Questions

1. One end of a light elastic string, of natural length $2a$ and modulus of elasticity $2mg$, is attached to a fixed point O. The other end of the string is attached to a particle P of mass m. The string is vertical with P directly above O. The particle P is pulled vertically upwards until the extension of the string is $2a$. The particle P is then released from rest.

(a) Find the speed of P when it is at a distance $\frac{3}{2}a$ above O. **[3]**

(b) Find the initial acceleration of P when it is released from rest. **[2]**

2. A light elastic string has natural length 2.5 metres and modulus of elasticity 15 newtons.

One end of the elastic string is attached to a particle of mass 0.75 kg.

The other end of the elastic string is attached to a fixed point O.

The particle is released from rest at a point A, which is 2.9 metres vertically below O.

(a) Calculate the elastic potential energy stored in the string when the particle is at A. **[2]**

(b) The point B is 4.0 metres vertically below O.
Calculate the gravitational potential energy lost by the particle as it moves from A to B. **[2]**

(c) Find the speed of the particle at B. **[3]**

(d) The point C is 3.3 metres vertically below O.
Explain, showing any calculations that you make, why the speed of the particle is increasing the first time that the particle is at C. **[3]**

3. A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus $2mg$. The other end of the string is attached to a fixed point O on a rough plane inclined at $30°$ to the horizontal.

The string lies along a line of greatest slope, with O below P. The particle is initially at rest on the plane with the string at its natural length.

The coefficient of friction between P and the plane is $\dfrac{1}{\sqrt{3}}$.

The particle is projected up the plane, away from O, with speed $\sqrt{\dfrac{3ga}{2}}$.

Find the greatest extension of the string during the subsequent motion. **[5]**

4. The points A and B are at the same horizontal level and are $24a$ apart. The ends of a light elastic string, of natural length $18a$ and modulus of elasticity λ, are attached to A and B. A particle P of mass m is attached to the midpoint of the string. The system is in equilibrium with P at a distance $5a$ below M, the midpoint of AB.

(a) Find λ in terms of m and g. **[3]**

The particle P is pulled down vertically and released from rest from a position $9a$ below M.

(b) Find, in terms of a and g, the speed of P as it passes through its equilibrium position. **[4]**

5. A light elastic string has natural length $3a$ and modulus of elasticity $9mg$.
One end of the string is attached to a fixed point O. A particle P of mass m is attached to the other end.
The particle hangs freely in equilibrium at the point E, vertically below O.

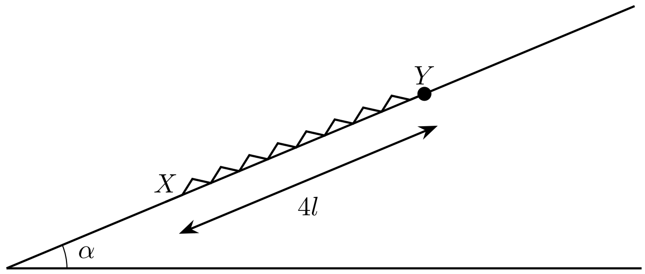
(a) Find the length OE. **[4]**

The particle is then pulled vertically downwards from E to a point A, where $OA = \dfrac{16a}{3}$, and released from rest.
The resistance to the motion of P is a constant force of magnitude $\dfrac{1}{4}mg$.

(b) Find, in terms of a and g, the speed of P as it passes through E. **[7]**

The particle is now held at O and released from rest.
The resistance to the motion of P is again a constant force of magnitude $\dfrac{1}{4}mg$.
The speed of P is greatest when it is at the point B.

(c) Find the distance OB. **[4]**



6. A light elastic spring has natural length $2l$ and modulus of elasticity mg. One end of the spring is attached to a fixed point X on a rough inclined plane. The other end is attached to a package P of mass m.

The plane is inclined to the horizontal at an angle α where

$$\tan\alpha = \frac{5}{12}$$

The package is initially held at the point Y on the plane, where $XY = 4l$. The point Y is higher than X and XY is a line of greatest slope of the plane, as shown. The package is released from rest at Y and moves down the plane.

The coefficient of friction between P and the plane is $\frac{1}{4}$.

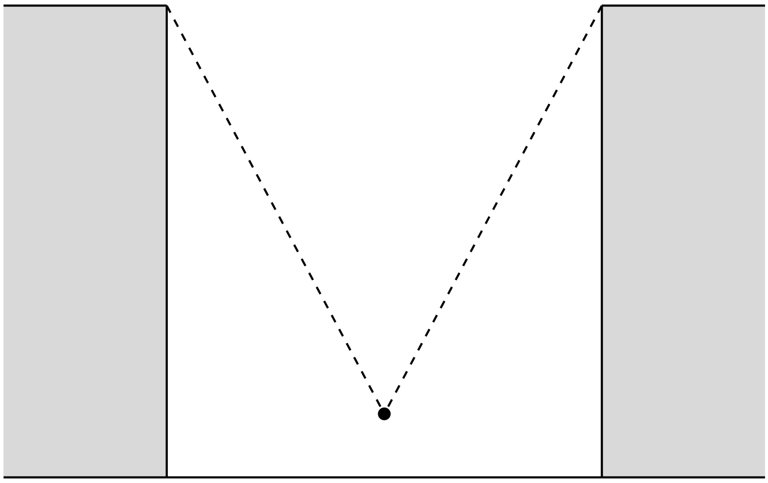
By modelling P as a particle,

(a) show that the acceleration of P at the instant when it is released is

$$\frac{15}{13}g$$

down the plane. **[5]**

(b) find, in terms of g and l, the speed of P at the instant when the spring first regains its natural length. **[6]**



7. A stunt performer is attached to two identical light elastic cords. One end of each cord is fixed to opposite sides of the rim of a vertical shaft, as shown in the diagram above.

The performer has mass 80 kg and is modelled as a particle. Initially the performer is held at rest at the centre of the shaft, 7 metres above the floor. The shaft is 52 metres deep and 48 metres wide. Each elastic cord has natural length 30 metres and modulus of elasticity 3150 N.

The performer is released from rest.

(a) Show that the speed of the performer when the cords first become slack is $25\,\mathrm{m\,s^{-1}}$ correct to two significant figures. **[6]**

(b) Determine whether the performer is moving up or down when the cords become taut again. **[5]**

8. A spring of natural length a is suspended from a fixed point A. A package P of mass m is attached to the lower end of the spring.

The package is initially at the point B, vertically below A, where $AB = a$.

The package is projected vertically downwards from B with speed $\sqrt{ag}$.

It first comes to instantaneous rest at the point C, where $AC = 2a$.

Air resistance is modelled as negligible. By modelling the spring as light and modelling P as a particle,

(a) show that the modulus of elasticity of the spring is $3mg$ **[5]**

(b) (i) Show that P attains its maximum speed when the extension of the spring is $\frac{1}{3}a$ **[2]**

(ii) Hence find the maximum speed, giving your answer in terms of a and g. **[4]**

(c) In reality, the spring is not light.
State one way in which this would affect your energy equation in part (b). **[1]**

9. A light elastic string with natural length l and modulus of elasticity kmg has one end attached to a fixed point A on a rough inclined plane. The other end of the string is attached to a package of mass m.
The plane is inclined at an angle θ to the horizontal, where

$$\tan\theta = \frac{3}{4}$$

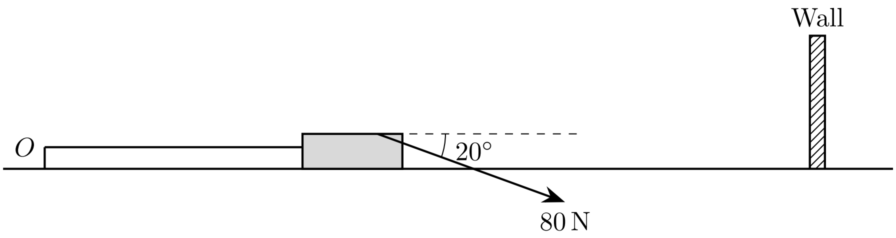
The package is initially held at the point C on a line of greatest slope of the plane below A, where $AC = 3l$. The package is then projected down the plane with speed $2\sqrt{gl}$ and first comes to rest at the point B, where $AB = 5l$.
The coefficient of friction between the package and the plane is $\frac{1}{2}$.
By modelling the package as a particle,

(a) show that

$$k = \frac{2}{5}$$

[6]

(b) find the acceleration of the package at the instant it starts to move back up the plane from the point B. **[5]**



10. A block of mass 6 kg rests on a rough horizontal surface and is attached to a fixed point O by a light elastic string. Initially the block is at rest 2.4 m from O.

The elastic string has natural length 2 m and modulus of elasticity 80 N.

The coefficient of friction between the block and the surface is 0.25.
A force of magnitude 80 N is applied to the block towards a vertical wall. The force acts at an angle of $20°$ below the horizontal, as shown in the diagram.

The block moves in a straight line perpendicular to the wall, and the wall is sufficiently far away that the block does not reach it during the motion considered.

(a) Show that the block has moved 0.94 m, correct to 2 significant figures, when the forces acting on it are in equilibrium. **[4]**

(b) State one limitation of the model. **[1]**

(c) Find the maximum speed of the block. **[4]**