Questions

1. A particle A of mass $2m$ and a particle B of mass $3m$ are moving along the same straight line on a smooth horizontal surface. The particles are moving in opposite directions towards each other when they collide directly.

Immediately before the collision, the speed of A is $4u$ and the speed of B is ku.
Immediately after the collision, A rebounds with speed v and the speed of B is v.

The magnitude of the impulse received by A in the collision is $12mu$.

(a) Find v in terms of u only. **[3]**

(b) Find the two possible values of k. **[5]**

2. The first diagram shows two spheres, A and B, of equal radii on a smooth horizontal surface. Their masses are 1 kg and 3 kg respectively.

Take motion from left to right in each diagram to be positive.

Initially, sphere A travels at speed $2\,\mathrm{m\,s^{-1}}$ towards B, which is at rest. The spheres collide and the coefficient of restitution between A and B is e.

(a) Show that, after the collision, the velocity of A is $\frac{1}{2}(1-3e)\,\mathrm{m\,s^{-1}}$, and find an expression for the velocity of B in terms of e. **[4]**

(b) During the collision, the kinetic energy of the system decreases by 48%. Determine the value of e. **[3]**

(c) Explain why the assumption that A and B have equal radii was needed in part (a). **[1]**

The second diagram shows two spheres, C and D, of equal radii on a smooth horizontal surface. Their masses are 2 kg and 5 kg respectively.

Sphere C is behind D and both move along the same straight line in the same direction, with C having speed $u\,\mathrm{m\,s^{-1}}$ and D having speed $1\,\mathrm{m\,s^{-1}}$, where $u > 1$.

The spheres collide and during the collision C exerts an impulse on D of magnitude $\frac{10}{7}(u-1)\,\mathrm{N\,s}$.

(d) Show that C and D have the same velocity after the collision. **[4]**

(e) Determine the fraction of kinetic energy lost due to the collision between C and D as $u \to \infty$. **[3]**

3. Two trolleys, A and B, are modelled as particles of masses 0.5 kg and 1.0 kg respectively, moving along the same straight horizontal track.

Trolley A is travelling at $7\,\mathrm{m\,s^{-1}}$ and catches up with trolley B, which is travelling in the same direction at $2\,\mathrm{m\,s^{-1}}$. The trolleys collide directly.

After the collision, trolley B moves at $5\,\mathrm{m\,s^{-1}}$ in the original direction of motion.

The coefficient of restitution between the trolleys is e.

(a) Assuming that the track is smooth, show that $e = 0.8$. **[5]**

(b) Describe one way in which the model could be refined. **[1]**

4. A particle P of mass 6 kg is projected vertically upwards from the ground with initial speed $15.8\,\mathrm{m\,s^{-1}}$

At the same instant a particle Q of mass 3 kg is projected vertically downwards with speed $4.2\,\mathrm{m\,s^{-1}}$ from a point 20 m above the ground.

During the subsequent motion P and Q collide. The coefficient of restitution between P and Q is 0.5

Determine the time between this collision and P subsequently hitting the ground. **[10]**

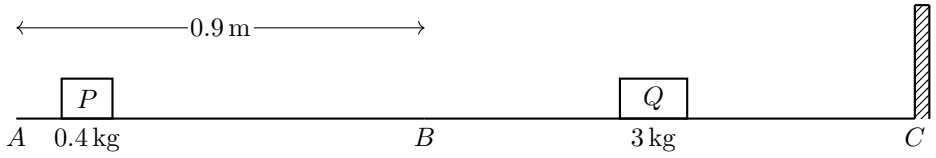
5. Two trolleys P and Q, of masses $2m$ and $5m$ respectively, move along the same straight line on a smooth horizontal track. Trolley P is behind trolley Q, and both are moving in the same direction. Trolley P catches up with trolley Q and they collide directly.

Immediately before the collision, the speed of P is ku and the speed of Q is u.
Immediately after the collision, trolley Q continues to move in the same direction with speed v and the speed of P is v.

The magnitude of the impulse received by Q in the collision is $10mu$.

(a) Find v in terms of u only. **[3]**

(b) Find the two possible values of k. **[5]**



6. The diagram shows two blocks P and Q of masses 0.4 kg and 3 kg respectively, on a horizontal surface. The points A, B and C lie on the surface in a straight line. The surface between A and B is rough and the surface between B and C is smooth. There is a wall at C.

The coefficient of friction between P and AB is $\frac{25}{49}$

Initially, P is at A and Q is at rest on BC. Block P is projected directly towards Q with speed $5\,\mathrm{m\,s^{-1}}$. The two blocks collide on BC. As a result of the collision, P changes direction and comes to rest at A. You may assume that P only collides with Q once.

(a) Determine the coefficient of restitution between P and Q. **[6]**

(b) Calculate the impulse exerted on P by Q during the collision. **[2]**

After the collision with P, block Q strikes the wall at C. The collision between Q and the wall is perfectly elastic. After rebounding from the wall, Q comes to rest at a point on AB that is 0.30 m from A.

(c) Determine the coefficient of friction between Q and AB. **[3]**

7. Particle P has mass m and particle Q has mass $7m$.

The particles are moving in the same direction along the same straight line on a smooth horizontal surface.
Particle P collides directly with particle Q.
Immediately before the collision, the speed of P is $9u$ and the speed of Q is u.
After the collision, the direction of motion of P is reversed.

The coefficient of restitution between P and Q is e.

(a) Find the complete range of possible values of e. **[7]**

(b) Given that $e = \frac{3}{4}$, find the total kinetic energy lost in the collision between P and Q. **[4]**

After the collision, P hits a smooth fixed vertical wall that is perpendicular to the direction of motion of P. Particle P rebounds.
The coefficient of restitution between P and the wall is f.

Given that there is a second collision between P and Q,

(c) find the complete range of possible values of f. **[3]**

8. A particle A of mass $3m$ is moving with speed $4u$ on a smooth horizontal plane towards a smooth fixed vertical wall. A particle B of mass m is on the same line between A and the wall, moving directly towards A with speed u.

Particles A and B collide directly. The coefficient of restitution between A and B is e, where $0 < e \leq 1$.

(a) Show that immediately after the collision B moves towards the wall with speed $\frac{u}{4}(11 + 15e)$. **[6]**

(b) Show that there will be a second collision between A and B after B has collided with the wall. **[3]**

The coefficient of restitution between B and the wall is $\frac{1}{3}$

Find, in simplified form, in terms of m, u and e,

(c) the magnitude of the impulse received by B from the wall, **[3]**

(d) the loss in kinetic energy of B due to its collision with the wall. **[3]**

9. A particle P of mass m is moving in a straight line with speed $5u$ on a smooth horizontal plane. It collides directly with a particle Q of mass $4m$ that is initially at rest on the plane.

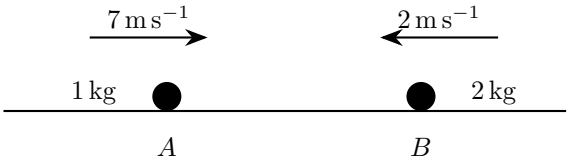
The coefficient of restitution between P and Q is e, where $e > \frac{2}{3}$.

(a) Show that the speed of P immediately after the collision is

$$(4e - 1)u$$ **[4]**

After the collision, P moves in the opposite direction and strikes a smooth fixed vertical wall that is perpendicular to the direction of motion of P. The coefficient of restitution between P and the wall is f.

(b) Find, in terms of e, the set of values of f for which there will be a second collision between P and Q. **[6]**



10. Two small uniform smooth spheres A and B have masses 1 kg and 2 kg respectively. The spheres are moving towards each other in the same straight line on a smooth horizontal surface. Sphere A is moving to the right with speed $7\,\mathrm{m\,s^{-1}}$ and sphere B is moving to the left with speed $2\,\mathrm{m\,s^{-1}}$. The spheres collide. After the collision, A moves with speed $1\,\mathrm{m\,s^{-1}}$.

Determine the possible speeds with which B moves after the collision. **[4]**

11. Particles P and Q, of masses m and $3m$ respectively, move on a smooth horizontal plane in the same straight line towards a vertical wall. Particle Q is in front of P. Initially, P has speed $6u$ and Q has speed u. Particle P catches up with Q and the particles collide before either particle reaches the wall.

The coefficient of restitution between the particles is e.

As a result of the collision, the direction of motion of P is reversed.

(a) Find, in terms of u and e, the speed of P after the collision. **[6]**

After the collision, Q continues to the wall and rebounds. The coefficient of restitution between Q and the wall is $\frac{1}{3}$.

Given that there is a second collision between P and Q,

(b) find the full range of possible values of e. **[5]**

12. Two small uniform smooth spheres A and B, of masses 2 kg and 6 kg respectively, are on a smooth horizontal surface. Sphere A is projected directly towards B with speed $4\,\mathrm{m\,s^{-1}}$. Sphere B is initially at rest. The coefficient of restitution between A and B is e.

(a) Show that the velocity of A after the collision, in the original direction of motion of A, is $(1-3e)\,\mathrm{m\,s^{-1}}$ and find a similar expression for the velocity of B. **[5]**

(b) The following three parts are independent of each other, and each considers a different scenario regarding the collision between A and B.

(i) In the collision between A and B the spheres coalesce to form a combined body C. State the speed of C after the collision. **[1]**

(ii) In the collision between A and B the direction of motion of A is reversed. Find the range of possible values of e. **[2]**

(iii) The total loss in kinetic energy due to the collision is 9 J. Determine the value of e. **[4]**