$\frac{\mathrm{d}^2u}{\mathrm{d}x^2} = 2y + 2x\frac{\mathrm{d}y}{\mathrm{d}x} + 2x\frac{\mathrm{d}y}{\mathrm{d}x} + x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	**M1**	Differentiates their $\frac{\mathrm{d}u}{\mathrm{d}x}$, using the product rule on $x^2\frac{\mathrm{d}y}{\mathrm{d}x}$, or differentiates $y = ux^{-2}$ twice by the product or quotient rule. Condone coefficient slips.
	$= x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y$	**A1**	Correct second derivative, oe unsimplified.
	$\frac{\mathrm{d}^2u}{\mathrm{d}x^2} - 4\left(\frac{\mathrm{d}u}{\mathrm{d}x} - 2xy\right) + 4u - 8xy = 4x^2$	**M1**	Substitutes their expressions into the equation, aiming for an equation in u and x only; y terms that cancel may remain. Condone slips.
	$\frac{\mathrm{d}^2u}{\mathrm{d}x^2} - 4\frac{\mathrm{d}u}{\mathrm{d}x} + 4u = 4x^2$	**A1 AG**	Correct substituted equation leading without error to the printed equation; combined steps accepted.
		[5]	
1(b)	$\lambda^2 - 4\lambda + 4 = 0 \Rightarrow \lambda = \ldots$	**M1**	Forms the auxiliary equation of $(**)$ and solves it by any method.
	$\lambda = 2$ (repeated) $(u =)\ (A + Bx)e^{2x}$	**A1**	
	$u = ax^2 + bx + c$	**B1**	Needs all three terms.
	$2a - 4(2ax + b) + 4\left(ax^2 + bx + c\right) = 4x^2$ $\Rightarrow a = \ldots,\ b = \ldots,\ c = \ldots$	**M1**	Differentiates their quadratic twice, substitutes into $(**)$ and equates coefficients of at least two powers of x. Condone slips.
	$u = x^2 + 2x + \frac{3}{2}$	**A1**	Or $a = 1$, $b = 2$, $c = \frac{3}{2}$ stated.
	$u = (A + Bx)e^{2x} + x^2 + 2x + \frac{3}{2}$	**A1ft**	ft their complementary function, with two arbitrary constants, plus their nonzero particular integral.
	$y = \frac{(A+Bx)e^{2x}}{x^2} + 1 + \frac{2}{x} + \frac{3}{2x^2}$	**B1ft**	ft their general solution for u, with every term divided by x^2. Must start $y =$; any equivalent form.
		[7]	
	Total for Question 1	**12**	

Question	Scheme	Marks	Notes
2(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{t}\frac{\mathrm{d}y}{\mathrm{d}t} - \frac{y}{t^2}$	**B1**	Correct first derivative, oe such as $y = tx \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t} = t\frac{\mathrm{d}x}{\mathrm{d}t} + x$.
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \frac{1}{t}\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - \frac{2}{t^2}\frac{\mathrm{d}y}{\mathrm{d}t} + \frac{2y}{t^3}$	**M1 A1**	**M1** Differentiates their first derivative again, using the product rule on at least one product term. Condone sign and coefficient slips. **A1** Correct second derivative, oe such as $\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = t\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\frac{\mathrm{d}x}{\mathrm{d}t}$.
	$\left(\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - \frac{2}{t}\frac{\mathrm{d}y}{\mathrm{d}t} + \frac{2y}{t^2}\right)$ $+ (4t+2)\left(\frac{1}{t}\frac{\mathrm{d}y}{\mathrm{d}t} - \frac{y}{t^2}\right)$ $+ (13t+4)\frac{y}{t} = 40\cos t$ $\Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 4\frac{\mathrm{d}y}{\mathrm{d}t} + 13y = 40\cos t$	**M1** **A1 AG**	**M1** Substitutes their expressions for x, $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ into $(*)$, or for $\frac{\mathrm{d}y}{\mathrm{d}t}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}t^2}$ into the printed equation. **A1 AG** Correct substituted equation leading without error to the printed equation; collected steps accepted.
		[5]	
2(b)	$\lambda^2 + 4\lambda + 13 = 0 \Rightarrow \lambda = \ldots$	**M1**	Forms the auxiliary equation and solves it by any method.
	$\lambda = -2 \pm 3i$ $(y =)\ e^{-2t}(A\cos 3t + B\sin 3t)$	**A1**	Correct complementary function in t, oe such as $Re^{-2t}\cos(3t+\alpha)$.
	$y = a\cos t + b\sin t$	**B1**	
	$-a\cos t - b\sin t + 4(-a\sin t + b\cos t)$ $+ 13(a\cos t + b\sin t) = 40\cos t$ $\Rightarrow 12a + 4b = 40,\ -4a + 12b = 0$	**M1 A1**	**M1** Differentiates their trial function twice, substitutes into $(**)$ and equates coefficients of $\cos t$ and $\sin t$. Condone sign slips. **A1** Both equations correct, oe.
	$a = 3,\ b = 1 \Rightarrow (y =)\ 3\cos t + \sin t$	**A1**	
	$y = e^{-2t}(A\cos 3t + B\sin 3t) + 3\cos t + \sin t$	**A1ft**	ft their complementary function plus their particular integral, with two arbitrary constants and y in terms of t.
	$x = \frac{e^{-2t}(A\cos 3t + B\sin 3t) + 3\cos t + \sin t}{t}$	**B1ft**	ft their general solution for y divided by t, giving x in terms of t.
		[8]	
	Total for Question 2	**13**	

Question	Scheme	Marks	Notes
3(a)	$\frac{\mathrm{d}t}{\mathrm{d}x} = 3x^2$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{\mathrm{d}t}{\mathrm{d}x}$	**M1**	Finds $\frac{\mathrm{d}t}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}t}$ and uses the chain rule for $\frac{\mathrm{d}y}{\mathrm{d}x}$. Condone coefficient slips.
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3t^{\frac{2}{3}} \frac{\mathrm{d}y}{\mathrm{d}t}$	**A1**	oe, in terms of t only.
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 6x\frac{\mathrm{d}y}{\mathrm{d}t} + 3x^2\frac{\mathrm{d}^2y}{\mathrm{d}t^2} \times \frac{\mathrm{d}t}{\mathrm{d}x}$	**M1**	Differentiates their $\frac{\mathrm{d}y}{\mathrm{d}x}$ by the product rule, in x or t, multiplying $\frac{\mathrm{d}^2y}{\mathrm{d}t^2}$ by their $\frac{\mathrm{d}t}{\mathrm{d}x}$. Condone coefficient slips.
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 9t^{\frac{4}{3}}\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 6t^{\frac{1}{3}}\frac{\mathrm{d}y}{\mathrm{d}t}$	**A1**	oe, in terms of t only.
		[4]	
3(b)	$t^{\frac{1}{3}}\left(9t^{\frac{4}{3}}\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 6t^{\frac{1}{3}}\frac{\mathrm{d}y}{\mathrm{d}t}\right)$ $+ (4t-2)\,3t^{\frac{2}{3}}\frac{\mathrm{d}y}{\mathrm{d}t} + 4t^{\frac{5}{3}}y = 50t^{\frac{5}{3}}e^t$	**M1**	Substitutes their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ from part (a) and $x = t^{\frac{1}{3}}$ into every term of $(*)$.
	$9\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 12\frac{\mathrm{d}y}{\mathrm{d}t} + 4y = 50e^t$	**A1 AG**	Correct substituted equation leading without error to the printed result; dividing by $t^{\frac{5}{3}}$ must be seen or implied.
		[2]	
3(c)	$9\lambda^2 + 12\lambda + 4 = 0 \Rightarrow \lambda = \ldots$	**M1**	Forms the three-term auxiliary equation and solves it by any method.
	$\lambda = -\frac{2}{3}$ (repeated) $(y =)\ (A + Bt)e^{-\frac{2}{3}t}$	**A1**	Correct complementary function in t, oe.
	$y = ae^t$	**B1**	
	$9ae^t + 12ae^t + 4ae^t = 50e^t \Rightarrow a = \ldots$	**M1**	Differentiates their ae^t twice, substitutes into $(**)$ and solves for a. Condone sign or coefficient slips.
	$y = 2e^t$	**A1**	Or $a = 2$.
	$y = (A + Bt)e^{-\frac{2}{3}t} + 2e^t$	**A1ft**	ft their complementary function with two arbitrary constants plus their nonzero particular integral; must start $y =$ and be in terms of t.
		[6]	

Question 3 continued

Question	Scheme	Marks	Notes
3(d)	$y = \left(A + Bx^3\right) e^{-\frac{2}{3}x^3} + 2e^{x^3}$	**B1ft**	ft their general solution from part (c) with t replaced by x^3 throughout; must start $y =$.
		[1]	
	Total for Question 3	**13**	

Question	Scheme	Marks	Notes
4(a)	$\frac{\mathrm{d}v}{\mathrm{d}x} = 2y\frac{\mathrm{d}y}{\mathrm{d}x}$	**B1**	
	$\frac{\mathrm{d}^2v}{\mathrm{d}x^2} = 2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + 2y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	**M1 A1**	**M1** Differentiates their $\frac{\mathrm{d}v}{\mathrm{d}x}$ by the product rule, giving terms in $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ and $y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$, or differentiates $y = v^{\frac{1}{2}}$ twice. Condone coefficient slips.
	$\frac{1}{2}\frac{\mathrm{d}^2v}{\mathrm{d}x^2} + \frac{\mathrm{d}v}{\mathrm{d}x} + 5v = 5x^2 + 2x + 6$	**M1**	Substitutes their derivatives and v into the given equation, leaving it in v and its derivatives only. Condone slips.
	$\frac{\mathrm{d}^2v}{\mathrm{d}x^2} + 2\frac{\mathrm{d}v}{\mathrm{d}x} + 10v = 10x^2 + 4x + 12$	**A1 AG**	Correct substituted equation leading without error to the printed result; combined steps accepted.
		[5]	
4(b)	$\lambda^2 + 2\lambda + 10 = 0 \Rightarrow \lambda = \ldots$	**M1**	Forms the auxiliary equation from the equation in v and solves it by any method.
	$\lambda = -1 \pm 3i$ $(v =)\ e^{-x}(A\cos 3x + B\sin 3x)$	**A1**	Correct complementary function, oe such as $Re^{-x}\cos(3x + \alpha)$.
	$v = ax^2 + bx + c$	**B1**	
	$2a + 2(2ax + b) + 10(ax^2 + bx + c)$ $= 10x^2 + 4x + 12 \Rightarrow a = \ldots$	**M1**	Differentiates their quadratic, substitutes into the equation in v and equates coefficients to find at least one constant. Condone slips.
	$v = x^2 + 1$	**A1**	
	$v = e^{-x}(A\cos 3x + B\sin 3x) + x^2 + 1$	**A1ft**	ft their complex-root complementary function plus their nonzero particular integral, in terms of v and x.
	$x = 0$: $v = 1$, $\frac{\mathrm{d}v}{\mathrm{d}x} = 2 \times 1 \times \frac{3}{2} = 3$	**B1**	Both values, seen or used.
	$A + 1 = 1 \Rightarrow A = \ldots\ [= 0]$	**M1**	Substitutes $x = 0$ and their v into their general solution to find a constant.
	$\frac{\mathrm{d}v}{\mathrm{d}x} = e^{-x}\left((3B - A)\cos 3x\right.$ $\left. - (3A + B)\sin 3x\right) + 2x$ $x = 0$: $3B - A = 3 \Rightarrow B = \ldots$	**M1**	Differentiates their general solution, with the product rule on the exponential terms, and substitutes $x = 0$ and their $\frac{\mathrm{d}v}{\mathrm{d}x}$ to find the other constant. Condone slips.

Question 4 continued

Question	Scheme	Marks	Notes
	$y = \sqrt{x^2 + 1 + e^{-x}\sin 3x}$	**A1**	cso. Positive root only, in terms of x; $y^2 = \ldots$ or $y = \pm\sqrt{\ldots}$ scores A0.
		[10]	
	Total for Question 4	**15**	

Question	Scheme	Marks	Notes
5(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2t}\frac{\mathrm{d}y}{\mathrm{d}t}$	**B1**	Or $\frac{\mathrm{d}y}{\mathrm{d}t} = 2t\frac{\mathrm{d}y}{\mathrm{d}x}$ oe.
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \left(\frac{1}{2t}\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - \frac{1}{2t^2}\frac{\mathrm{d}y}{\mathrm{d}t}\right)\frac{1}{2t}$ $= \frac{1}{4t^2}\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - \frac{1}{4t^3}\frac{\mathrm{d}y}{\mathrm{d}t}$	**M1 A1**	**M1** Differentiates their first derivative with the product rule, including a factor $\frac{\mathrm{d}t}{\mathrm{d}x}$ on the $\frac{\mathrm{d}^2y}{\mathrm{d}t^2}$ term. Condone sign and coefficient slips. **A1** Correct second derivative in any equivalent form, in terms of t or x.
	$4t^2\left(\frac{1}{4t^2}\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - \frac{1}{4t^3}\frac{\mathrm{d}y}{\mathrm{d}t}\right) + 2(1-6t)\frac{1}{2t}\frac{\mathrm{d}y}{\mathrm{d}t} + 8y = 12e^t$	**M1**	Substitutes their two derivatives and $x = t^2$, $\sqrt{x} = t$ into $(*)$, with every term present.
	$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - 6\frac{\mathrm{d}y}{\mathrm{d}t} + 8y = 12e^t$	**A1 AG**	Correct substituted equation leading without error to the printed result; the cancelling $\frac{1}{t}\frac{\mathrm{d}y}{\mathrm{d}t}$ terms may be combined in one step.
		[5]	
5(b)	$\lambda^2 - 6\lambda + 8 = 0 \Rightarrow \lambda = \ldots$	**M1**	Forms the auxiliary equation and solves it by any method.
	$(y =)\ Ae^{2t} + Be^{4t}$	**A1**	
	$y = ae^t$	**B1**	
	$ae^t - 6ae^t + 8ae^t = 12e^t \Rightarrow a = \ldots$	**M1**	Differentiates their trial function twice, substitutes into $(**)$ and solves for the coefficient. Condone sign slips.
	$4e^t$	**A1**	
	$y = Ae^{2t} + Be^{4t} + 4e^t$	**A1ft**	ft their complementary function with two arbitrary constants plus their nonzero particular integral. Needs $y =$ in terms of t.
		[6]	
5(c)	$t = \ln 2$: $4A + 16B + 8 = 32$ $t = \ln 3$: $9A + 81B + 12 = 111 \Rightarrow A = \ldots,\ B = \ldots$	**M1**	Uses both conditions, with $t = \ln 2$ and $t = \ln 3$, in their general solution with two constants, and solves the resulting equations for both constants.
	$y = 2e^{2\sqrt{x}} + e^{4\sqrt{x}} + 4e^{\sqrt{x}}$	**A1**	cao. Must be in terms of x.
		[2]	
	Total for Question 5	**13**	

Question	Scheme	Marks	Notes
6(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}u}{\mathrm{d}x}\cos x - u\sin x$	**B1**	
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{\mathrm{d}^2u}{\mathrm{d}x^2}\cos x - 2\frac{\mathrm{d}u}{\mathrm{d}x}\sin x - u\cos x$	**M1 A1**	**M1** Applies the product rule to at least one term of their $\frac{\mathrm{d}y}{\mathrm{d}x}$. Condone sign slips.
	$\frac{\mathrm{d}u}{\mathrm{d}x}$: $-2\sin x + 2(1+\tan x)\cos x = 2\cos x$ u : $-\cos x - 2(1+\tan x)\sin x$ $+ 2\left(2+\tan x+\sec^2 x\right)\cos x$ $= 3\cos x + \frac{2\left(1-\sin^2 x\right)}{\cos x} = \ldots$	**M1**	Substitutes their derivatives into the equation and simplifies a coefficient using $\tan x = \frac{\sin x}{\cos x}$ with $\sec^2 x$ or $\sin^2 x + \cos^2 x = 1$. Condone sign slips.
	$\cos x\frac{\mathrm{d}^2u}{\mathrm{d}x^2} + 2\cos x\frac{\mathrm{d}u}{\mathrm{d}x} + 5u\cos x = 25x\cos x$ $\Rightarrow \frac{\mathrm{d}^2u}{\mathrm{d}x^2} + 2\frac{\mathrm{d}u}{\mathrm{d}x} + 5u = 25x$	**A1 AG**	Correct substituted equation, with all three coefficients simplified, leading without error to the printed equation.
		[5]	
6(b)	$\lambda^2 + 2\lambda + 5 = 0 \Rightarrow \lambda = \ldots$	**M1**	Forms the auxiliary equation and solves it by any method.
	$\lambda = -1 \pm 2i$ $(u =)\ e^{-x}\left(A\cos 2x + B\sin 2x\right)$	**A1**	Correct complementary function, oe such as $Re^{-x}\cos(2x+\alpha)$.
	$u = ax + b$	**B1**	
	$2a + 5(ax+b) = 25x \Rightarrow a = \ldots,\ b = \ldots$	**M1**	Substitutes their linear form into $(**)$ and equates coefficients of x and constants. Condone slips.
	$a = 5,\ b = -2$	**A1**	
	$u = e^{-x}\left(A\cos 2x + B\sin 2x\right) + 5x - 2$	**A1ft**	ft their complementary function, with two arbitrary constants, plus their particular integral.
	$y = \cos x\left[e^{-x}\left(A\cos 2x + B\sin 2x\right) + 5x - 2\right]$	**B1ft**	ft their u, of the form complementary function plus particular integral, multiplied by $\cos x$. Needs $y =$ in terms of x only.
		[7]	
	Total for Question 6	**12**	

Question	Scheme	Marks	Notes
7(a)	$\frac{\mathrm{d}t}{\mathrm{d}x} = e^x$ $\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{\mathrm{d}h}{\mathrm{d}t} \times \frac{\mathrm{d}t}{\mathrm{d}x}$	**M1**	Uses the chain rule with $\frac{\mathrm{d}t}{\mathrm{d}x} = e^x$, or $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{t}$ from $x = \ln t$.
	$\frac{\mathrm{d}h}{\mathrm{d}x} = e^x \frac{\mathrm{d}h}{\mathrm{d}t} = t\frac{\mathrm{d}h}{\mathrm{d}t}$	**A1**	Correct relation with e^x replaced by t.
	$\frac{\mathrm{d}^2h}{\mathrm{d}x^2} = \frac{\mathrm{d}t}{\mathrm{d}x}\frac{\mathrm{d}h}{\mathrm{d}t} + t\frac{\mathrm{d}^2h}{\mathrm{d}t^2}\frac{\mathrm{d}t}{\mathrm{d}x}$	**M1**	Differentiates their first-derivative relation again, in either variable, by the product rule with the chain rule on the derivative term. Condone sign slips.
	$\frac{\mathrm{d}^2h}{\mathrm{d}x^2} = t\frac{\mathrm{d}h}{\mathrm{d}t} + t^2\frac{\mathrm{d}^2h}{\mathrm{d}t^2}$ $\Rightarrow t^2\frac{\mathrm{d}^2h}{\mathrm{d}t^2} = \frac{\mathrm{d}^2h}{\mathrm{d}x^2} - \frac{\mathrm{d}h}{\mathrm{d}x}$	**A1 AG**	Correct differentiated relation leading without error to the printed result; combined steps accepted.
		[4]	
7(b)	$\left(\frac{\mathrm{d}^2h}{\mathrm{d}x^2} - \frac{\mathrm{d}h}{\mathrm{d}x}\right) - 4\frac{\mathrm{d}h}{\mathrm{d}x} + 6h = 6e^x + 12$	**M1**	Substitutes both relations from part (a) into $(*)$ and replaces t by e^x on the right-hand side.
	$\frac{\mathrm{d}^2h}{\mathrm{d}x^2} - 5\frac{\mathrm{d}h}{\mathrm{d}x} + 6h = 6e^x + 12$	**A1 AG**	Correct substituted equation leading without error to the printed equation.
		[2]	
7(c)	$\lambda^2 - 5\lambda + 6 = 0 \Rightarrow \lambda = \ldots$	**M1**	Forms the auxiliary equation and solves it by any method.
	$\lambda = 2,\ 3$ $(h =)\ Ae^{2x} + Be^{3x}$	**A1**	
	$(h =)\ ae^x + b$	**B1**	
	$ae^x - 5ae^x + 6\,(ae^x + b) = 6e^x + 12$ $\Rightarrow a = \ldots,\ b = \ldots$	**M1**	Differentiates their trial function twice, substitutes into $(**)$ and equates coefficients of e^x or constants. Condone sign slips.
	$a = 3,\ b = 2$ $h = Ae^{2x} + Be^{3x} + 3e^x + 2$	**A1**	Correct particular integral $3e^x + 2$.
	$e^x = t \Rightarrow e^{2x} = t^2,\ e^{3x} = t^3$	**M1**	Replaces e^x by t throughout a general solution of the form their complementary function plus their particular integral.
	$h = At^2 + Bt^3 + 3t + 2$	**A1 AG**	cso. Correct complementary function and particular integral leading to the printed result.
		[7]	

Question 7 continued

Question	Scheme	Marks	Notes
7(d)	$\frac{\mathrm{d}h}{\mathrm{d}t} = 2At + 3Bt^2 + 3$ $2A + 3B + 3 = 4, \quad 4A + 8B + 6 + 2 = \frac{28}{3}$	**M1**	Differentiates h and uses both conditions to form two linear equations in A and B. Condone one slip.
	$A = 1,\ B = -\frac{1}{3}$	**A1**	
	$2t - t^2 + 3 = 0 \Rightarrow t = \ldots$	**M1**	Sets their $\frac{\mathrm{d}h}{\mathrm{d}t}$ equal to zero and solves the three-term quadratic by any method.
	$t = 3$	**A1**	$t = 3$ only; $t = -1$ rejected or not given.
	$\frac{\mathrm{d}^2h}{\mathrm{d}t^2} = 2 - 2t = -4 < 0$ at $t = 3$	**M1**	Finds the sign of their $\frac{\mathrm{d}^2h}{\mathrm{d}t^2}$ at their t, or of $\frac{\mathrm{d}h}{\mathrm{d}t}$ either side of it.
	$h = 3^2 - \frac{1}{3} \times 3^3 + 3 \times 3 + 2 = 11$ (m), maximum	**A1**	cso. Height 11 with a correct second derivative or sign test and a conclusion that it is a maximum.
		[6]	
	Total for Question 7	**19**	

Question	Scheme	Marks	Notes
8(a)	$\frac{\mathrm{d}t}{\mathrm{d}x} = e^x = t$ $\Rightarrow \frac{\mathrm{d}N}{\mathrm{d}x} = t\frac{\mathrm{d}N}{\mathrm{d}t}$	**B1**	Or $\frac{\mathrm{d}N}{\mathrm{d}t} = e^{-x}\frac{\mathrm{d}N}{\mathrm{d}x}$, oe.
	$\frac{\mathrm{d}^2N}{\mathrm{d}x^2} = \frac{\mathrm{d}t}{\mathrm{d}x}\frac{\mathrm{d}N}{\mathrm{d}t} + t\frac{\mathrm{d}^2N}{\mathrm{d}t^2}\frac{\mathrm{d}t}{\mathrm{d}x}$	**M1**	Differentiates their first-derivative relation again, with respect to x or t, using the product rule and the chain rule. Condone sign slips.
	$t^2\frac{\mathrm{d}^2N}{\mathrm{d}t^2} = \frac{\mathrm{d}^2N}{\mathrm{d}x^2} - \frac{\mathrm{d}N}{\mathrm{d}x}$	**A1**	Correct relation, oe such as $\frac{\mathrm{d}^2N}{\mathrm{d}t^2} = e^{-2x}\left(\frac{\mathrm{d}^2N}{\mathrm{d}x^2} - \frac{\mathrm{d}N}{\mathrm{d}x}\right)$.
	$\left(\frac{\mathrm{d}^2N}{\mathrm{d}x^2} - \frac{\mathrm{d}N}{\mathrm{d}x}\right) + 2\frac{\mathrm{d}N}{\mathrm{d}x} - 2N$ $= 4e^{2x}$	**M1**	Substitutes their expressions for both derivative terms into $(*)$, with $4t^2$ written in terms of x.
	$\frac{\mathrm{d}^2N}{\mathrm{d}x^2} + \frac{\mathrm{d}N}{\mathrm{d}x} - 2N = 4e^{2x}$	**A1 AG**	Correct substituted equation leading without error to $(**)$.
		[5]	
8(b)	$\lambda^2 + \lambda - 2 = 0 \Rightarrow \lambda = \ldots$	**M1**	Forms the auxiliary equation of $(**)$ and solves it by any method.
	$\lambda = 1,\ -2$ $(N =)\ Ae^x + Be^{-2x}$	**A1**	
	$N = ae^{2x}$	**B1**	
	$4ae^{2x} + 2ae^{2x} - 2ae^{2x} = 4e^{2x} \Rightarrow a = \ldots$	**M1**	Differentiates their trial form twice, substitutes into $(**)$ and solves for its constant. Condone slips in the derivatives.
	$a = 1$	**A1**	
	$N = Ae^x + Be^{-2x} + e^{2x}$	**A1ft**	ft their complementary function, with two arbitrary constants, plus their nonzero particular integral.
	$N = At + \frac{B}{t^2} + t^2$	**B1ft**	ft their general solution in x, with $e^x = t$ used in every term.
		[7]	
8(c)	$\frac{\mathrm{d}N}{\mathrm{d}t} = A - \frac{2B}{t^3} + 2t$ $A + B + 1 = 37,\ A - 2B + 2 = -58$ $\Rightarrow A = \ldots,\ B = \ldots$	**M1**	Differentiates their general solution in t, uses both conditions at $t = 1$ and solves for both constants. Condone slips in the derivative.
	$A = 4,\ B = 32$	**A1**	

Question 8 continued

Question	Scheme	Marks	Notes
	$4 - \frac{64}{t^3} + 2t = 0 \Rightarrow t^4 + 2t^3 - 32 = 0$ $\Rightarrow t = \ldots$	**M1**	Sets their $\frac{\mathrm{d}N}{\mathrm{d}t}$ equal to zero and solves for t by any method, a calculator included.
	$t = 2$ [only root with $t > 0$]	**A1**	$t = 2$, with no other root accepted in $1 \leq t \leq 10$.
	$\frac{\mathrm{d}^2N}{\mathrm{d}t^2} = \frac{192}{t^4} + 2 = 14$ at $t = 2$	**M1**	Finds the sign of their second derivative at their t, or of their $\frac{\mathrm{d}N}{\mathrm{d}t}$ either side of it.
	$14 > 0$, minimum $N = 20$, so 20 000 bacteria	**A1**	cso. 20 000 bacteria, or $N = 20$ with thousands stated, and a minimum concluded from a correct second derivative value or correct signs.
		[6]	
Total for Question 8		**18**	

Question	Scheme	Marks	Notes
9(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = u + t\frac{\mathrm{d}u}{\mathrm{d}t}$	**B1**	Correct first derivative, oe such as $u = \frac{x}{t} \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}t} = \frac{1}{t}\frac{\mathrm{d}x}{\mathrm{d}t} - \frac{x}{t^2}$.
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 2\frac{\mathrm{d}u}{\mathrm{d}t} + t\frac{\mathrm{d}^2u}{\mathrm{d}t^2}$	**M1 A1**	**M1** Differentiates their $\frac{\mathrm{d}x}{\mathrm{d}t}$ again, using the product rule on the $t\frac{\mathrm{d}u}{\mathrm{d}t}$ term, or differentiates $u = \frac{x}{t}$ twice by the quotient or product rule. Condone one slip.
	$t^2\left(2\frac{\mathrm{d}u}{\mathrm{d}t} + t\frac{\mathrm{d}^2u}{\mathrm{d}t^2}\right)$ $- 2t(2t+1)\left(u + t\frac{\mathrm{d}u}{\mathrm{d}t}\right)$ $+ \left(3t^2 + 4t + 2\right)tu = -6t^3e^{2t}$	**M1**	Substitutes $x = tu$ and their derivatives into $(*)$, or their $\frac{\mathrm{d}u}{\mathrm{d}t}$ and $\frac{\mathrm{d}^2u}{\mathrm{d}t^2}$ into the printed equation, collecting like terms. Condone sign slips.
	$t^3\frac{\mathrm{d}^2u}{\mathrm{d}t^2} - 4t^3\frac{\mathrm{d}u}{\mathrm{d}t} + 3t^3u = -6t^3e^{2t}$ $\Rightarrow \frac{\mathrm{d}^2u}{\mathrm{d}t^2} - 4\frac{\mathrm{d}u}{\mathrm{d}t} + 3u = -6e^{2t}$	**A1 AG**	Correct substituted equation leading without error to the printed result; combined or collected steps accepted.
		[5]	
9(b)	$\lambda^2 - 4\lambda + 3 = 0 \Rightarrow \lambda = \ldots$	**M1**	Forms the auxiliary equation and solves it by any method.
	$\lambda = 1,\ 3$ $(u =)\ Ae^t + Be^{3t}$	**A1**	
	$(u =)\ ae^{2t}$	**B1**	
	$4ae^{2t} - 8ae^{2t} + 3ae^{2t} = -6e^{2t} \Rightarrow a = \ldots$	**M1**	Differentiates their trial function twice, substitutes into $(**)$ and solves for a. Condone sign slips.
	$a = 6$	**A1**	
	$u = Ae^t + Be^{3t} + 6e^{2t}$	**A1ft**	ft their complementary function, with two arbitrary constants, plus their particular integral.
	$x = t\left(Ae^t + Be^{3t} + 6e^{2t}\right)$	**B1ft**	ft t times their u, with two arbitrary constants. Needs x in terms of t.
		[7]	
9(c)	$t = \ln 2 \Rightarrow 2A + 8B + 24 = 0$ $t = \ln 4 \Rightarrow 4A + 64B + 96 = 0$ $\Rightarrow A = \ldots,\ B = \ldots$	**M1**	Substitutes both conditions into their general solution, using $e^{\ln k} = k$, and solves for both constants. Condone sign slips.
	$x = t\left(6e^{2t} - 8e^t - e^{3t}\right)$	**A1**	oe, with $A = -8$, $B = -1$.

Question 9 continued

Question	Scheme	Marks	Notes
	$x = -te^t\left(e^t - 2\right)\left(e^t - 4\right)$	**M1**	Considers the sign of their x, such as by factorising as a quadratic in e^t, using $t > 0$.
	$\ln 2 < t < \ln 4$	**A1**	Both ends with strict inequalities, or awrt $0.693 < t < 1.39$.
		[4]	
	Total for Question 9	**16**	

Question	Scheme	Marks	Notes
10(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = \cos t = \sqrt{1-x^2}$ $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t}$	**M1**	Uses the chain rule with $\frac{\mathrm{d}x}{\mathrm{d}t} = \cos t$, or $\frac{\mathrm{d}t}{\mathrm{d}x} = \frac{1}{\sqrt{1-x^2}}$, and $\cos t = \sqrt{1-x^2}$ oe.
	$\frac{\mathrm{d}y}{\mathrm{d}t} = \sqrt{1-x^2}\,\frac{\mathrm{d}y}{\mathrm{d}x}$	**A1**	Printed result from correct working. The sign of $\cos t$ need not be justified.
	$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = -\sin t\,\frac{\mathrm{d}y}{\mathrm{d}x} + \cos t\,\frac{\mathrm{d}^2y}{\mathrm{d}x^2} \times \frac{\mathrm{d}x}{\mathrm{d}t}$	**M1**	Differentiates their $\frac{\mathrm{d}y}{\mathrm{d}t}$ with respect to t, in t or x, by the product rule with the chain rule on $\frac{\mathrm{d}y}{\mathrm{d}x}$. Condone sign slips.
	$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = \cos^2 t\,\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - \sin t\,\frac{\mathrm{d}y}{\mathrm{d}x}$ $= (1-x^2)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - x\frac{\mathrm{d}y}{\mathrm{d}x}$	**A1 AG**	Correct second derivative, with $\sin t = x$ and $\cos^2 t = 1 - x^2$ used, leading without error to the printed result.
		[4]	
10(b)	$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 4y = 8\sin^2 t = 4(1-\cos 2t)$	**M1**	Uses part (a) for the left-hand side of $(*)$ and a double angle formula on $8\sin^2 t$. Condone a sign slip in the identity.
	$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 4y = 4 - 4\cos 2t$	**A1 AG**	Printed equation reached without error from correct substitution and identity.
		[2]	
10(c)	$\lambda^2 + 4 = 0 \Rightarrow \lambda = \ldots$	**M1**	Forms the auxiliary equation and solves it by any method.
	$\lambda = \pm 2i$ $(y =)\ A\cos 2t + B\sin 2t$	**A1**	Any correct equivalent form in t.
	$\frac{\mathrm{d}y}{\mathrm{d}t} = q\sin 2t + 2qt\cos 2t$ $\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = 4q\cos 2t - 4qt\sin 2t$ $4q\cos 2t - 4qt\sin 2t + 4(p + qt\sin 2t)$ $= 4 - 4\cos 2t$	**M1**	Differentiates $p + qt\sin 2t$ twice, using the product rule, and substitutes into $(**)$. Condone sign and coefficient slips.
	$p = 1, \quad q = -1$	**A1**	
	$y = A\cos 2t + B\sin 2t + 1 - t\sin 2t$	**A1ft**	ft their complementary function, with two arbitrary constants, plus $p + qt\sin 2t$ with their p and q. Must be in t.
		[5]	
10(d)	$x = 0 \Rightarrow t = 0,\ \frac{\mathrm{d}y}{\mathrm{d}t} = \sqrt{1-0^2} \times 4 = 4$	**B1**	

Question 10 continued

Question	Scheme	Marks	Notes
	$A+1=3$ $\frac{\mathrm{d}y}{\mathrm{d}t} = -2A\sin 2t + 2B\cos 2t - \sin 2t - 2t\cos 2t$ $t=0 \Rightarrow 2B =$ "4"	**M1**	Uses $y=3$ and their $\frac{\mathrm{d}y}{\mathrm{d}t}$ at $t=0$ in their general solution, with two constants, and its derivative, or equivalent conditions in x. Condone differentiation slips.
	$A=2, \quad B=2$	**A1**	
	$y = 2\cos 2t + 2\sin 2t + 1 - t\sin 2t$ with $t = \arcsin x$, $\cos 2t = 1-2x^2$, $\sin 2t = 2x\sqrt{1-x^2}$	**M1**	Replaces t by $\arcsin x$, and $\cos 2t$ and $\sin 2t$ by double angle forms in x, in their solution. Condone sign slips.
	$y = 3 - 4x^2 + 4x\sqrt{1-x^2} - 2x\sqrt{1-x^2}\arcsin x$	**A1**	oe, in x alone with no trigonometric function of $\arcsin x$ remaining, as the question requires.
		[5]	
	Total for Question 10	**16**	

Question	Scheme	Marks	Notes
11(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \cosh u \frac{\mathrm{d}u}{\mathrm{d}x}$	**B1**	
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \cosh u \frac{\mathrm{d}^2u}{\mathrm{d}x^2} + \sinh u \left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)^2$	**M1 A1**	**M1** Differentiates their $\frac{\mathrm{d}y}{\mathrm{d}x}$ by the product rule, with the chain rule on $\cosh u$, giving two terms. Condone sign slips.
	$\sqrt{1+\sinh^2 u} = \cosh u$ $\left(\cosh u \frac{\mathrm{d}^2u}{\mathrm{d}x^2} + \sinh u \left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)^2\right) + 3\cosh u \frac{\mathrm{d}u}{\mathrm{d}x} +$ $2\sinh u = 10\sin x$ $\Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 3\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 10\sin x$	**M1** **A1 AG**	**M1** Replaces $\sqrt{1+\sinh^2 u}$ by $\cosh u$ and substitutes their derivatives and y for $\sinh u$, in either direction. **A1 AG** Correct derivatives and substitution leading without error to the printed equation; combined steps accepted.
		[5]	
11(b)	$\lambda^2 + 3\lambda + 2 = 0 \Rightarrow \lambda = \ldots$	**M1**	Forms the auxiliary equation and solves it by any method.
	$\lambda = -1,\ -2$ $(y =)\ Ae^{-x} + Be^{-2x}$	**A1**	
	$(y =)\ a\sin x + b\cos x$	**B1**	Both terms needed; $a\sin x + b\cos x + c$ accepted.
	$(-a\sin x - b\cos x) + 3(a\cos x - b\sin x)$ $+ 2(a\sin x + b\cos x) = 10\sin x$ $\Rightarrow a - 3b = 10, \quad 3a + b = 0$	**M1 A1**	**M1** Differentiates their trial function twice, substitutes into the equation in y and equates coefficients of $\sin x$ and $\cos x$. Condone sign slips. **A1** Both equations correct, oe.
	$a = 1,\ b = -3$	**A1**	Correct particular integral $\sin x - 3\cos x$, or both values.
	$y = Ae^{-x} + Be^{-2x} + \sin x - 3\cos x$	**A1ft**	ft their complementary function plus their particular integral, as $y = \ldots$ with two arbitrary constants.
	$x = 0:\ y = 0,\ \frac{\mathrm{d}y}{\mathrm{d}x} = \cosh 0 \times 2 = 2$	**B1**	Both conditions on y, soi.
	$A + B - 3 = 0, \quad -A - 2B + 1 = 2$ $\Rightarrow A = \ldots,\ B = \ldots$	**M1**	Uses both conditions in their general solution with two constants and its derivative, and solves for both constants. Condone sign slips.
	$y = 7e^{-x} - 4e^{-2x} + \sin x - 3\cos x$	**A1**	

Question 11 continued

Question	Scheme	Marks	Notes
	$u = \operatorname{arsinh}\left(7e^{-x} - 4e^{-2x} + \sin x - 3\cos x\right)$	**B1ft**	ft their particular solution for y. Accept $\ln\left(y + \sqrt{y^2+1}\right)$ form. Must be $u = \ldots$ in terms of x.
		[11]	
	Total for Question 11	**16**	

Question	Scheme	Marks	Notes
12(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = \cosh t = \sqrt{1+x^2}$ $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t}$	**M1**	Differentiates $x = \sinh t$, or $t = \operatorname{arsinh} x$, and uses the chain rule, with $\cosh t$ written as $\sqrt{1+x^2}$.
	$\frac{\mathrm{d}y}{\mathrm{d}t} = \sqrt{1+x^2}\,\frac{\mathrm{d}y}{\mathrm{d}x}$	**A1**	Printed relation from correct working.
	$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = \sinh t\,\frac{\mathrm{d}y}{\mathrm{d}x} + \cosh t\,\frac{\mathrm{d}^2y}{\mathrm{d}x^2} \times \frac{\mathrm{d}x}{\mathrm{d}t}$	**M1**	Differentiates their first result again by the product rule, using the chain rule to convert between $\frac{\mathrm{d}}{\mathrm{d}t}$ and $\frac{\mathrm{d}}{\mathrm{d}x}$. Condone sign slips.
	$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = x\frac{\mathrm{d}y}{\mathrm{d}x} + \left(1+x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	**A1 AG**	Correct differentiated expression with $\sinh t = x$ and $\cosh^2 t = 1 + x^2$, leading without error to the printed result.
		[4]	
12(b)	$\left(1+x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + x\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}^2y}{\mathrm{d}t^2}$ $6x = 6\sinh t$	**M1**	Uses part (a) to replace the derivative terms of $(*)$ and writes $6x$ as $6\sinh t$.
	$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - 4y = 6\sinh t$	**A1 AG**	Printed equation from a correct substitution, with no errors.
		[2]	
12(c)	$\lambda^2 - 4 = 0 \Rightarrow \lambda = \ldots$	**M1**	Forms the auxiliary equation and solves it by any method.
	$\lambda = \pm 2$ $(y =)\ Ae^{2t} + Be^{-2t}$	**A1**	Correct complementary function, oe such as $A\cosh 2t + B\sinh 2t$.
	$y = a\sinh t + b\cosh t$	**B1**	oe, such as $pe^{t} + qe^{-t}$. Condone $a\sinh t$ alone.
	$a\sinh t + b\cosh t - 4\left(a\sinh t + b\cosh t\right) = 6\sinh t$ $\Rightarrow -3a = 6,\ -3b = 0$	**M1**	Substitutes their particular integral and its second derivative into $(**)$ and equates coefficients to find at least one constant. Condone sign slips.
	$y = -2\sinh t$	**A1**	oe, such as $e^{-t} - e^{t}$.
	$y = Ae^{2t} + Be^{-2t} - 2\sinh t$	**A1ft**	ft their complementary function and particular integral, with two arbitrary constants, as $y = \ldots$ in terms of t.
		[6]	

Question 12 continued

Question	Scheme	Marks	Notes
12(d)	$t = 0 \Rightarrow A + B = 3$ $\frac{\mathrm{d}y}{\mathrm{d}t} = 2Ae^{2t} - 2Be^{-2t} - 2\cosh t$ $2A - 2B - 2 = 0 \Rightarrow A = \ldots,\ B = \ldots$	**M1**	Applies both conditions to their general solution, with $t = 0$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = 0$ or equivalent, and solves for both constants. Condone slips in differentiating.
	$A = 2,\ B = 1$	**A1**	
	$e^t = x + \sqrt{1+x^2}, \quad e^{-t} = \sqrt{1+x^2} - x$	**M1**	Expresses e^t or e^{-t}, or $\cosh 2t$ and $\sinh 2t$, in terms of x using $\sinh t = x$ and $\cosh t = \sqrt{1+x^2}$. Condone sign slips.
	$y = 2\left(2x^2 + 1 + 2x\sqrt{1+x^2}\right)$ $\quad + \left(2x^2 + 1 - 2x\sqrt{1+x^2}\right) - 2x$	**dM1**	Substitutes their expressions for every exponential or hyperbolic term, with $\sinh t = x$, into their particular solution to give y in terms of x only.
	$y = 6x^2 - 2x + 3 + 2x\sqrt{1+x^2}$	**A1**	oe, with no hyperbolic or exponential functions.
		[5]	
	Total for Question 12	**17**	