OPTIONAL PURE / CONIC SECTIONS

Parabolas and Rectangular Hyperbolas

Mark scheme 11 questions · 121 marks

EXAM BOARDS Edexcel FP1 9FM0/3A

Questions

Question	Marks by part	Marks	Page
Question 1	(a) 4 (b) 2 (c) 3	9	4
Question 2	(a) 1 (b) 2 (c) 4 (d) 6	13	5
Question 3	(a) 4 (b) 3 (c) 4	11	7
Question 4	(a) 4 (b) 3 (c) 3	10	8
Question 5	(a) 4 (b) 1 (c) 6	11	9
Question 6	(a) 1 (b) 2 (c) 5	8	10
Question 7	(a) 4 (b) 4 (c) 5	13	11
Question 8		11	12
Question 9	(a) 4 (b) 5 (c) 3	12	13
Question 10		9	14
Question 11	(a) 1 (b) 4 (c) 9	14	15
	Total	**121**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last accuracy mark of the working that reaches a printed result: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	$y = 16x^{-1} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -16x^{-2}$	**M1**	Differentiates to the form kx^{-2}, or uses implicit or parametric differentiation. Condone sign and coefficient slips.
	At P: $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{t^2}$	**A1**	Correct gradient in terms of t, oe unsimplified.
	$y - \frac{4}{t} = \text{“}-\frac{1}{t^2}\text{”}(x - 4t)$	**M1**	Uses a straight line through P with their gradient in terms of t. If $y = mx + c$ is used, needs c found.
	$t^2y - 4t = -x + 4t \Rightarrow x + t^2y = 8t$	**A1 AG**	Correct substituted line equation leading without error to the printed result; combined steps accepted.
		[4]	
1(b)	$t = 2$: $x + 4y = 16 \Rightarrow k + 12 = 16$	**M1**	Finds the tangent at Q, using $t = 2$ or otherwise, and substitutes $x = k$, $y = 3$. Condone one arithmetic slip.
	$k = 4$	**A1**	
		[2]	
1(c)	“4” $+ 3t^2 = 8t$	**M1**	Substitutes T, with their k, into the tangent at a general point of H, forming a quadratic in one variable.
	$3t^2 - 8t + 4 = 0 \Rightarrow t = \ldots$ Rejects $t = 2$, the parameter of Q: $t = \frac{2}{3}$	**dM1**	Solves their three-term quadratic by any method, including a calculator, and rejects the root that gives Q.
	$R\left(\frac{8}{3}, 6\right)$	**A1**	Both coordinates, oe, with x awrt 2.67 accepted.
		[3]	
Total for Question 1		**9**	

Question	Scheme	Marks	Notes
2(a)	$S = (2, 0)$	**B1**	
		[1]	
2(b)	$P(8, -8)$, $T(-2, -8)$ Area $= \frac{1}{2} \times 10 \times 8 = \ldots$	**M1**	Finds P from $y = -8$ and T on their directrix, then uses $\frac{1}{2} \times PT \times$ height from S, or any valid area method.
	(Area =) 40	**A1**	
		[2]	
2(c)	$2y\frac{\mathrm{d}y}{\mathrm{d}x} = 8$	**M1**	Differentiates implicitly to $ky\frac{\mathrm{d}y}{\mathrm{d}x} = c$, or $y = -\sqrt{8x}$ to the form $kx^{-\frac{1}{2}}$, or parametrically. Condone coefficient slips.
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{-8} = -\frac{1}{2}$	**A1**	$-\frac{1}{2}$ at $P(8, -8)$ from a correct derivative.
	$y + 8 =$ "2"$(x - 8)$	**M1**	Uses gradient $-1 \div$ their tangent gradient in a line through $(8, -8)$, in any form, including $y = mx + c$ with c found.
	$y + 8 = 2(x - 8) \Rightarrow y = 2x - 24$	**A1 AG**	Normal gradient 2 and a correct substituted line leading without error to $y = 2x - 24$.
		[4]	
2(d)	$x(2x - 24) = 16$	**M1**	Substitutes the line into $xy = 16$ to form a quadratic in x, or in y using $x = \frac{y+24}{2}$.
	$x^2 - 12x - 8 = 0$	**A1**	oe, or $y^2 + 24y - 32 = 0$ oe.
	$(x - 6)^2 = 44 \Rightarrow x = \ldots$	**M1**	Solves their three-term quadratic by the formula with their values, completing the square, or factorising with the product of the constants equal to the constant term.
	$x = 6 \pm 2\sqrt{11}$	**A1**	Both values, any exact form such as $6 \pm \sqrt{44}$, or $y = -12 \pm 4\sqrt{11}$ from a quadratic in y.
	$y = 2\left(6 \pm 2\sqrt{11}\right) - 24 = \ldots$	**M1**	Substitutes their exact roots into the line or $xy = 16$ for the other coordinates.

Question 2 continued

Question	Scheme	Marks	Notes
	$\left(6+2\sqrt{11},\ -12+4\sqrt{11}\right)$ $\left(6-2\sqrt{11},\ -12-4\sqrt{11}\right)$	**A1**	Both points, correctly paired, in simplest surd form. Either point may be L.
		[6]	
	Total for Question 2	**13**	

Question	Scheme	Marks	Notes
3(a)	$y = \frac{9}{x} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{9}{x^2}$	**M1**	Differentiates to the form kx^{-2}, or uses implicit or parametric differentiation. Condone a sign slip.
	$x = 3t \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{t^2}$	**A1**	Correct gradient at P, oe.
	$y - \frac{3}{t} = \text{“}-\frac{1}{t^2}\text{”}(x - 3t)$	**M1**	Forms a straight line through P with their gradient in terms of t, by either standard form.
	$t^2y - 3t = -x + 3t \Rightarrow x + t^2y = 6t$	**A1 AG**	Correct substituted line leading without error to the printed equation; combined steps accepted.
		[4]	
3(b)	$y = 0 \Rightarrow x = 6t$ $Q\left(6t, \frac{3}{2t}\right)$	**B1**	Both coordinates of Q, oe.
	$x + (2t)^2y = 6(2t)$	**M1**	Replaces t by $2t$ in the result of (a), or forms a line through their Q with gradient $-\frac{9}{x^2}$ at their x.
	$x + 4t^2y = 12t$	**A1**	oe.
		[3]	
3(c)	$3t^2y = 6t \Rightarrow y = \ldots$	**M1**	Solves their two tangent equations simultaneously, eliminating x or y to find one coordinate in terms of t.
	$y = \frac{2}{t}, \quad x = 4t$	**A1**	Both coordinates of R.
	$xy = 4t \times \frac{2}{t}$	**M1**	Eliminates t between their coordinates of R, by any method.
	$xy = 8$, a rectangular hyperbola	**A1**	$xy = 8$ oe, with a statement that the locus is a rectangular hyperbola.
		[4]	
	Total for Question 3	**11**	

Question	Scheme	Marks	Notes
4(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6}{6t} = \frac{1}{t}$	**M1 A1**	**M1** Divides $\frac{\mathrm{d}y}{\mathrm{d}t}$ by $\frac{\mathrm{d}x}{\mathrm{d}t}$, or differentiates $y^2 = 12x$ implicitly or explicitly. Condone coefficient slips.
	$y - 6t = -t\left(x - 3t^2\right)$	**M1**	Forms a line through P with gradient -1 divided by their tangent gradient in terms of t, in any form. Condone sign slips.
	$y + tx = 6t + 3t^3$	**A1 AG**	Correct substituted line equation leading without error to the printed result; collected steps accepted.
		[4]	
4(b)	$S(3, 0)$	**B1**	
	$y = \frac{1}{t}(x - 3) \Rightarrow x = 3 + ty$ $y + t(3 + ty) = 6t + 3t^3 \Rightarrow y = \ldots$	**M1**	Forms the line through their S perpendicular to the normal, with gradient the negative reciprocal of $-t$, and solves it with the normal for one coordinate. Condone sign slips.
	$Q\left(3 + 3t^2,\, 3t\right)$	**A1**	Both coordinates simplified, oe such as $x = 3\left(1 + t^2\right)$.
		[3]	
4(c)	$t = \frac{y}{3} \Rightarrow x = 3 + 3\left(\frac{y}{3}\right)^2$	**M1**	Substitutes t in terms of y, or t^2 in terms of x, from their Q of the form $\left(p + qt^2,\, rt\right)$ to eliminate t.
	$y^2 = 3(x - 3)$, a parabola	**A1**	Correct equation oe, with a statement that Q lies on a parabola, such as a translation of $y^2 = 3x$. A0 if the conclusion is missing.
	$\left(\frac{15}{4},\, 0\right)$	**B1ft**	ft their equation of the form $y^2 = k(x - h)$, giving $\left(h + \frac{k}{4},\, 0\right)$.
		[3]	
	Total for Question 4	**10**	

Question	Scheme	Marks	Notes
5(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{12}{12t} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \ldots$	**M1**	Obtains $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t, by $\frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}$ or by differentiating $y^2 = 24x$. Condone coefficient slips.
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{t}$	**A1**	Correct gradient at P, oe such as $\frac{12}{y}$ with $y = 12t$.
	$y - 12t = -t\left(x - 6t^2\right)$	**M1**	Forms a line through P with gradient the negative reciprocal of their tangent gradient, in any form.
	$y + tx = 12t + 6t^3$	**A1 AG**	Correct substituted equation leading without error to the printed equation; combined steps accepted.
		[4]	
5(b)	$(6, 0)$	**B1**	
		[1]	
5(c)	Replaces y by $24t - y$ in the normal: $(24t - y) + tx = 12t + 6t^3$	**M1**	Replaces y by $24t - y$ in the normal, or forms a line through P with gradient t, or equates the gradient of PS to t. Condone sign slips.
	$y = tx + 12t - 6t^3$	**A1**	Correct equation of ℓ, oe such as $y - 12t = t\left(x - 6t^2\right)$, or $\frac{2t}{t^2-1} = t$, which also earns the next M1 A1.
	$0 = 6t + 12t - 6t^3$	**M1**	Substitutes their focus into their equation of ℓ, which must involve t.
	$18t - 6t^3 = 0$	**A1**	Correct equation in t, oe.
	$6t\left(3 - t^2\right) = 0,\ t > 0 \Rightarrow t = \ldots\ [= \sqrt{3}]$	**dM1**	Solves their equation in t and selects a positive nonzero root.
	$P\left(18, 12\sqrt{3}\right)$	**A1**	cso. Exact coordinates only; A0 if a second point is also given as the answer.
		[6]	
	Total for Question 5	**11**	

Question	Scheme	Marks	Notes
6(a)	$\frac{2ap}{ap^2} = \frac{2}{p}$	**B1**	
		[1]	
6(b)	$\frac{2}{p} \times \frac{2}{q} = -1$	**M1**	Uses perpendicularity of OP and OQ, product of gradients -1 or scalar product zero. Condone slips in the gradients.
	$\frac{4}{pq} = -1 \Rightarrow pq = -4$	**A1 AG**	Correct equation leading without error to $pq = -4$; a scalar-product route must discard $pq = 0$.
		[2]	
6(c)	$x = \frac{ap^2+aq^2}{3}$ $y = \frac{2ap+2aq}{3}$	**M1 A1**	**M1** Forms either coordinate of G as the sum of the three vertices' coordinates divided by 3. **A1** Both coordinates correct, in any form.
	$\frac{3x}{a} = p^2 + \frac{16}{p^2}$ $\frac{3y}{2a} = p - \frac{4}{p}$	**M1**	Uses $pq = -4$ in their coordinates, substituting $q = -\frac{4}{p}$ or $p = -\frac{4}{q}$, or writing $p^2 + q^2 = (p+q)^2 + 8$. Condone sign slips.
	$\frac{9y^2}{4a^2} = p^2 - 8 + \frac{16}{p^2}$ $= \frac{3x}{a} - 8$	**dM1**	Squares their y-expression and uses their x-expression to remove the parameter, reaching an equation in x, y and a only. Condone slips.
	$9y^2 = 12ax - 32a^2$ $= 4a(3x - 8a)$	**A1 AG**	Correct elimination leading without error to the printed equation; combined steps accepted.
		[5]	
	Total for Question 6	**8**	

Question	Scheme	Marks	Notes
7(a)	$y = c^2x^{-1} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -c^2x^{-2}$	**M1**	Differentiates to the form kx^{-2}, or by implicit or parametric differentiation to an equivalent form in t.
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{c^2}{c^2t^2} = -\frac{1}{t^2}$	**A1**	
	$y - \frac{c}{t} = t^2(x - ct)$	**M1**	Uses the negative reciprocal of their tangent gradient in a straight line through P, in any form.
	$ty - c = t^3x - ct^4$ $\Rightarrow t^3x - ty = c\left(t^4 - 1\right)$	**A1 AG**	Correct line through P with gradient t^2, leading without error to the printed equation; combined steps accepted.
		[4]	
7(b)	$c^2 = 36 \Rightarrow c = 6,\ t = \frac{2}{3}$ $\frac{8}{27}x - \frac{2}{3}y = 6\left(\frac{16}{81} - 1\right)$	**M1**	Finds c and t from $(4, 9)$ and substitutes both into the normal from (a), or finds the normal at $(4, 9)$ directly.
	$4x - 9y = -65$	**A1**	oe, need not be simplified.
	$\frac{y^2}{4} - 9y + 65 = 0$ $\Rightarrow y^2 - 36y + 260 = 0 \Rightarrow y = \ldots$	**M1**	Eliminates x or y between their normal and $y^2 = 16x$ to form a three-term quadratic, and solves it by any means.
	$A\left(\frac{25}{4}, 10\right),\ B\left(\frac{169}{4}, 26\right)$	**A1**	Both points, correctly labelled; 6.25 and 42.25 accepted.
		[4]	
7(c)	$R(4, 0)$	**B1**	
	$x = \frac{9y}{40} + 4 \Rightarrow y^2 = \frac{18y}{5} + 64$ $\Rightarrow 5y^2 - 18y - 320 = 0 \Rightarrow y = \ldots$	**M1**	Forms line AR from their A and R, substitutes into $y^2 = 16x$ and solves the quadratic, or any valid method for the second intersection.
	$T\left(\frac{64}{25}, -\frac{32}{5}\right)$	**A1**	
	$AT = \left(\text{“}\frac{25}{4}\text{”} + 4\right) + \left(\text{“}\frac{64}{25}\text{”} + 4\right)$	**M1**	Adds focus-to-directrix distances $x + 4$ for their A and T, or uses the distance formula with their A and T.
	$AT = \frac{1681}{100}$	**A1**	oe, such as 16.81.
		[5]	
	Total for Question 7	**13**	

Question	Scheme	Marks	Notes
8	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-ct^{-2}}{c} = -\frac{1}{t^2}$	**M1 A1**	**M1** Differentiates parametrically, implicitly or $y = c^2x^{-1}$ to a gradient at P. Condone sign slips. **A1** $-\frac{1}{t^2}$ oe, such as $-\frac{c^2}{x^2}$ at P. A correct $x + t^2y = 2ct$ quoted earns M1 A1 and the next M1.
	$y - \frac{c}{t} = \text{“}-\frac{1}{t^2}\text{”}(x - ct)$ $y = 0 \Rightarrow x = \ldots$	**M1**	Forms a line through P with their tangent gradient in terms of t and sets $y = 0$.
	$A(2ct, 0)$	**A1**	
	$y - \frac{c}{t} = \text{“}t^2\text{”}(x - ct)$ $y = 0 \Rightarrow x = \ldots$	**M1**	Forms a line through P with gradient the negative reciprocal of their tangent gradient and sets $y = 0$. A correct normal quoted earns this mark.
	$N\left(ct - \frac{c}{t^3}, 0\right)$	**A1**	Or exact equivalent, such as $\frac{c\left(t^4-1\right)}{t^3}$.
	$PA^2 = c^2t^2 + \frac{c^2}{t^2}$, $PN^2 = \frac{c^2}{t^6} + \frac{c^2}{t^2}$ $PA^2 = 4PN^2 \Rightarrow t^4 = \ldots$	**M1**	Uses $PA = 2PN$, or $PA^2 = 4PN^2$, with lengths from their A and N, or any valid route, to an equation in t alone. Condone slips.
	$t^4 = 4 \Rightarrow t = \sqrt{2}$	**A1**	$t = \sqrt{2}$ only, any negative root rejected.
	$\frac{1}{2}\left(\text{“}ct + \frac{c}{t^3}\text{”}\right) \times \frac{c}{t} = 40$ $\Rightarrow \frac{5c^2}{8} = 40 \Rightarrow c = \ldots$	**M1**	Sets $\frac{1}{2}\times$ their $NA \times \frac{c}{t}$, or another valid area of the triangle, equal to 40 with their t and solves for c or c^2.
	$c = 8$	**A1**	$c = 8$ only.
	$P\left(8\sqrt{2}, 4\sqrt{2}\right)$	**A1**	cso. Exact equivalents accepted, such as $\frac{8}{\sqrt{2}}$ for $4\sqrt{2}$.
	Total for Question 8	**11**	

Question	Scheme	Marks	Notes
9(a)	$y = c^2x^{-1} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -c^2x^{-2}$	**M1**	Differentiates to the form kx^{-2}, or differentiates implicitly or parametrically. Condone sign slips.
	$x = cp \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{p^2}$	**A1**	
	$y - \frac{c}{p} = \text{“}p^2\text{”}(x - cp)$	**M1**	Forms a line through P with gradient the negative reciprocal of their tangent gradient, in any form.
	$py - c = p^3x - cp^4$ $\Rightarrow p^3x - py = c\left(p^4 - 1\right)$	**A1 AG**	Correct normal through P with gradient p^2, leading without error to the printed result.
		[4]	
9(b)	$p^3cq - \frac{pc}{q} = c\left(p^4 - 1\right)$ $\Rightarrow p^3q^2 + \left(1 - p^4\right)q - p = 0$	**M1**	Substitutes $\left(cq, \frac{c}{q}\right)$, or eliminates x or y using $xy = c^2$, forming a quadratic. Condone sign slips.
	$(q - p)\left(p^3q + 1\right) = 0 \Rightarrow q = \ldots$	**dM1**	Uses $q = p$ (or $x = cp$) as a root, by factorising, sum or product of roots, or otherwise, to find the other root.
	$q = -\frac{1}{p^3}$, $Q\left(-\frac{c}{p^3}, -cp^3\right)$	**A1**	Both coordinates of Q correct.
	$PQ^2 = \left(cp + \text{“}\frac{c}{p^3}\text{”}\right)^2$ $+ \left(\frac{c}{p} + \text{“}cp^3\text{”}\right)^2$	**M1**	Uses the distance formula with P and their Q in terms of p and c. Condone sign slips.
	$PQ^2 = \frac{c^2\left(1+p^4\right)^2}{p^6} + \frac{c^2\left(1+p^4\right)^2}{p^2}$ $= \frac{c^2\left(1+p^4\right)^3}{p^6}$ $= c^2\left(p^2 + \frac{1}{p^2}\right)^3$	**A1 AG**	Correct Q and substituted distance expression, leading without error to the printed result; combined steps accepted.
		[5]	
9(c)	$p^2 + \frac{1}{p^2} = \left(p - \frac{1}{p}\right)^2 + 2 \geq 2$ least when $p = \ldots$	**M1**	Finds the least value of $p^2 + \frac{1}{p^2}$ or PQ^2, or the p giving it, by any valid method, such as a derivative set to zero.
	$PQ^2 = 8c^2 \Rightarrow PQ = 2\sqrt{2}\,c$	**A1**	$2\sqrt{2}\,c$ oe, or awrt $2.83c$.
	$p = \pm 1$: $P(c, c)$ and $P(-c, -c)$	**A1**	Both points and no others.
		[3]	
	Total for Question 9	**12**	

Question	Scheme	Marks	Notes
10	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6}{y} = \frac{1}{p}$ $y - 6p = \frac{1}{p}\left(x - 3p^2\right) \Rightarrow py = x + 3p^2$	**M1**	Differentiates by any method for a gradient at P in terms of p, and uses it in a line through P. Need not be simplified.
	$y - 6p = -p\left(x - 3p^2\right)$ $6q - 6p = -p\left(3q^2 - 3p^2\right)$	**M1**	Forms the normal at P with gradient $-1\div$ their tangent gradient, and substitutes $(3q^2, 6q)$ or solves with $y^2 = 12x$. Condone sign slips.
	$\Rightarrow p + q = -\frac{2}{p}$	**A1**	oe, such as $q = -p - \frac{2}{p}$.
	$qy = x + 3q^2$ $(p - q)y = 3\left(p^2 - q^2\right) \Rightarrow y = \ldots$	**M1**	Writes the tangent at Q in the form of their tangent at P and eliminates x, or substitutes $y = -3$ into both. Condone slips.
	$y = 3(p + q)$	**A1**	Or $p + q = -1$ from $y = -3$ in both tangents.
	$3(p + q) = -3 \Rightarrow$ "$-\frac{2}{p}$" $= -1$ $\Rightarrow p = \ldots,\ q = \ldots$	**dM1**	Combines their $p + q = -1$, from the y-coordinate -3 of R, with their relation from the normal to find p and q.
	$p = 2,\ q = -3$ $P(12, 12),\ Q(27, -18),\ R(-18, -3)$	**A1**	All three points correct.
	Uses the right angle at P: $PQ = \sqrt{15^2 + 30^2},\ PR = \sqrt{30^2 + 15^2}$ Area $= \frac{1}{2} \times PQ \times PR$	**M1**	Uses $\frac{1}{2} \times PQ \times PR$ with lengths from their points, or a coordinate formula such as the shoelace formula with their three points. Condone arithmetic slips.
	Area $= \frac{1}{2} \times \sqrt{1125} \times \sqrt{1125} = \frac{1125}{2}$	**A1 AG**	Correct substituted formula leading without error to $\frac{1125}{2}$; by $\frac{1}{2} \times PQ \times PR$, needs the right angle at P stated or shown.
	Total for Question 10	**9**	

Question	Scheme	Marks	Notes
11(a)	$y^2 = \left(-\frac{24}{p}\right)^2 = \frac{576}{p^2}$ $12x = 12 \times \frac{48}{p^2} = \frac{576}{p^2}$, so Q lies on the parabola	**B1**	Both sides shown equal to $\frac{576}{p^2}$ oe, with a conclusion.
		[1]	
11(b)	$\frac{6p + \frac{24}{p}}{3p^2 - \frac{48}{p^2}} = \ldots$	**M1**	Uses difference in y over difference in x with the coordinates of P and Q. Condone sign slips.
	$= \frac{6p\left(p^2+4\right)}{3\left(p^2-4\right)\left(p^2+4\right)} = \frac{2p}{p^2-4}$	**A1**	$\frac{2p}{p^2-4}$ oe, or the factor p^2+4 cancelled at a later step of the chord working.
	$y - 6p = \text{“}\frac{2p}{p^2-4}\text{”}\left(x - 3p^2\right)$ $y = 0 \Rightarrow -6p\left(p^2-4\right) = 2p\left(x-3p^2\right) \Rightarrow x = \ldots$	**M1**	Forms the chord through P or Q with their gradient and sets $y = 0$ or substitutes $(12, 0)$, or compares their gradient with that from P to $(12, 0)$.
	$-3p^2 + 12 = x - 3p^2 \Rightarrow x = 12$, so PQ passes through $(12, 0)$	**A1 AG**	Correct chord equation leading without error to $x = 12$ or an identity, with a conclusion. The case $p^2 = 4$ need not be considered.
		[4]	
11(c)	$2y\frac{\mathrm{d}y}{\mathrm{d}x} = 12 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6}{y}$	**M1**	Differentiates implicitly, explicitly or parametrically to a gradient of the form $\frac{k}{y}$, $kx^{-\frac{1}{2}}$ or $\frac{k}{p}$. Condone coefficient slips.
	Normal gradients: $-p$ at P, $\frac{4}{p}$ at Q	**A1**	Both, soi.
	$y - 6p = -p\left(x - 3p^2\right)$	**M1 A1**	**M1** Forms the line through P with gradient the negative reciprocal of their tangent gradient at P. **A1** Correct, any form.
	$y + \frac{24}{p} = \frac{4}{p}\left(x - \frac{48}{p^2}\right)$	**M1 A1**	**M1** Forms the line through Q with gradient the negative reciprocal of their tangent gradient at Q. **A1** Correct, any form.
	$-px + 6p + 3p^3 = \frac{4x}{p} - \frac{192}{p^3} - \frac{24}{p}$ $\Rightarrow p^2\left(p^2+4\right)x = 3p^6 + 6p^4 + 24p^2 + 192$ $\Rightarrow x = \ldots$	**dM1**	Eliminates y between their two normals and collects the x terms to reach $x = \ldots$. Condone sign slips.
	$x = 3p^2 - 6 + \frac{48}{p^2}$	**A1**	Or $\frac{3\left(p^4-2p^2+16\right)}{p^2}$. Not with the factor p^2+4 uncancelled.

Question 11 continued

Question	Scheme	Marks	Notes
	$y = 12p - \frac{48}{p}$	**A1**	Or $\frac{12\left(p^2-4\right)}{p}$, from correct working.
		[9]	
	Total for Question 11	**14**	