FURTHER MECHANICS / WORK, ENERGY AND POWER

Elastic Potential Energy

Mark scheme 10 questions · 101 marks

EXAM BOARDS Edexcel FM1 9FM0/3C · AQA 7367/3M · OCR A Y543 · OCR MEI Mech Major Y421 CIE 9231 Paper 3

Questions

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	Total	**101**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last mark of a part whose answer is printed in the question: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [***n***] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	$EPE_0 = \frac{2mg(2a)^2}{2(2a)} = 2mga$ String slack at $\frac{3}{2}a$, so $EPE_1 = 0$	**B1**	Needs both; EPE zero at $\frac{3}{2}a$ is implied by an energy equation with no elastic term there.
	$\frac{1}{2}mv^2 = mg\left(\frac{5}{2}a\right) + \text{“}2mga\text{”}$	**M1 A1**	**M1** Equates energies at the two positions, with kinetic, gravitational and elastic terms, the elastic of the form $\frac{\lambda x^2}{2l}$. Condone sign slips. **A1** Any correct form, e.g. $\frac{1}{2}mv^2 + \frac{3}{2}mga = 4mga + 2mga$.
	$v = 3\sqrt{ag}$	**A1**	Or $\sqrt{9ag}$ oe.
		[4]	
1(b)	$m\alpha = \frac{2mg(2a)}{2a} + mg$	**M1**	Uses $F = ma$ with a tension of the form $\frac{\lambda x}{l}$ and the weight. Condone sign slips.
	$\alpha = 3g$, downwards	**A1**	$3g$ with its direction, downwards or towards O; A0 without a direction.
		[2]	
Total for Question 1		**6**	

Question	Scheme	Marks	Notes
2(a)	$EPE = \frac{15\times0.4^2}{2\times2.5}$	**M1**	Uses $\frac{\lambda x^2}{2l}$ with $x = 0.4$. Not for using the length 2.9 as the extension.
	$= 0.48\,\text{J}$	**A1**	
		[2]	
2(b)	GPE lost $= 0.75 \times 9.8 \times 1.1$	**M1**	Uses mgh with $h = 4.0 - 2.9$. Not for $h = 4.0$.
	$= 8.085\,\text{J}$	**A1**	awrt 8.09, or 8.1.
		[2]	
2(c)	$EPE_B = \frac{15\times1.5^2}{2\times2.5} = 6.75\,\text{J}$	**B1**	
	$\frac{1}{2}(0.75)v^2 + 6.75 = 0.48 + 8.085$	**M1 A1**	**M1** Forms an energy equation from A to B with KE at B, GPE lost and both EPE terms. Condone sign slips and their values from (a), (b). **A1** Correct equation, oe.
	$v^2 = 4.84 \Rightarrow v = 2.2\,\text{m s}^{-1}$	**A1**	2.2 or 2.20.
		[4]	
2(d)	$T = \frac{15\times0.8}{2.5}$, compared with 0.75×9.8	**M1**	Uses Hooke's law to compare tension at C with weight, or finds the zero-resultant extension or length and compares with C.
	$T = 4.8\,\text{N}$, weight $= 7.35\,\text{N}$	**A1**	Both correct, or resultant $2.55\,\text{N}$; alternatively equilibrium extension $1.225\,\text{m}$ or length $3.725\,\text{m}$, compared with C (1.25, 3.75 for $g = 10$).
	Resultant force (acceleration) downwards, and the particle is moving downwards, so its speed increases.	**E1**	States acceleration and motion are downwards at C, so speed increases; alternatively C is above the equilibrium position and motion is downwards. Needs the A1.
		[3]	
	Total for Question 2	**11**	

Question	Scheme	Marks	Notes
3	$F = \frac{1}{\sqrt{3}} \times mg\cos 30° = \frac{1}{2}mg$	**B1**	Or work against friction $\frac{1}{2}mgx$ seen, including in the energy equation.
	$EPE = \frac{2mg\,x^2}{2a} = \frac{mgx^2}{a}$	**B1**	soi in the energy equation.
	$\frac{1}{2}m\left(\frac{3ga}{2}\right) - \frac{1}{2}mgx$ $= mg\left(\frac{x}{2}\right) + \frac{mgx^2}{a}$	**M1 A1**	**M1** Forms an energy equation with initial KE, GPE gain, EPE and work against friction, the last three in x. Condone sign slips and sin/cos confusion. **A1** Correct equation, oe.
	$x^2 + ax - \frac{3}{4}a^2 = 0$ $\Rightarrow x = \frac{-a \pm \sqrt{a^2+3a^2}}{2} = \ldots$	**M1**	Solves their quadratic by factorisation with the correct constant-term product, the quadratic formula with their coefficients, or completing the square. Condone slips.
	$x = \frac{a}{2}$ (reject $-\frac{3a}{2}$)	**A1**	cso. Rejects the negative root if shown.
	Total for Question 3	**6**	

Question	Scheme	Marks	Notes
4(a)	$2T\sin\theta = mg$, $\sin\theta = \frac{5}{13}$ $T = \frac{\lambda(\text{“}26a\text{”}-18a)}{18a}$	**M1**	Resolves vertically with the vertical components of both tensions and mg, each tension $\frac{\lambda x}{l}$ with their extension. Condone sin/cos confusion.
	$2\left(\frac{4\lambda}{9}\right)\left(\frac{5}{13}\right) = mg$	**A1**	Any correct form, e.g. $\frac{40\lambda}{117} = mg$.
	$\lambda = \frac{117mg}{40}$	**A1**	oe, e.g. $2.925mg$.
		[3]	
4(b)	$AP = \sqrt{(12a)^2 + (9a)^2} = 15a$ $\Rightarrow$ extension $= 30a - 18a = 12a$	**B1**	Or $6a$ for each half of natural length $9a$.
	EPE at release $= \frac{\lambda(12a)^2}{2(18a)} = 4\lambda a$	**B1**	oe, in terms of λ or as $11.7mga$; two halves $2\times\frac{\lambda(6a)^2}{2(9a)}$ accepted.
	EPE at equilibrium $= \frac{\lambda(8a)^2}{2(18a)} = \frac{16\lambda a}{9}$	**B1**	oe, in terms of λ or as $5.2mga$.
	“$4\lambda a$” = “$\frac{16\lambda a}{9}$” $+ mg(4a) + \frac{1}{2}mv^2$	**M1**	Forms an energy equation with both their EPE terms, a GPE change of mg times a distance, and $\frac{1}{2}mv^2$. Condone sign slips.
	$4\lambda a = \frac{16\lambda a}{9} + 4mga + \frac{1}{2}mv^2$	**A1**	Any correct form, with λ or with $\lambda = \frac{117mg}{40}$ substituted.
	$v^2 = 5ag \Rightarrow v = \sqrt{5ag}$	**A1**	
		[6]	
Total for Question 4		**9**	

Question	Scheme	Marks	Notes
5(a)	$\frac{9mg\,x}{3a} = mg$	**M1 A1**	**M1** Equates a tension of the form $\frac{\lambda x}{l}$, with their λ and l, to mg. Condone x written as a length minus $3a$. **A1** Correct equation, oe.
	$x = \frac{a}{3}$ $\Rightarrow OE = 3a + \frac{a}{3} = \frac{10a}{3}$	**A1**	$\frac{10a}{3}$ oe.
		[3]	
5(b)	EPE at A: $\frac{9mg}{2(3a)}\left(\frac{7a}{3}\right)^2$ $= \frac{49}{6}mga$	**B1**	Need not be simplified; soi in the energy equation.
	EPE at E: $\frac{9mg}{2(3a)}\left(\frac{a}{3}\right)^2 = \frac{1}{6}mga$	**B1**	Need not be simplified; soi in the energy equation.
	Work against resistance $= \frac{1}{4}mg \times 2a = \frac{1}{2}mga$	**B1**	Need not be simplified; soi in the energy equation.
	$\frac{49}{6}mga - \frac{1}{2}mga$ $= \frac{1}{2}mv^2 + mg(2a) + \frac{1}{6}mga$	**M1 A1**	**M1** Forms an energy equation from A to E with KE, a GPE change, two EPE terms of the form $\frac{\lambda x^2}{2l}$ and the work against resistance. Condone sign slips. **A1** Correct equation, oe, with any zero level for GPE.
	$v = \sqrt{11ag}$	**A1**	$\sqrt{11ag}$ oe.
		[6]	
5(c)	$mg - \frac{1}{4}mg - \frac{9mg\,x}{3a} = 0$	**M1 A1**	**M1** Sets to zero a resultant of the weight, the resistance and a tension of the form $\frac{\lambda x}{l}$. Condone sign slips. Or sets the derivative of their energy expression for v^2 to zero. **A1** Correct equation, oe, with the resistance opposing the weight.
	$x = \frac{a}{4}$	**A1**	Correct extension, or implied by a correct OB from an equation in the length.
	$OB = 3a + \frac{a}{4} = \frac{13a}{4}$	**A1**	$\frac{13a}{4}$ oe.
		[4]	
	Total for Question 5	**13**	

Question	Scheme	Marks	Notes
6(a)	$T = \frac{mg(2l)}{2l} = mg$	**B1**	
	$R = mg\cos\alpha = \frac{12mg}{13}$ $F = \frac{1}{4} \times \frac{12mg}{13} = \frac{3mg}{13}$	**B1**	Needs $R = \frac{12mg}{13}$ or $mg\cos\alpha$ used; $F = \frac{3mg}{13}$ or exact equivalent.
	$ma =$ "T" $+ mg\sin\alpha -$ "F"	**M1**	Forms an equation of motion along the plane with tension, a resolved weight component and friction. Condone sign slips and sin/cos confusion.
	$ma = mg + \frac{5mg}{13} - \frac{3mg}{13}$	**A1**	Correct equation with $\sin\alpha = \frac{5}{13}$ substituted, or $\sin\alpha$ left in.
	$ma = \frac{15mg}{13} \Rightarrow a = \frac{15}{13}g$	**A1 AG**	Correct substituted equation of motion leading without error to $a = \frac{15}{13}g$; combined simplification accepted.
		[5]	
6(b)	$EPE = \frac{mg(2l)^2}{2(2l)} = mgl$	**B1**	
	GPE lost $= mg \times 2l\sin\alpha = \frac{10mgl}{13}$	**B1**	
	Work against friction $= \frac{3mg}{13} \times 2l = \frac{6mgl}{13}$	**B1**	
	$\frac{1}{2}mv^2 =$ "mgl" $+$ "$\frac{10mgl}{13}$" $-$ "$\frac{6mgl}{13}$"	**M1**	Forms an energy equation with kinetic energy, EPE, GPE and work against friction, all dimensionally correct. Condone sign slips.
	$\frac{1}{2}mv^2 = mgl + \frac{10mgl}{13} - \frac{6mgl}{13}$	**A1**	
	$v = \sqrt{\frac{34gl}{13}}$	**A1**	Any exact equivalent, or awrt $1.62\sqrt{gl}$.
		[6]	
	Total for Question 6	**11**	

Question	Scheme	Marks	Notes
7(a)	$R = 6(9.8) + 80\sin 20°$ $\Rightarrow F = 0.25R = 21.54$	**B1**	Correct $R = 6g + 80\sin 20°$, seen or used, and $F = 0.25R$, awrt 21.5 N for $g = 9.8$.
	Resolves horizontally: $80\cos 20° = T + F$ with $T = \frac{80}{2}e$	**M1**	Equates horizontal force to Hooke's law tension plus friction, using total extension or distance moved as the variable. Condone sign slips.
	$80\cos 20° = 40(0.4 + x) + 21.54$	**A1**	Correct equation, oe, including $80\cos 20° = 40x + 21.54$ if x is total extension.
	$40x = 37.64 \Rightarrow x = 0.9409$ $= 0.94$ (2 s.f.)	**A1 AG**	Correct substituted force equation leading without error to distance moved 0.94 m; at least 3 s.f. seen before rounding, including $x - 0.4$ if x is total extension.
		[4]	
7(b)	The coefficient of friction is assumed constant.	**E1**	States any valid limitation, e.g. block modelled as a particle, air resistance ignored, string light or obeying Hooke's law throughout.
		[1]	
7(c)	Maximum speed where the resultant force is zero $\Rightarrow x = 0.9409$	**B1**	Stated or used at distance moved 0.94 m or more accurate, or total extension 1.34 m or more accurate; may use (a).
	$EPE = \frac{80(0.4)^2}{2\times 2} = 3.2$ and $\frac{80(1.3409)^2}{2\times 2} = 35.96$	**B1**	Both, or their difference, as expressions or values; final EPE uses x^2 if x denotes total extension.
	Work by force $= 80\cos 20° \times 0.9409$ [= 70.73]	**B1**	Uses distance moved, $x - 0.4$ if x denotes total extension.
	Work against friction $= 21.54 \times 0.9409$ [= 20.27]	**B1**	Uses distance moved, $x - 0.4$ if x denotes total extension.
	"3.2" + "70.73" − "20.27" $= \frac{1}{2}(6)v^2 +$ "35.96"	**M1**	Forms an equation with kinetic energy, both elastic potential energies and both work terms. Condone sign slips.
	$53.66 = 3v^2 + 35.96$	**A1**	Correct equation, oe.
	$v = 2.43\,\text{m s}^{-1}$	**A1**	awrt 2.43.
		[7]	
	Total for Question 7	**12**	

Question	Scheme	Marks	Notes
8(a)	EPE at $C = \frac{\lambda a^2}{2a}$ $\left[= \frac{\lambda a}{2}\right]$	**B1**	Extension a used in $\frac{\lambda x^2}{2a}$.
	$\frac{1}{2}m\left(\sqrt{ag}\right)^2 + mga = \frac{\lambda a}{2}$	**M1 A1**	**M1** Forms an energy equation from B to C with KE, GPE and EPE terms, each dimensionally correct. Condone sign slips. **A1** Correct equation, oe, with any zero level for GPE.
	$\frac{3}{2}mga = \frac{\lambda a}{2} \Rightarrow \lambda = 3mg$	**A1 AG**	Correct substituted energy equation leading without error to $\lambda = 3mg$; combined rearrangement accepted.
		[4]	
8(b)(i)	Acceleration zero at maximum speed: $mg = \frac{\lambda x}{a} = \frac{3mgx}{a}$	**M1**	Equates weight and tension $\frac{\lambda x}{a}$ with $\lambda = 3mg$, using zero acceleration. Condone sign slips.
	$x = \frac{a}{3}$	**A1 AG**	From $mg = \frac{3mgx}{a}$ oe with no errors. Need not show that the speed increases then decreases.
		[2]	
8(b)(ii)	EPE $= \frac{3mg}{2a}\left(\frac{a}{3}\right)^2$ $\left[= \frac{1}{6}mga\right]$	**B1**	Or $\frac{\lambda a}{18}$.
	$\frac{1}{2}m\left(\sqrt{ag}\right)^2 + mg\left(\frac{a}{3}\right)$ $= \frac{1}{2}mv^2 +$ "$\frac{1}{6}mga$"	**M1 A1**	**M1** Forms an energy equation from B or C to extension $\frac{a}{3}$ with all relevant nonzero KE, GPE and EPE terms, including EPE at C, $\frac{\lambda a}{2}$, if working from C. Condone sign slips. **A1** Correct equation, with λ or $3mg$.
	$v^2 = \frac{4ag}{3} \Rightarrow v = 2\sqrt{\frac{ag}{3}}$	**A1**	Any exact form, e.g. $\sqrt{\frac{4ag}{3}}$. Positive root only.
		[4]	
8(c)	The kinetic energy of the spring must be included.	**E1**	States that a term for the spring's kinetic energy, or its gravitational potential energy, is needed. Not for a vague effect such as "less energy".
		[1]	
	Total for Question 8	**11**	

Question	Scheme	Marks	Notes
9(a)	GPE lost $= mg(2l)\sin\theta = \frac{6}{5}mgl$	**B1**	
	$R = \frac{4}{5}mg \Rightarrow$ work against friction $= \frac{1}{2} \times \frac{4}{5}mg \times 2l = \frac{4}{5}mgl$	**B1**	
	EPE at $C = \frac{kmg(2l)^2}{2l} = 2kmgl$ EPE at $B = \frac{kmg(4l)^2}{2l} = 8kmgl$	**B1**	Both needed.
	$\frac{1}{2}m\left(2\sqrt{gl}\right)^2 + \frac{6}{5}mgl + 2kmgl$ $= \frac{4}{5}mgl + 8kmgl$	**M1 A1**	**M1** Forms an energy equation with initial KE, GPE, both EPE terms and work against friction. Condone sign slips and errors in individual terms. **A1** Correct equation, oe.
	$6kmgl = \frac{12}{5}mgl \Rightarrow k = \frac{2}{5}$	**A1 AG**	Correct substituted energy equation leading without error to $k = \frac{2}{5}$; combined collection, rearrangement and solution accepted.
		[6]	
9(b)	$T = \frac{kmg(4l)}{l} = \frac{8}{5}mg$	**B1**	Accept $4kmg$, with $k = \frac{2}{5}$ used later.
	$F = \frac{1}{2} \times \frac{4}{5}mg = \frac{2}{5}mg$, down the plane	**B1**	Needs the magnitude and the direction; the direction may be implied by the equation of motion.
	$\frac{8}{5}mg - \frac{3}{5}mg - \frac{2}{5}mg = ma$	**M1 A1**	**M1** Resolves along the plane with tension, a weight component and friction, all three terms present. Condone sign slips. **A1** Correct equation, oe.
	$a = \frac{3}{5}g$, up the plane	**A1**	$\frac{3}{5}g$ or awrt 5.88, with the direction up the plane.
		[5]	
	Total for Question 9	**11**	

Question	Scheme	Marks	Notes
10(a)	$\sqrt{24^2+45^2}=51 \Rightarrow x=51-30=21$	**B1**	Both the length 51 and the extension 21.
	$24^2+(52-h)^2=30^2 \Rightarrow h=34$	**B1**	Height 34 m above the floor, 18 m below the rim, or a rise of 27 m.
	$EPE=2\times\frac{3150\times21^2}{2\times30}=46\,305$	**B1**	For both cords; awrt 46 300.
	"46 305" $=\frac{1}{2}(80)v^2+80g(27)$	**M1 A1**	**M1** Forms an energy equation from release to slack with KE, GPE at their two heights (or their change in height) and their EPE. Condone sign slips and EPE of one cord. **A1** Correct equation, oe.
	$40v^2=25\,137 \Rightarrow v=25.07\ldots=25\,\text{m s}^{-1}$	**A1 AG**	Correct substituted energy equation leading without error to $v=25\,\text{m s}^{-1}$ to 2 s.f.; v to 3 s.f. or better (awrt 25.1, or 24.9 with $g=10$) must be seen before it is rounded.
		[6]	
10(b)	$\frac{1}{2}(80)(\text{"628.4"})=80g(H-34) \Rightarrow H=\ldots$	**M1**	Equates their KE at slack to the gain in GPE, with no EPE, or uses $0=$ their v^2-2gs, with their v^2 from (a) or the printed $v=25$. Condone sign slips.
	$H=66.1$ m, i.e. 14.1 m above the rim	**A1**	awrt 66.1 m above the floor (14.1 above the rim), or 65.9 from the printed $v=25$; with $g=10$, 64.9, or 65.25 from $v=25$. Equivalents above the rim or slack point accepted.
	$24^2+y^2=30^2 \Rightarrow y=\ldots$	**M1**	Uses Pythagoras or symmetry to find the taut height above the rim; alternatively uses $\sqrt{24^2+(H-52)^2}$ for cord length at maximum height.
	$y=18$ m above the rim (70 m above the floor)	**A1**	Or cord length awrt 27.8 m at maximum height; accept the correct length using any accepted first-A1 height.
	$14.1<18$, so the cords are not taut on the way up: moving down	**E1**	Compares accepted maximum and taut heights, or establishes an above-rim maximum with cord length below 30 m, and concludes moving down. A bare "down" scores nothing.
		[5]	
	Total for Question 10	**11**	