Questions

1. A delivery van of mass 2000 kg is travelling up a straight road inclined at angle α to the horizontal, where $\sin\alpha = 0.05$.

The van's engine can produce power at a maximum rate of 64 kW.

The total resistance force due to friction and air resistance is modelled as $(200 + kv)$ N, where $v\,\mathrm{m\,s^{-1}}$ is the speed of the van and k is a constant.

The greatest possible constant speed of the van up the hill is $20\,\mathrm{m\,s^{-1}}$.

Find the maximum possible acceleration of the van when it is travelling uphill at $16\,\mathrm{m\,s^{-1}}$.

Fully justify your answer. **[7]**

2. A delivery van has mass 1300 kg and travels on a straight horizontal road.

In a simple model, the van experiences a constant rolling resistance of 600 newtons and an additional air-resistance force whose magnitude is proportional to the speed of the van.

During a test, the van moves at a steady speed of $15\,\mathrm{m\,s^{-1}}$ when the engine provides a driving force of 1200 newtons.

In this model, the engine can deliver a constant maximum power. The greatest speed the van can attain on this road is $30\,\mathrm{m\,s^{-1}}$.

(a) Show that the maximum power of the engine is 54 000 W **[2]**

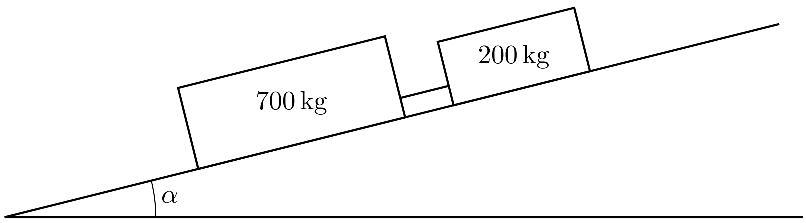
(b) Find the maximum acceleration of the van when it is travelling at $20\,\mathrm{m\,s^{-1}}$ **[3]**

(c) Find the maximum acceleration of the van when it is travelling at $4\,\mathrm{m\,s^{-1}}$ **[1]**

(d) Comment on the validity of the model in the context of your answers to parts (b) and (c). **[2]**

3. A maintenance vehicle of mass 1000 kg moves on a straight slope inclined at an angle θ to the horizontal, where $\sin\theta = 0.08$. When the motor is switched off, the vehicle coasts down the slope and reaches a limiting speed of $14\,\mathrm{m\,s^{-1}}$. The resistance to motion is proportional to the square of the speed. The motor is then switched on and works at a constant power P W, allowing the vehicle to travel up the same slope with a limiting speed of $10\,\mathrm{m\,s^{-1}}$.

Find the value of P and determine the resistance to motion when the vehicle is travelling at $12\,\mathrm{m\,s^{-1}}$ **[10]**



4. A car of mass 700 kg is descending a straight road which is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{10}$. The car is connected to a trailer of mass 200 kg by a light rigid towbar which is parallel to the road, as shown in the diagram.

The resistance to the motion of the car is modelled as a constant force of magnitude 240 N.
The resistance to the motion of the trailer is modelled as a constant force of magnitude 90 N.

The brakes of the car dissipate energy at a constant rate of 9 kW.

Find the force in the towbar at the instant when the speed of the car is $12\,\mathrm{m\,s^{-1}}$, stating whether the towbar is in tension or compression. **[8]**

5. An airport tug of mass 1800 kg has an engine that can deliver a maximum power of 72 kW.

In an initial model of the motion of the tug, the total resistance to motion is assumed to be constant.

(i) Given that the greatest steady speed of the tug on a straight horizontal runway is $18\,\mathrm{m\,s^{-1}}$, find the magnitude of the resistance force. **[2]**

(ii) The tug is connected to a small aircraft of mass 2200 kg by a light rigid horizontal tow bar. The greatest steady speed of the tug and aircraft on the runway is now $12\,\mathrm{m\,s^{-1}}$. The resistance to motion of the aircraft may also be assumed constant.

Find the magnitude of the resistance force on the aircraft. **[2]**

(iii) The tug and aircraft again travel along the runway. At one instant their speed is $10\,\mathrm{m\,s^{-1}}$ and their acceleration is $0.25\,\mathrm{m\,s^{-2}}$.

(a) Find the power of the engine of the tug at this instant. **[4]**
(b) Find the magnitude of the force in the tow bar at this instant. **[2]**

(iv) In a refined model of the motion of the tug and aircraft, the resistance to motion of each is assumed to be zero until they reach a speed of $6\,\mathrm{m\,s^{-1}}$. When the speed is $6\,\mathrm{m\,s^{-1}}$ or above, the same constant resistance forces as in the first model are assumed to apply to each.

The tug and aircraft start from rest on the runway and accelerate, using maximum power.

Without carrying out any further calculations,
(a) explain whether the time taken to attain a speed of $10\,\mathrm{m\,s^{-1}}$ would be predicted to be lower, the same or higher using the refined model compared with the original model. **[2]**
(b) explain whether the greatest steady speed of the system would be predicted to be lower, the same or higher using the refined model compared with the original model. **[2]**

6. A snowmobile of mass 800 kg tows a rescue sledge of mass 200 kg up a straight slope that is inclined at angle θ to the horizontal. The sledge is attached to the snowmobile by a light inextensible rope. The resistance to the motion of the snowmobile from non-gravitational forces is modelled as a constant force of magnitude 216 N. The resistance to the motion of the sledge from non-gravitational forces is modelled as a constant force of magnitude 294 N.

When the engine of the snowmobile is working at a constant rate of 12 kW, the snowmobile and the sledge move up the slope with constant speed $12\,\mathrm{m\,s^{-1}}$.

(a) Find the value of $\sin\theta$. **[4]**

(b) Later, at the instant when the snowmobile and the sledge are travelling down the slope with speed $14\,\mathrm{m\,s^{-1}}$, the rope snaps. When the rope snaps the sledge is at the point A. The sledge continues to travel down the slope before coming to instantaneous rest at the point B. The resistance to the motion of the sledge from non-gravitational forces is again modelled as a constant force of magnitude 294 N.

Use the work-energy principle, and your answer to part (a), to find the distance AB. **[5]**

7. A lorry of mass 3200 kg tows a trailer of mass 1800 kg down a line of greatest slope of a straight road inclined at an angle θ to the horizontal, where $\sin\theta = 0.06$.

The power developed by the lorry's engine is constant. The resistances to motion of the lorry and the trailer are constant and equal to 800 N and 500 N respectively. When the lorry passes through a point A on the road, the speed of the combination is $14\,\mathrm{m\,s^{-1}}$ and its acceleration is $1.2\,\mathrm{m\,s^{-2}}$.

(a) Calculate the power developed by the engine. **[5]**

(b) The lorry later passes through a point B on the road with speed $22\,\mathrm{m\,s^{-1}}$. The time taken for the lorry to travel from A to B is 7.93 s.

Find the distance AB. **[5]**

8. A lorry of mass 2500 kg travels up a straight road inclined at an angle α to the horizontal, where $\tan\alpha = \dfrac{7}{24}$.

The engine of the lorry supplies a constant power of P W. The total resistance to motion is a constant force of magnitude R N.

When the lorry is moving at $12\,\mathrm{m\,s^{-1}}$, its acceleration is $2.4\,\mathrm{m\,s^{-2}}$. When it is moving at $18\,\mathrm{m\,s^{-1}}$, its acceleration is $0.4\,\mathrm{m\,s^{-2}}$.

Find the value of P and the value of R **[8]**

9. A van of mass 1000 kg is driven up a straight road inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{20}$. The engine is working at a constant rate of 12 kW and the van moves at a constant speed of $15\,\mathrm{m\,s^{-1}}$.

(a) Find the resistance to the motion of the van. **[4]**

(b) The van now tows a trailer of mass 600 kg up the same road using a tow rope parallel to the line of motion. The resistance to the motion of the van is the same as in part (a), and the resistance to the motion of the trailer is a constant force of magnitude 106 N.

The engine is still working at a constant rate of 12 kW.

The tow rope is modelled as light and inextensible.

Using the model, find the tension in the tow rope at the instant when the speed of the van is $7.5\,\mathrm{m\,s^{-1}}$. **[5]**

10. A lorry of mass 2400 kg is towing a trailer of mass 600 kg up a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{49}$.

When the lorry and trailer move at speed $v\,\text{m}\,\text{s}^{-1}$, the total resistance to motion has magnitude $(300 + 60v)$ N.

The greatest constant speed that the lorry and trailer can maintain on this hill is $15\,\text{m}\,\text{s}^{-1}$.

Find the maximum power output of the lorry's engine.

Fully justify your answer. **[5]**

11. A maintenance train has mass 10 000 kg and the maximum power of its engine is 200 kW. In a model of the train's motion, it is assumed that while the train is moving the total resistance to motion is constant.

On a straight horizontal track, the greatest constant speed that the train can maintain when the engine is working at maximum power is $20\,\mathrm{m\,s^{-1}}$.

(a) Find the resistance to motion. **[2]**

(b) While the train is still on horizontal track, the engine is delivering 60 kW when the speed of the train is $8\,\mathrm{m\,s^{-1}}$.
Find the acceleration of the train at this instant. **[2]**

(c) The train then climbs a straight track inclined at an angle θ above the horizontal. When the engine is working at maximum power, the greatest constant speed that the train can maintain is $10\,\mathrm{m\,s^{-1}}$.
Determine the value of θ. **[3]**

12. A rail maintenance vehicle of mass 2000 kg travels in a straight line along a horizontal section of track at a constant speed of $20\,\mathrm{m\,s^{-1}}$.

The resistance to the motion of the vehicle is modelled as a constant force of magnitude 2000 N.

The engine is working at a constant rate of P kW.

Using the model,

(a) find the value of P. **[3]**

The vehicle now travels up a straight section of track which is inclined to the horizontal at an angle α, where $\sin\alpha = \dfrac{1}{20}$.

In a refined model, the resistance to the motion of the vehicle from non-gravitational forces is modelled as a force of magnitude $(220 + 2v^2)$ N, where $v\,\mathrm{m\,s^{-1}}$ is the speed.

At the instant when the engine is working at the same constant rate as in part (a) and the vehicle is moving up the track at $10\,\mathrm{m\,s^{-1}}$, the acceleration of the vehicle is $a\,\mathrm{m\,s^{-2}}$.

Using the refined model,

(b) find the value of a. **[4]**

Later, when the engine is again working at this constant rate, the vehicle is moving up the track at a constant speed $U\,\mathrm{m\,s^{-1}}$.

Using the refined model,

(c) find the value of U. **[5]**