OPTIONAL PURE / CONIC SECTIONS

Parabolas and Rectangular Hyperbolas

Questions by topic 11 questions · 121 marks

EXAM BOARDS Edexcel FP1 9FM0/3A

Questions

Question 1	*9 marks*	2
Question 2	*13 marks*	3
Question 3	*11 marks*	4
Question 4	*10 marks*	5
Question 5	*11 marks*	6
Question 6	*8 marks*	7
Question 7	*13 marks*	8
Question 8	*11 marks*	9
Question 9	*12 marks*	10
Question 10	*9 marks*	11
Question 11	*14 marks*	12

1. A rectangular hyperbola H has equation $xy = 16$.
The point $P\left(4t, \frac{4}{t}\right)$, $t \neq 0$, is a general point on H.

(a) Show that an equation of the tangent to H at P is $x + t^2y = 8t$. **[4]**

The distinct points Q and R lie on H. The tangent to H at Q and the tangent to H at R meet at the point $T(k, 3)$, where k is a constant. The point Q has coordinates $(8, 2)$.

(b) Find the value of k. **[2]**

(c) Find the coordinates of R. **[3]**

2. The parabola C has equation $y^2 = 8x$, and the point S is the focus of C.
The point P lies on C and has y-coordinate -8. The point T lies on the directrix of C, and the line segment PT is parallel to the x-axis.

(a) Write down the coordinates of S. **[1]**

(b) Find the area of triangle PST. **[2]**

(c) Use calculus to show that the normal to C at the point P has equation

$$y = 2x - 24$$ **[4]**

The rectangular hyperbola H has equation $xy = 16$. The normal to C at P meets H at the points L and M.

(d) Find the exact coordinates of the points L and M, giving your answers in their simplest form. **[6]**

3. The rectangular hyperbola H has equation $xy = 9$.

(a) Use calculus to show that an equation of the tangent to H at the point $P\left(3t, \frac{3}{t}\right)$, where $t \neq 0$, is

$$x + t^2 y = 6t$$ **[4]**

The tangent to H at P meets the x-axis at the point A. The point Q lies on H and has the same x-coordinate as A.

(b) Find the coordinates of Q in terms of t, and find an equation of the tangent to H at Q. **[3]**

The tangent at P and the tangent at Q meet at the point R.

(c) Show that, as t varies, the locus of R is a rectangular hyperbola, and state its equation. **[4]**

4. The parabola C has equation $y^2 = 12x$ and focus S.
The point $P(3t^2, 6t)$, where $t \neq 0$, lies on C.

(a) Show that an equation of the normal to C at P is

$$y + tx = 6t + 3t^3$$ **[4]**

The line ℓ passes through S and is perpendicular to the normal to C at P.
The line ℓ and the normal to C at P intersect at the point Q.

(b) Determine, in simplest form in terms of t, the coordinates of Q. **[3]**

(c) Hence show that, as t varies, the point Q lies on a parabola, and find the coordinates of the focus of this parabola. **[3]**

5. The parabola C has equation $y^2 = 24x$.
The point $P\left(6t^2, 12t\right)$ lies on C, where t is a positive constant.

(a) Use calculus to show that an equation of the normal to C at P is

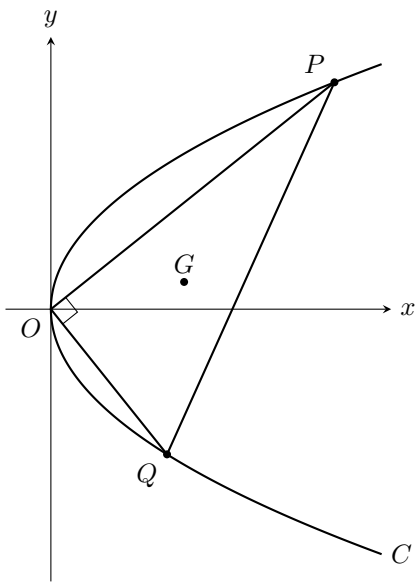
$$y + tx = 12t + 6t^3$$

[4]

(b) Write down the coordinates of the focus of C. **[1]**

The line ℓ is the reflection of the normal to C at P in the line through P parallel to the x-axis.

(c) Given that ℓ passes through the focus of C, determine the exact coordinates of P. **[6]**



6. The diagram above shows the parabola C with equation $y^2 = 4ax$, where a is a positive constant. The point $P(ap^2, 2ap)$, where $p > 0$, lies on C, and O is the origin.

(a) Write down the gradient of OP in terms of p. **[1]**

A second point $Q(aq^2, 2aq)$ on C is such that $\angle POQ = 90°$.

(b) Show that $pq = -4$. **[2]**

(c) The point G is the centroid of triangle OPQ, the point whose coordinates are the means of the coordinates of O, P and Q.
Prove that, as P varies, the locus of G has equation

$$9y^2 = 4a(3x - 8a)$$ **[5]**

7. The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant.
The point $P\left(ct, \frac{c}{t}\right)$, where $t > 0$, lies on H.

(a) Use calculus to show that an equation of the normal to H at P is

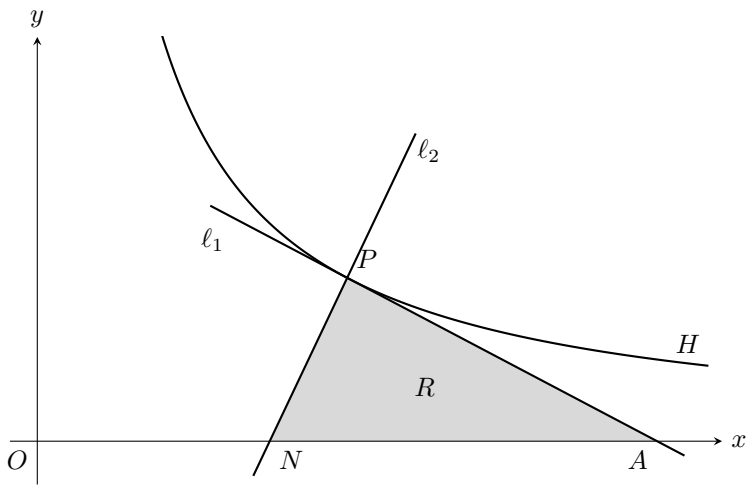
$$t^3x - ty = c\left(t^4 - 1\right)$$ **[4]**

The parabola C has equation $y^2 = 16x$.
The point $(4, 9)$ lies on H. The normal to H at $(4, 9)$ meets C at the points A and B, where the x-coordinate of A is less than the x-coordinate of B.

(b) Determine the exact coordinates of A and of B. **[4]**

The point R is the focus of C. The line through A and R meets C again at the point T.

(c) Determine the exact length of AT. **[5]**



8. The diagram above shows a sketch of part of the rectangular hyperbola H with equation

$$xy = c^2 \qquad x > 0$$

where c is a positive constant.
The point $P\left(ct, \frac{c}{t}\right)$, where $t > 0$, lies on H.
The line ℓ_1 is the tangent to H at P, and it crosses the x-axis at the point A. The line ℓ_2 is the normal to H at P, and it crosses the x-axis at the point N.
The region R, shown shaded in the diagram above, is the triangle PNA.
Given that the length PA is twice the length PN, and that the area of R is 40, find the exact coordinates of the point P. **[11]**

9. The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant.
The point $P\left(cp, \frac{c}{p}\right)$, where $p \neq 0$, lies on H.

(a) Use calculus to show that an equation of the normal to H at P is

$$p^3x - py = c\left(p^4 - 1\right) \quad \textbf{[4]}$$

The normal to H at P meets H again at the point Q.

(b) Show that

$$PQ^2 = c^2\left(p^2 + \frac{1}{p^2}\right)^3 \quad \textbf{[5]}$$

(c) Hence find, in terms of c, the least possible length of PQ, and the coordinates of each point P for which this least length occurs. **[3]**

10. The parabola C has equation $y^2 = 12x$.
The point $P(3p^2, 6p)$, where $p \neq 0$, lies on C. The normal to C at P meets C again at the point Q.
The tangent to C at P and the tangent to C at Q meet at the point R, whose y-coordinate is -3.
Show that the area of triangle PQR is $\frac{1125}{2}$. **[9]**

11. The point $P(3p^2, 6p)$, where $p \neq 0$, lies on the parabola with equation $y^2 = 12x$.

(a) Show that the point $Q\left(\frac{48}{p^2}, -\frac{24}{p}\right)$ also lies on the parabola. **[1]**

(b) Show that the chord PQ passes through the point $(12, 0)$. **[4]**

The normal to the parabola at P and the normal to the parabola at Q meet at the point N.

(c) Determine, in simplest form, the coordinates of N in terms of p. **[9]**