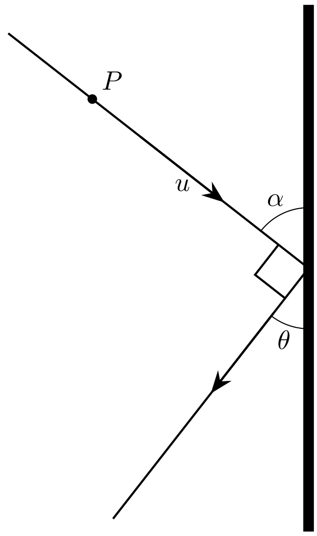
Questions



1. A particle P of mass m is moving with speed u on a fixed smooth horizontal surface. It collides with a fixed smooth vertical barrier. Before impact its path makes an angle α with the barrier, and after impact its path makes an angle θ with the barrier. The incident path and the outgoing path are perpendicular. The coefficient of restitution between P and the barrier is e. The particle loses 36% of its kinetic energy in the collision.

Find the value of e and the exact value of $\tan\alpha$. **[5]**

2. A hockey puck moving on a smooth horizontal rink collides with a fixed straight side-board.
Let **i** be a unit vector parallel to the side-board and let **j** be a unit vector perpendicular to the side-board, directed away from it.

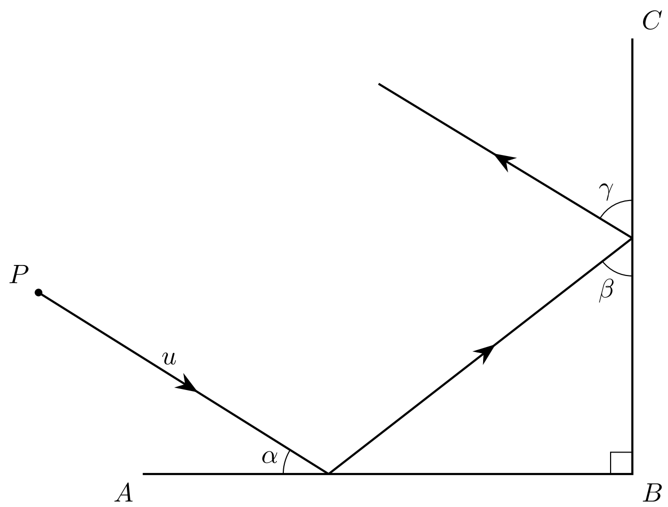
Immediately before the collision, the puck has velocity $(9\mathbf{i} - 12\mathbf{j})\,\text{m}\,\text{s}^{-1}$.

Immediately after the collision, the puck moves with speed $10\,\text{m}\,\text{s}^{-1}$ in the direction of the vector $3\mathbf{i} + 4\mathbf{j}$.

Model the puck as a particle.

(a) Find the coefficient of restitution between the puck and the side-board. **[3]**

(b) Determine whether or not the side-board is smooth.
Fully justify your answer. **[3]**



3. The smooth walls AB and BC are at right angles to each other. A particle P moves on a smooth horizontal floor with speed u and strikes the wall AB at an angle α to AB. It rebounds, then strikes the wall BC at an angle β to BC. After the second impact it rebounds at an angle γ to BC, as shown in the diagram. The coefficient of restitution between P and each wall is e.

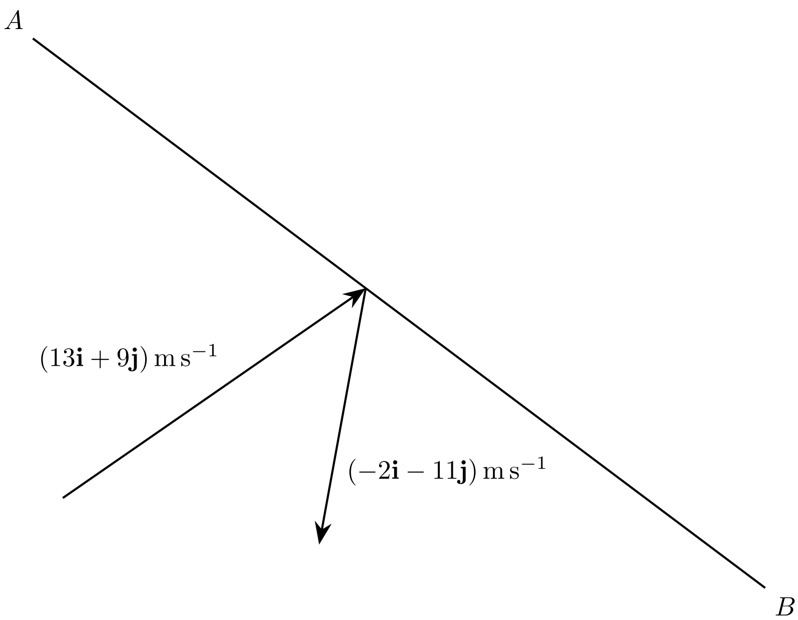
(a) Show that

$$\tan\alpha\tan\beta = \frac{1}{e}$$ **[3]**

(b) Express γ in terms of α and explain what this implies about the final direction of motion of P. **[4]**

During the first impact the particle loses four times as much kinetic energy as it loses during the second impact. After the second impact the kinetic energy of P is $\frac{9}{25}$ of its initial kinetic energy.

(c) Find the value of e and the value of $\tan\alpha$. **[4]**



4. The diagram above shows the plan view of part of a smooth horizontal table. The line segment AB represents a fixed smooth vertical wall.

A small ball of mass 0.4 kg moves across the table and rebounds after striking the wall.

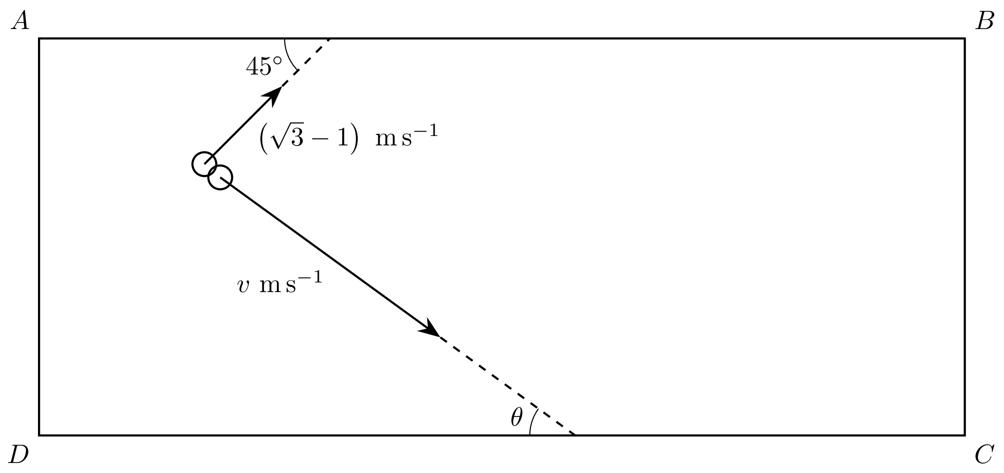
Immediately before impact its velocity is $(13\mathbf{i} + 9\mathbf{j})\,\mathrm{m\,s^{-1}}$.

Immediately after impact its velocity is $(-2\mathbf{i} - 11\mathbf{j})\,\mathrm{m\,s^{-1}}$.

The coefficient of restitution between the ball and the wall is e.

(a) Show that AB is parallel to $(4\mathbf{i} - 3\mathbf{j})$. **[4]**

(b) Find the value of e. **[5]**



5. Two snooker balls, one white and one red, have equal mass.
The balls are on a horizontal table $ABCD$.

The white ball is struck so that it moves at a speed of $2\,\mathrm{m\,s^{-1}}$ in a direction making an angle of 15° with AB, towards CD.

The white ball hits a stationary red ball.

After the collision, the white ball moves at a speed of $(\sqrt{3}-1)\,\mathrm{m\,s^{-1}}$ and at an angle of 45° to AB.

After the collision, the red ball moves at a speed $v\,\mathrm{m\,s^{-1}}$ and at an angle θ to CD.

When the collision takes place, the point of impact is twice as far from CD as it is from AB.

The diagram below shows the velocities of the balls after the collision.

After the collision, the white ball hits AB and the red ball hits CD.

Model the balls as particles and neglect any resistance.

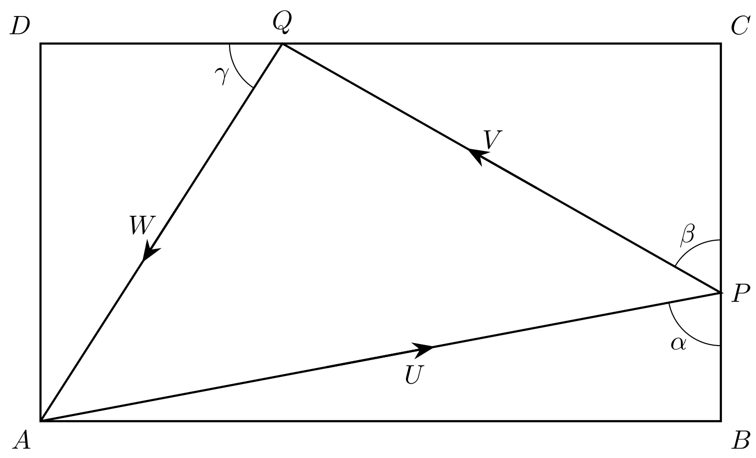
(a) Explain why the two balls hit the sides of the table at the same time. **[2]**

(b) Show that $\theta = 36.2°$ correct to one decimal place. **[4]**

(c) Find v. **[2]**

(d) Determine which ball travels the greater distance after the collision and before hitting the side of the table.
Fully justify your answer. **[2]**

(e) State one possible refinement to the model that you have used. **[1]**



6. A small smooth billiard ball is projected from the corner A of a horizontal rectangular table $ABCD$.

The ball first hits the side BC at the point P, then hits the side CD at the point Q, and then returns to A.

Angle $APB = \alpha$, angle $CPQ = \beta$ and angle $DQA = \gamma$.

The ball moves along AP with speed U, along PQ with speed V and along QA with speed W, as shown in the diagram.

The coefficient of restitution between the ball and side BC is e_1.

The coefficient of restitution between the ball and side CD is e_2.

The ball is modelled as a particle.

(a) Show that $\tan\beta = e_1 \tan\alpha$. **[4]**

(b) Hence show that $e_1 \tan\alpha \tan\gamma = e_2$. **[3]**

(c) By considering $\angle PAQ$ or otherwise, show that it is possible for the ball to return to A only if $e_2 > e_1$. **[6]**

If instead $e_1 = e_2$, the ball would not return to A.

(d) Given that $e_1 = e_2$, use the result from part (b) to describe the path of the ball after it hits CD at Q, explaining your answer. **[1]**

7. A particle P is moving with velocity $(7\mathbf{i} - 4\mathbf{j})\,\mathrm{m\,s^{-1}}$ on a smooth horizontal plane. It collides with a smooth vertical wall which is parallel to the direction vector $2\mathbf{i}+\mathbf{j}$ and rebounds with velocity $(2\mathbf{i}+6\mathbf{j})\,\mathrm{m\,s^{-1}}$.

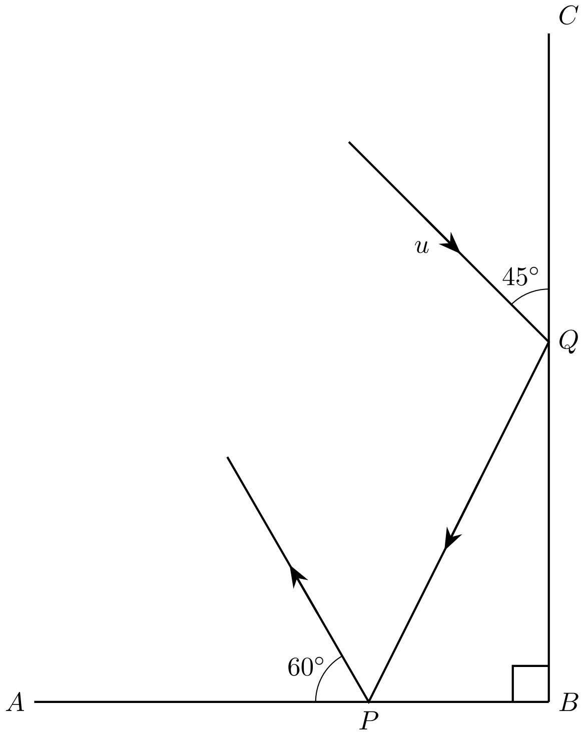
The coefficient of restitution between P and this wall is e.

(a) Find the value of e. **[5]**

After this collision, P goes on to hit a second smooth vertical wall which is parallel to the direction vector $\mathbf{i} - 2\mathbf{j}$. The coefficient of restitution between P and this second wall is $\frac{1}{2}$.

The angle through which the direction of motion of P is deflected by its collision with this second wall is $\alpha°$.

(b) Find the value of α, giving your answer to the nearest whole number. **[5]**



8. The diagram above represents the plan view of part of a horizontal floor, where AB and BC are perpendicular vertical walls.
The floor and the walls are modelled as smooth.

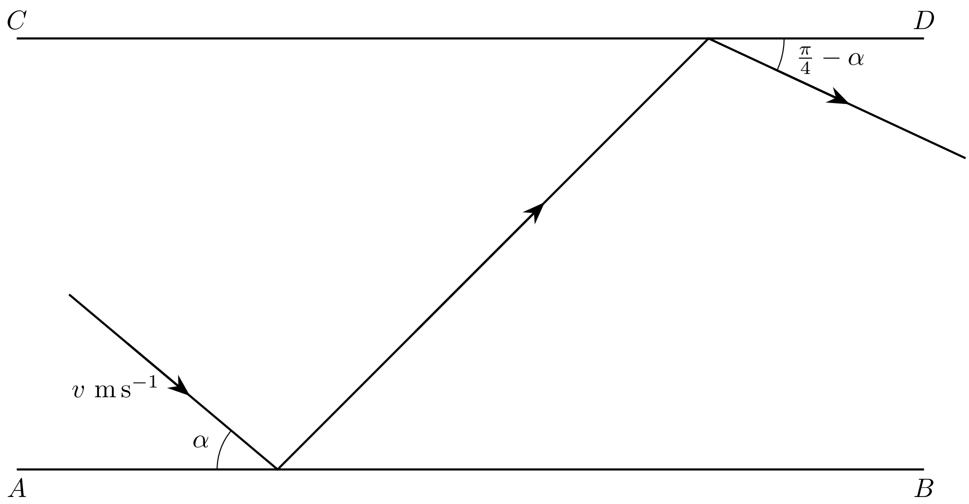
A ball is projected along the floor towards BC with speed $u\,\mathrm{m\,s^{-1}}$ on a path at an angle of 45° to BC. The ball hits BC at Q and then hits AB at P. After striking AB, the ball moves away on a path at an angle of 60° to AB.

The ball is modelled as a particle.

The coefficient of restitution between the ball and wall BC is $\frac{1}{2}$.

(a) Find the coefficient of restitution between the ball and wall AB and hence show that, using this model, the final kinetic energy of the ball is 50% of the initial kinetic energy of the ball. **[8]**

(b) In practice the walls and the floor may not be smooth. Explain how this model is likely to affect the percentage of kinetic energy remaining. **[1]**



9. The diagram above represents the plan view of part of a horizontal floor, where AB and CD represent fixed vertical walls, with AB parallel to CD.

A small ball is projected along the floor towards wall AB. Immediately before hitting wall AB, the ball is moving with speed $v\,\text{m}\,\text{s}^{-1}$ at an angle α to AB, where $0 < \alpha < \frac{\pi}{4}$.

The ball hits wall AB and then hits wall CD.

Immediately after the impact with wall CD, the ball is moving at an angle $\frac{\pi}{4} - \alpha$ to CD.

The coefficient of restitution between the ball and wall AB is $\frac{3}{5}$.

The coefficient of restitution between the ball and wall CD is $\frac{1}{2}$.

The floor and the walls are modelled as being smooth. The ball is modelled as a particle.

(a) Show that

$$\tan \alpha = \frac{2}{3}$$ **[7]**

(b) Find the percentage of the ball's initial kinetic energy that is lost in the two impacts. **[4]**

10. A particle P of mass m is moving downwards and to the right in a direction making an angle $45°$ with the horizontal when it strikes a fixed smooth inclined plane. The plane is inclined to the horizontal at an angle α, where $0 < \alpha < 45°$.

At the instant immediately before the impact, the speed of P is u.

At the instant immediately after the impact, P is moving upwards and to the right in a direction making an angle $45°$ with the horizontal, with speed v.

(a) Show that the magnitude of the impulse exerted on the plane by P is $mu\sec(45° - \alpha)$. **[5]**

The coefficient of restitution between P and the plane is e, where $0 < e < 1$.

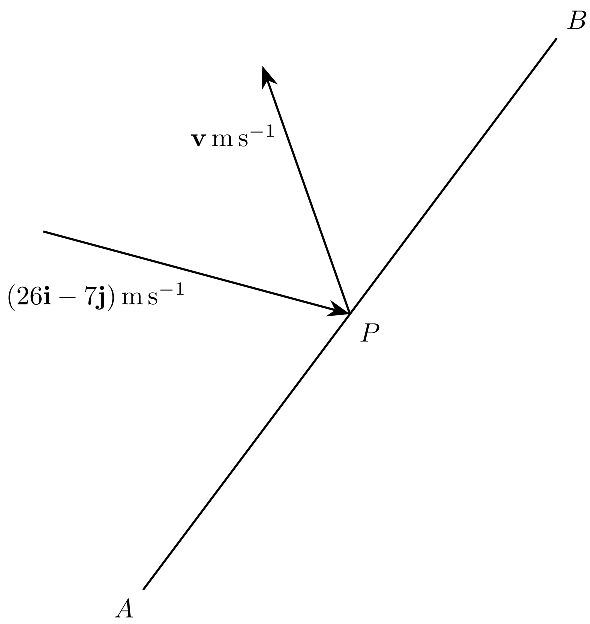
(b) Show that **[3]**

$$v^2 = u^2\left(\cos^2(45° + \alpha) + e^2\sin^2(45° + \alpha)\right)$$

(c) Show that the kinetic energy lost by P in the impact is

$$\frac{1}{2}mu^2(1 - e^2)\sin^2(45° + \alpha)$$ **[2]**

(d) Hence find, in terms of m, u and e only, the kinetic energy lost by P in the impact. **[2]**



11. The diagram above represents the plan view of part of a smooth horizontal billiards table, where AB is a fixed smooth cushion.

The direction of $\overrightarrow{AB}$ is parallel to the vector $(3\mathbf{i} + 4\mathbf{j})$.

A small disc of mass 0.2 kg is moving on the table when it strikes the cushion AB.

Immediately before impact with AB, the velocity of the disc is $(26\mathbf{i} - 7\mathbf{j})\,\mathrm{m\,s^{-1}}$.

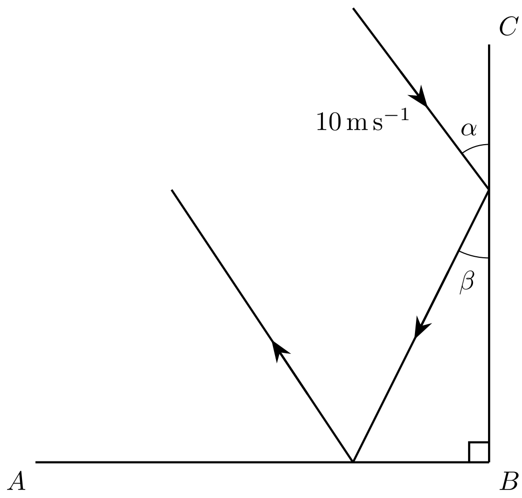
Immediately after impact with AB, the velocity of the disc is $\mathbf{v}\,\mathrm{m\,s^{-1}}$.

The coefficient of restitution between the disc and the cushion is $\frac{3}{5}$.

By modelling the disc as a particle,

(a) show that $\mathbf{v} = -6\mathbf{i} + 17\mathbf{j}$ **[6]**

(b) Find the magnitude of the impulse exerted by the cushion on the disc in the impact. **[3]**



12. The diagram above represents the plan view of one corner of a smooth air-hockey table, where AB and BC are fixed cushions with AB perpendicular to BC.

A small puck is struck along the table towards BC with speed $10\,\mathrm{m\,s^{-1}}$ on a path that makes an angle α with BC, where $\tan\alpha = \frac{3}{4}$. The puck hits BC and then hits AB.

Immediately after hitting BC, the puck is moving at an angle β to BC, where $\tan\beta = \frac{1}{2}$.

The coefficient of restitution between the puck and BC is e.

The coefficient of restitution between the puck and AB is $\frac{3}{4}$.

By modelling the puck as a particle and the table and cushions as being smooth,

(a) show that $e = \frac{2}{3}$ **[5]**

(b) find the speed of the puck immediately after it hits AB. **[4]**

(c) Suggest two ways in which the model could be refined to make it more realistic. **[2]**