OPTIONAL PURE / CONIC SECTIONS

Ellipses and Hyperbolas

Mark scheme 12 questions · 126 marks

EXAM BOARDS Edexcel FP1 9FM0/3A

Questions

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	Total	**126**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last accuracy mark of the working that reaches a printed result: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	$\frac{x^2}{5} + \frac{y^2}{9} = 1$ $5 = 9\left(1 - e^2\right) \Rightarrow e = \ldots$	**M1**	Uses $b^2 = a^2\left(1 - e^2\right)$ oe with $a^2 = 9$ and $b^2 = 5$, the larger denominator as a^2. Condone slips in rearranging.
	$e = \frac{2}{3}$	**A1**	Positive value only, oe or awrt 0.667.
	$(0, \pm ae) = \left(0, \pm 3 \times \text{“}\frac{2}{3}\text{”}\right)$	**M1**	Uses $(0, \pm ae)$ with $a = 3$ and their e, or $\sqrt{a^2 - b^2}$, foci on the y-axis. One focus is enough.
	$(0, 2)$ and $(0, -2)$	**A1**	Both foci.
		[4]	
1(b)	$y = \pm\frac{a}{e} = \pm\frac{3}{\text{“}\frac{2}{3}\text{”}}$	**M1**	Uses $y = \pm\frac{a}{e}$ with $a = 3$ and their e. One directrix is enough; not for lines $x = \ldots$.
	$y = \frac{9}{2}$ and $y = -\frac{9}{2}$	**A1**	Both equations, oe such as $y = \pm 4.5$.
		[2]	
Total for Question 1		**6**	

Question	Scheme	Marks	Notes
2(a)	$\left(\frac{x}{3}\right)^2 - \left(\frac{y}{2}\right)^2 = 1$	**M1**	Uses $\sec^2\theta - \tan^2\theta = 1$, or $1 + \tan^2\theta = \sec^2\theta$, with $\sec\theta$ and $\tan\theta$ in terms of x and y. Condone slips in squaring.
	$\frac{x^2}{9} - \frac{y^2}{4} = 1$	**A1**	oe, such as $4x^2 - 9y^2 = 36$.
	"4" = "9" $\left(e^2 - 1\right) \Rightarrow e = \ldots$ Foci $(\pm ae, 0)$	**M1**	Uses $b^2 = a^2(e^2 - 1)$, or $a^2 + b^2$ as the focal distance squared, with a, b from their equation of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, giving foci on the x-axis.
	$\left(\sqrt{13}, 0\right)$ and $\left(-\sqrt{13}, 0\right)$	**A1**	Both foci, oe such as $(\pm\sqrt{13}, 0)$, or awrt $(\pm 3.61, 0)$.
		[4]	
2(b)	$x = \pm\frac{3}{\text{"}\sqrt{13}/3\text{"}}$	**M1**	Uses $x = \pm\frac{a}{e}$, or $\frac{a^2}{ae}$, with their a and e from (a). Condone one sign only. Not for $x = \pm ae$.
	$x = \pm\frac{9\sqrt{13}}{13}$	**A1**	Both equations, exact, oe such as $x = \pm\frac{9}{\sqrt{13}}$. Not for a compound fraction left unsimplified.
		[2]	
	Total for Question 2	**6**	

Question	Scheme	Marks	Notes
3(a)	$\frac{b}{a} = \frac{15}{8}$	**B1**	Or equivalent, such as $b = \frac{15}{8}a$ or $\frac{b^2}{a^2} = \frac{225}{64}$.
	$e^2 = 1 + \frac{b^2}{a^2} = 1 + \left(\text{“}\frac{15}{8}\text{”}\right)^2$ $\Rightarrow e = \ldots$	**M1**	Uses $b^2 = a^2(e^2 - 1)$ oe with their ratio, or with values of a and b in it, to find e.
	$e = \frac{17}{8}$	**A1**	Or 2.125. A0 if $-\frac{17}{8}$ is also given as a value of e.
		[3]	
3(b)	$\frac{a}{e} = \frac{32}{17} \Rightarrow a = \frac{32}{17} \times \text{“}\frac{17}{8}\text{”}$ $= \ldots \ [= 4]$	**M1**	Uses directrix $x = \frac{a}{e}$ with their e to find a. Not for ae or $\frac{e}{a}$.
	$b = \frac{15}{8} \times \text{“4”} = \ldots \ \left[= \frac{15}{2}\right]$	**dM1**	Uses their a with $\frac{b}{a} = \frac{15}{8}$ or $b^2 = a^2(e^2 - 1)$ to find b or b^2.
	$\frac{x^2}{16} - \frac{4y^2}{225} = 1$ $\Rightarrow 225x^2 - 64y^2 = 3600$	**A1**	Or any integer multiple, such as $450x^2 - 128y^2 = 7200$. Not for $\frac{x^2}{16} - \frac{4y^2}{225} = 1$ alone.
	$(\pm ae, 0) = \left(\pm\frac{17}{2}, 0\right)$	**B1ft**	Both foci, ft their a and e as $(\pm ae, 0)$. Coordinates needed, not $x = \pm\frac{17}{2}$ alone.
		[4]	
	Total for Question 3	**7**	

Question	Scheme	Marks	Notes
4(a)	$16 = 9\left(e_1^2 - 1\right) \Rightarrow e_1^2 = \ldots$	**M1**	Uses $b^2 = a^2(e^2 - 1)$, or $e^2 = 1 + \frac{b^2}{a^2}$, with $a^2 = 9$ and $b^2 = 16$ for the hyperbola.
	$e_1 = \frac{5}{3}$	**A1**	Or $e_1^2 = \frac{25}{9}$.
	$e_2 = \text{“}\frac{3}{5}\text{”} \Rightarrow b^2 = a^2\left(1 - \frac{9}{25}\right)$	**M1**	Uses $e_2 = \frac{1}{e_1}$, or $e_2^2 = \frac{1}{e_1^2}$, with their e_1 in $b^2 = a^2(1 - e_2^2)$ or an equivalent ellipse relation.
	$b^2 = \frac{16a^2}{25} \Rightarrow 16a^2 = 25b^2$	**A1 AG**	Correct substituted relation leading without error to $16a^2 = 25b^2$; combined steps accepted.
		[4]	
4(b)	$y = 0 \Rightarrow x = \pm 3$ $ae_2 = 3 \Rightarrow \frac{3a}{5} = 3 \Rightarrow a = \ldots$	**M1**	Uses foci $(\pm 3, 0)$ in $ae_2 = 3$ with their e_2 from (a), or in $a^2 - b^2 = 9$ with (a), and solves for a or b.
	$\frac{x^2}{25} + \frac{y^2}{16} = 1$	**A1**	Or equivalent, such as $16x^2 + 25y^2 = 400$.
	$x = \pm\frac{a}{e_2} = \pm\frac{\text{“5”}}{\text{“3/5”}} = \ldots$	**M1**	Uses $x = \pm\frac{a}{e_2}$ with their a and e_2. One sign is enough.
	$x = \pm\frac{25}{3}$	**A1**	Both directrices, as equations.
		[4]	
Total for Question 4		**8**	

Question	Scheme	Marks	Notes
5(a)	$16 = 25\left(1 - e^2\right) \Rightarrow e^2 = \ldots$	**M1**	Uses $b^2 = a^2\left(1 - e^2\right)$ oe with $a = 5$ and $b = 4$.
	$e = \frac{3}{5}$	**A1**	$\frac{3}{5}$ or 0.6 only.
		[2]	
5(b)	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = -5\sin\theta, \quad \frac{\mathrm{d}y}{\mathrm{d}\theta} = 4\cos\theta$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{4\cos\theta}{5\sin\theta}$	**M1**	Finds $\frac{\mathrm{d}y}{\mathrm{d}x}$ at P by calculus, parametrically or implicitly. Condone sign slips.
	Normal gradient $\frac{5\sin\theta}{4\cos\theta}$	**A1**	oe, from a correct derivative.
	$y - 4\sin\theta = \text{“}\frac{5\sin\theta}{4\cos\theta}\text{”}\left(x - 5\cos\theta\right)$	**M1**	Forms a line through P with their normal gradient, the negative reciprocal of their tangent gradient.
	$4y\cos\theta - 16\sin\theta\cos\theta$ $\quad = 5x\sin\theta - 25\sin\theta\cos\theta$ $\Rightarrow 5x\sin\theta - 4y\cos\theta = 9\sin\theta\cos\theta$	**A1 AG**	Correct equation through P leading without error to the printed result; combined steps accepted.
		[4]	
5(c)	$y = 0 \Rightarrow x = \ldots, \quad x = 0 \Rightarrow y = \ldots$	**M1**	Substitutes $y = 0$ and $x = 0$ into the equation of ℓ to find both intercepts.
	$A\left(\frac{9\cos\theta}{5}, 0\right), \quad B\left(0, -\frac{9\sin\theta}{4}\right)$	**A1**	
	$M\left(\frac{9\cos\theta}{10}, -\frac{9\sin\theta}{8}\right)$	**A1ft**	ft their A and B, halving each coordinate; oe
		[3]	
5(d)	$\cos\theta = \frac{10x}{9}, \quad \sin\theta = -\frac{8y}{9}$ $\Rightarrow \left(\frac{10x}{9}\right)^2 + \left(\frac{8y}{9}\right)^2 = 1$	**M1**	Uses $\cos^2\theta + \sin^2\theta = 1$ with their coordinates of M to eliminate θ. Condone sign slips.
	$\left(\frac{9}{10}\right)^2 = \left(\frac{9}{8}\right)^2\left(1 - e^2\right) \Rightarrow e^2 = \ldots$	**M1**	Uses $b^2 = a^2\left(1 - e^2\right)$ oe with their semi-axes, the larger as a.
	$100x^2 + 64y^2 = 81$ $e = \frac{3}{5}$, the same as E	**A1 AG**	Correct equation oe and $e = \frac{3}{5}$ reached without error, with a statement that it equals the eccentricity of E.
		[3]	
	Total for Question 5	**12**	

Question	Scheme	Marks	Notes
6(a)	$b^2 = 36\left(\frac{25}{9} - 1\right)$	**M1**	Uses $b^2 = a^2(e^2 - 1)$ with $a = 6$ and $e = \frac{5}{3}$. Condone a slip in substitution.
	$b^2 = 64 \Rightarrow b = 8$	**A1 AG**	Correct substituted relation leading without error to $b = 8$.
	$S(10, 0)$, $S'(-10, 0)$	**B1**	Both foci, correctly labelled, or $(\pm 10, 0)$.
		[3]	
6(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{8\sec^2\theta}{6\sec\theta\tan\theta}$ $\left[= \frac{4}{3\sin\theta}\right]$	**M1**	Divides their $\frac{\mathrm{d}y}{\mathrm{d}\theta}$, of the form $k\sec^2\theta$, by their $\frac{\mathrm{d}x}{\mathrm{d}\theta}$, of the form $k\sec\theta\tan\theta$; or differentiates the Cartesian equation implicitly and substitutes P.
	Normal gradient $= -\frac{3\sin\theta}{4}$	**A1**	oe, such as $-\frac{6\sec\theta\tan\theta}{8\sec^2\theta}$.
	$y - 8\tan\theta = \text{“}-\frac{3\sin\theta}{4}\text{”}(x - 6\sec\theta)$	**M1**	Forms a line through P whose gradient is the negative reciprocal of their $\frac{\mathrm{d}y}{\mathrm{d}x}$, in terms of θ.
	$4y - 32\tan\theta = -3x\sin\theta + 18\tan\theta$ $\Rightarrow 3x\sin\theta + 4y = 50\tan\theta$	**A1 AG**	Correct line equation leading without error to the printed result; combined steps accepted.
		[4]	
6(c)	$Q(6\cos\theta, 0)$	**B1**	
		[1]	
6(d)	$R\left(\frac{50}{3\cos\theta}, 0\right)$	**B1**	oe, such as $\frac{50}{3}\sec\theta$ for the x-coordinate.
		[1]	
6(e)	$SQ = 10 - 6\cos\theta$, $QS' = 10 + 6\cos\theta$ $SR = \frac{50}{3\cos\theta} - 10$, $RS' = \frac{50}{3\cos\theta} + 10$	**M1 A1**	**M1** Finds all four distances as differences of x-coordinates, using S, S' and their Q and R in terms of θ. Condone sign slips. **A1** All four correct, oe.
	$\frac{SQ}{QS'} = \frac{10-6\cos\theta}{10+6\cos\theta}$ $\frac{SR}{RS'} = \frac{50-30\cos\theta}{50+30\cos\theta}$	**M1**	Clears the fractions in $\frac{SR}{RS'}$ to write both ratios in $\cos\theta$ alone, or cross-multiplies to compare $SQ \times RS'$ with $QS' \times SR$.

Question 6 continued

Question	Scheme	Marks	Notes
	$\frac{SQ}{QS'} = \frac{SR}{RS'} = \frac{5-3\cos\theta}{5+3\cos\theta}$	**A1 AG**	Both ratios reduced without error to the same expression, or both products expanded and shown equal, with a conclusion.
		[4]	
	Total for Question 6	**13**	

Question	Scheme	Marks	Notes
7(a)	$y = \pm\frac{2}{3}x$	**B1**	Both, oe such as $2x \pm 3y = 0$.
		[1]	
7(b)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 3\sinh t,\ \frac{\mathrm{d}y}{\mathrm{d}t} = 2\cosh t \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \ldots$	**M1**	Differentiates parametrically, or implicitly, and forms $\frac{\mathrm{d}y}{\mathrm{d}x}$ at P in terms of t. Condone sign and coefficient slips.
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\cosh t}{3\sinh t}$	**A1**	oe, such as $\frac{2}{3}\coth t$.
	$y - 2\sinh t = \text{“}\frac{2\cosh t}{3\sinh t}\text{”}(x - 3\cosh t)$	**M1**	Forms a straight line through P with their gradient in terms of t, in any form.
	$3y\sinh t - 6\sinh^2 t = 2x\cosh t - 6\cosh^2 t$ $\Rightarrow 2x\cosh t - 3y\sinh t = 6$	**A1 AG**	Correct tangent rearranged without error to the printed equation, using $\cosh^2 t - \sinh^2 t = 1$; combined steps accepted.
		[4]	
7(c)	$y = \frac{2}{3}x \Rightarrow 2x(\cosh t - \sinh t) = 6$ $\Rightarrow xe^{-t} = 3 \Rightarrow x = \ldots$	**M1**	Substitutes either asymptote into the tangent and solves for x. Condone sign slips.
	$Q\,(3e^t,\ 2e^t)$	**A1**	Both coordinates in exponentials; A0 for a form in $\cosh t$ and $\sinh t$.
	$R\,(3e^{-t},\ -2e^{-t})$	**A1**	Both coordinates in exponentials.
		[3]	
7(d)	$OQ = \sqrt{(3e^t)^2 + (2e^t)^2} = \sqrt{13}\,e^t$ $OR = \sqrt{(3e^{-t})^2 + (2e^{-t})^2} = \sqrt{13}\,e^{-t}$	**M1**	Uses Pythagoras for both OQ and OR from their Q and R in exponentials.
	$OQ \times OR = 13$, independent of t	**A1**	cso, with a statement that it is independent of t.
	$\sin\angle QOR = 2 \times \frac{2}{\sqrt{13}} \times \frac{3}{\sqrt{13}} = \frac{12}{13}$ Area $= \frac{1}{2} \times$ “13” $\times \frac{12}{13}$	**M1**	Uses $\frac{1}{2}\times$ their $OQ \times OR \times \sin\angle QOR$, with the sine found from the gradients $\pm\frac{2}{3}$ by any method. Condone slips in the sine.
	Area $= 6$	**A1**	
		[4]	
	Total for Question 7	**12**	

Question	Scheme	Marks	Notes
8(a)	$y = k(x-4)$ $x^2 + 4k^2(x-4)^2 = 8$ $\Rightarrow x^2 + 4k^2\left(x^2 - 8x + 16\right) - 8 = 0$	**M1**	Substitutes their line through $(4,0)$ into the ellipse and expands $(x-4)^2$ to three terms. Condone sign slips.
	$(4k^2+1)x^2 - 32k^2x + 64k^2 - 8 = 0$	**A1 AG**	Correct substituted equation leading without error to the printed quadratic.
		[2]	
8(b)	$(-32k^2)^2 - 4(4k^2+1)(64k^2-8) > 0$	**M1**	Uses $b^2 - 4ac > 0$ with the coefficients of the quadratic in (a). Condone $\geq$ and slips in expanding.
	$32 - 128k^2 > 0 \Rightarrow k^2 < \frac{1}{4}$ $\Rightarrow -\frac{1}{2} < k < \frac{1}{2}$	**A1 AG**	Correct discriminant inequality leading without error to the printed range, with strict inequalities.
		[2]	
8(c)	Halves the sum of the roots: $x = \frac{1}{2} \times \frac{32k^2}{4k^2+1}$	**M1**	Uses half the sum of the roots of the quadratic in (a), with their coefficients, or solves it and averages the roots.
	$x = \frac{16k^2}{4k^2+1}$	**A1**	
	$y = k\left(\frac{16k^2}{4k^2+1} - 4\right)$ $= -\frac{4k}{4k^2+1}$	**A1**	Single simplified fraction, oe.
		[3]	
8(d)	$\frac{256k^4}{(4k^2+1)^2} + \frac{16pk^2}{(4k^2+1)^2} = \frac{16qk^2}{4k^2+1}$	**M1**	Substitutes their x and y from (c) into $x^2 + py^2 = qx$, squaring both coordinates, or eliminates k between them. Condone slips.
	$\frac{256k^4+16pk^2}{(4k^2+1)^2} = \frac{64qk^4+16qk^2}{(4k^2+1)^2}$	**A1**	Both sides correct over a common denominator, or the equation multiplied through by $(4k^2+1)^2$, or a correct equation in x and y alone, oe.
	Equates coefficients of k^4 and k^2: $256 = 64q, \quad 16p = 16q$	**dM1**	Equates coefficients of k^4 and k^2 in their identity, or substitutes two values of k, to form equations in p and q.
	$q = 4$	**A1**	

Question 8 continued

Question	Scheme	Marks	Notes
	$p = 4$	**A1**	
		[5]	
	Total for Question 8	**12**	

Question	Scheme	Marks	Notes
9(a)(i)	$(0, 6e)$ and $(0, -6e)$	**B1**	Both, oe $(0, \pm 6e)$.
		[1]	
9(a)(ii)	$y = \frac{6}{e}$ and $y = -\frac{6}{e}$	**B1**	Both, oe $y = \pm\frac{6}{e}$.
		[1]	
9(b)(i)	$PS^2 = x^2 + (y - 6e)^2$	**B1**	oe, including expanded.
		[1]	
9(b)(ii)	$PM^2 = \left(\frac{6}{e} - y\right)^2$	**B1**	oe, such as $\left(y - \frac{6}{e}\right)^2$ or expanded.
		[1]	
9(c)	$x^2 + (y - 6e)^2 = e^2\left(\frac{6}{e} - y\right)^2$ $\Rightarrow x^2 + y^2 - 12ey + 36e^2$ $\quad = 36 - 12ey + e^2y^2$	**M1**	Uses $PS^2 = e^2PM^2$, or $PS = e\,PM$, with their expressions from (b) and expands both sides. Condone sign slips.
	$x^2 + \left(1 - e^2\right)y^2 = 36\left(1 - e^2\right)$ $\Rightarrow \frac{x^2}{36(1-e^2)} + \frac{y^2}{36} = 1$ $\Rightarrow b^2 = 36\left(1 - e^2\right)$	**A1 AG**	Correct substituted equation leading without error to the printed result, compared with the equation of E; collected steps accepted.
		[2]	
9(d)	$\frac{12}{e} = 18 \Rightarrow e = \ldots$ $b^2 = 36\left(1 - \left(\text{“}\frac{2}{3}\text{”}\right)^2\right) = \ldots$	**M1**	Uses the distance between their directrices from (a)(ii) equal to 18 to find e, and substitutes it into $b^2 = 36(1 - e^2)$.
	$e = \frac{2}{3}$, $b = 2\sqrt{5}$	**A1**	Both, with b exact, such as $\sqrt{20}$.
		[2]	
9(e)	$PM = \frac{PS}{e} = \frac{3}{\text{“}\frac{2}{3}\text{”}}$ $\left[= \frac{9}{2}\right]$ $9 - y = \frac{9}{2} \Rightarrow y = \ldots$	**M1**	Uses $PS = e\,PM$ with their e and their directrix $y = \frac{6}{e}$ to reach $y = \ldots$, or solves $PS^2 = 9$ with the equation of E.
	$y = \frac{9}{2}$	**A1**	
	$\frac{x^2}{20} + \frac{\left(\frac{9}{2}\right)^2}{36} = 1 \Rightarrow x = \ldots$ Area $= \frac{1}{2} \times \frac{9}{2} \times \text{“}\frac{\sqrt{35}}{2}\text{”}$	**M1**	Substitutes their y into $\frac{x^2}{b^2} + \frac{y^2}{36} = 1$ with their b^2 to find x, then uses $\frac{1}{2} \times PM \times x$ or another complete area method.

Question 9 continued

Question	Scheme	Marks	Notes
	$\frac{9\sqrt{35}}{8}$	**A1**	Any exact equivalent.
		[4]	
	Total for Question 9	**12**	

Question	Scheme	Marks	Notes
10(a)	$9x^2 - 16(mx + c)^2 = 144$	**M1**	Substitutes for y (or x) from the line into H and expands to a three-term quadratic. Condone sign slips.
	$\left(9 - 16m^2\right)x^2 - 32mcx - \left(16c^2 + 144\right) = 0$	**A1**	Correct quadratic, oe with terms on one side.
	$(-32mc)^2 - 4\left(9 - 16m^2\right)\left(-16c^2 - 144\right) = 0$	**M1**	Uses discriminant $= 0$ with their three coefficients. Condone sign slips.
	$\Rightarrow 9c^2 = 144m^2 - 81$ $\Rightarrow c^2 = 16m^2 - 9$	**A1 AG**	Correct substituted discriminant equation simplified without error to the printed result; collected steps accepted.
		[4]	
10(b)	$1 = 4m + c \Rightarrow c = 1 - 4m$ $(1 - 4m)^2 = 16m^2 - 9 \Rightarrow m = \ldots$	**M1**	Uses P on the line to eliminate c (or m) in the result of (a) and solves. Condone sign slips.
	$m = \frac{5}{4}$	**A1**	
	$c = -4, \quad y = \frac{5}{4}x - 4$	**A1**	Or any equivalent equation, such as $4y = 5x - 16$.
	Tests the vertical line through P in H $144 - 16y^2 = 144 \Rightarrow y = \ldots$	**M1**	Substitutes $x = 4$ into the equation of H and solves for y, or identifies $(4, 0)$ as a vertex of H.
	$16y^2 = 0$, repeated root $y = 0$ so $x = 4$ is a tangent	**A1**	$x = 4$ meets H only at $(4, 0)$, by a repeated root or as the vertical tangent at the vertex, concluding that it is a tangent.
	$x = 4$ has no gradient, so is not of the form $y = mx + c$	**E1**	States that $x = 4$ is vertical or has no gradient, so the result of (a) cannot apply to it.
		[6]	
10(c)	$9x^2 - 16\left(\text{“}\frac{5}{4}x - 4\text{”}\right)^2 = 144$ $\Rightarrow x^2 - 10x + 25 = 0 \Rightarrow x = \ldots$	**M1**	Substitutes their non-vertical tangent $y = mx + c$ from (b) into H, or uses the quadratic of (a), and solves for x. Condone sign slips.
	$\left(5, \frac{9}{4}\right)$	**A1**	
	$(4, 0)$	**B1**	
		[3]	
	Total for Question 10	**13**	

Question	Scheme	Marks	Notes
11(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4\cosh\theta}{3\sinh\theta}$	**M1 A1**	**M1** Uses their $\frac{\mathrm{d}y}{\mathrm{d}\theta} \div \frac{\mathrm{d}x}{\mathrm{d}\theta}$, or differentiates implicitly and substitutes P. Condone sign slips. **A1** Correct gradient, oe such as $\frac{4}{3}\coth\theta$.
	$y - 4\sinh\theta = \text{“}\frac{4\cosh\theta}{3\sinh\theta}\text{”}(x - 3\cosh\theta)$	**M1**	Forms a straight line through P with their gradient, in any form.
	$4x\cosh\theta - 3y\sinh\theta = 12$	**A1**	Correct equation, oe, with $\cosh^2\theta - \sinh^2\theta$ replaced by 1.
	$y = 0 \Rightarrow x = \ldots, \quad x = 0 \Rightarrow y = \ldots$	**M1**	Substitutes $y = 0$ and $x = 0$ into their tangent to find both intercepts.
	$A\left(\frac{3}{\cosh\theta}, 0\right), \quad C\left(0, -\frac{4}{\sinh\theta}\right)$	**A1**	Both points correct, oe.
	$M\left(\frac{3}{2\cosh\theta}, -\frac{2}{\sinh\theta}\right)$	**A1ft**	ft their A and C, halving each coordinate.
	$\cosh\theta = \frac{3}{2x}, \quad \sinh\theta = -\frac{2}{y}$ $\Rightarrow \frac{9}{4x^2} - \frac{4}{y^2} = 1$	**M1**	Uses $\cosh^2\theta - \sinh^2\theta = 1$ with their coordinates of M to form an equation in x and y only. Condone sign slips.
	$y^2 = \frac{16x^2}{9-4x^2}$	**A1 AG**	Correct equation in x and y leading without error to the printed result; combined steps accepted.
	$0 < x < \frac{3}{2}$, so $p = 0$, $q = \frac{3}{2}$	**B1**	Both values, or $0 < x < \frac{3}{2}$ stated.
		[10]	
11(b)	$0 < x < \frac{3}{2} \Rightarrow 0 < 9 - 4x^2 < 9$	**M1**	Uses $0 < x < \frac{3}{2}$ to compare $9 - 4x^2$ with 9, or makes any valid comparison of y^2 with $\frac{16x^2}{9}$.
	$y^2 = \frac{16x^2}{9-4x^2} > \frac{16x^2}{9}$ $\Rightarrow \lvert y\rvert > \frac{4}{3}x$	**A1 AG**	Needs $9 - 4x^2 > 0$ and $x > 0$ used, and a correct inequality for y^2 leading without error to the printed result.
	Asymptotes $y = \pm\frac{4}{3}x$ Locus lies between the asymptotes and the y-axis, on the other side of each asymptote from H	**E1**	States the asymptotes $y = \pm\frac{4}{3}x$ and that the locus lies beyond them, away from H, never meeting them. Accept equivalent wording; not for the asymptotes alone.
		[3]	
	Total for Question 11	**13**	

Question	Scheme	Marks	Notes
12(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{b\cos\theta}{-a\sin\theta}$ $= -\frac{b\cos\theta}{a\sin\theta}$	**M1 A1**	**M1** Divides their $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ by their $\frac{\mathrm{d}x}{\mathrm{d}\theta}$, or differentiates the Cartesian equation implicitly. Condone sign slips. **A1** Correct gradient, oe.
	$y - b\sin\theta = \text{“}-\frac{b\cos\theta}{a\sin\theta}\text{”}\,(x - a\cos\theta)$	**M1**	Uses a straight line through P with their gradient, in any form.
	$ay\sin\theta - ab\sin^2\theta = -bx\cos\theta + ab\cos^2\theta$ $\Rightarrow bx\cos\theta + ay\sin\theta = ab$	**A1 AG**	Correct line through P leading without error to the printed equation, using $\sin^2\theta + \cos^2\theta = 1$.
		[4]	
12(b)	$y = 0 \Rightarrow x = \ldots,\ x = 0 \Rightarrow y = \ldots$	**M1**	Sets $y = 0$ and $x = 0$ in the tangent equation to find both intercepts.
	$A\,(a\sec\theta,\,0),\ B\,(0,\,b\,\mathrm{cosec}\,\theta)$	**A1**	Both, oe such as $\frac{a}{\cos\theta}$ and $\frac{b}{\sin\theta}$.
	$AB^2 = a^2\sec^2\theta + b^2\,\mathrm{cosec}^2\,\theta$ $= a^2 + b^2 + a^2\tan^2\theta + b^2\cot^2\theta$	**M1 A1**	**M1** Uses Pythagoras with their intercepts and rewrites $\sec^2\theta$ and $\mathrm{cosec}^2\,\theta$ in $\tan\theta$ and $\cot\theta$. Condone sign slips. **A1** Correct AB^2, oe.
	$AB^2 = (a+b)^2 + (a\tan\theta - b\cot\theta)^2$	**dM1**	Completes the square, using $\tan\theta\cot\theta = 1$, to reach $(a+b)^2$ plus a square, or sets their $\frac{\mathrm{d}(AB^2)}{\mathrm{d}\theta} = 0$ to reach $\tan^2\theta = \frac{b}{a}$. Condone sign slips.
	$(a\tan\theta - b\cot\theta)^2 \geq 0 \Rightarrow AB \geq a + b$	**A1**	Minimum $a + b$ reached without error from a correct completed square (or derivative), stating that the square is never negative (or that it is a minimum).
	$a\tan\theta = b\cot\theta \Rightarrow \tan^2\theta = \frac{b}{a}$ $\Rightarrow \cos\theta = \ldots,\ \sin\theta = \ldots$	**M1**	Sets their square to zero for $\tan^2\theta$ and uses it to find $\cos\theta$ and $\sin\theta$, or their squares.
	$P\left(\frac{a\sqrt{a}}{\sqrt{a+b}},\ \frac{b\sqrt{b}}{\sqrt{a+b}}\right)$	**A1**	oe, such as $\left(a\sqrt{\frac{a}{a+b}},\ b\sqrt{\frac{b}{a+b}}\right)$. Positive coordinates only.
		[8]	
	Total for Question 12	**12**	