FURTHER MECHANICS / MOMENTUM & COLLISIONS

Vector Momentum and Impulse

Mark scheme 8 questions · 49 marks

EXAM BOARDS Edexcel FM1 9FM0/3C · AQA 7367/3M · OCR A Y543 · OCR MEI Mech Minor Y431 OCR MEI Mech Major Y421

Questions

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	Total	**49**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last mark of a part whose answer is printed in the question: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	$\mathbf{u} = \frac{13}{13}(5\mathbf{i} - 12\mathbf{j}) = 5\mathbf{i} - 12\mathbf{j}$ Using impulse = change in momentum: $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ $45\mathbf{i} + 90\mathbf{j} = 5\mathbf{v} - 5(5\mathbf{i} - 12\mathbf{j})$ $\Rightarrow \mathbf{v} = \ldots$	**M1**	Uses $m\mathbf{v} - m\mathbf{u}$ with mass 5 and $\mathbf{u}$ along $5\mathbf{i} - 12\mathbf{j}$. Condone a scaling slip and sign slips.
	$\mathbf{v} = 14\mathbf{i} + 6\mathbf{j}$	**A1**	
	$\lvert\mathbf{v}\rvert = \sqrt{\text{“14”}^2 + \text{“6”}^2} = \ldots$	**M1**	Uses Pythagoras on both components of their $\mathbf{v}$, of the form $a\mathbf{i} + b\mathbf{j}$.
	$\lvert\mathbf{v}\rvert = 2\sqrt{58}\ [\approx 15.2]\,\text{m s}^{-1}$	**A1**	$2\sqrt{58}$, $\sqrt{232}$ or awrt 15.2.
		[4]	
1(b)	Using the scalar product: $\cos\alpha = \frac{\mathbf{u}\cdot\mathbf{v}}{\lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert}$ $= \frac{(5)(\text{“14”}) + (-12)(\text{“6”})}{13 \times \text{“}2\sqrt{58}\text{”}}$ $\Rightarrow \alpha = \ldots$	**M1**	Uses the scalar product of $\mathbf{u}$ and their $\mathbf{v}$, of the form $a\mathbf{i} + b\mathbf{j}$, over their magnitudes; or takes the difference of their two direction angles.
	$\alpha = 90.6$	**A1**	awrt 90.6.
		[2]	
	Total for Question 1	**6**	

Question	Scheme	Marks	Notes
2(a)	Using impulse = change in momentum: $\mathbf{J} = m\mathbf{v} - m\mathbf{u}$ $\mathbf{J} = 2(8\mathbf{i} + 15\mathbf{j}) - 2(9\mathbf{i} + 12\mathbf{j})$	**M1**	Uses $m\mathbf{v} - m\mathbf{u}$ with mass 2 and each velocity along its given direction. Condone a scaling slip and sign slips.
	$\mathbf{J} = -2\mathbf{i} + 6\mathbf{j}$	**A1**	
	$\lvert\mathbf{J}\rvert = \sqrt{(\text{“}-2\text{”})^2 + \text{“}6\text{”}^2}$	**M1**	Uses Pythagoras on both components of their two-component $\mathbf{J}$.
	$\lvert\mathbf{J}\rvert = 2\sqrt{10}\ [\approx 6.32]\,\text{N s}$	**A1**	$2\sqrt{10}$, $\sqrt{40}$ or awrt 6.32.
		[4]	
2(b)	Using the scalar product: $\cos\alpha = \frac{\mathbf{J}\cdot\mathbf{u}}{\lvert\mathbf{J}\rvert\lvert\mathbf{u}\rvert}$ $= \frac{(\text{“}-2\text{”})(9)+(\text{“}6\text{”})(12)}{(\text{“}2\sqrt{10}\text{”})(15)}$	**M1**	Divides the scalar product of their $\mathbf{J}$ and a vector along $3\mathbf{i} + 4\mathbf{j}$ by both magnitudes, or finds the difference of the two vectors' angles with $\mathbf{i}$.
	$\alpha = 55.3$	**A1**	awrt 55.3.
		[2]	
	Total for Question 2	**6**	

Question	Scheme	Marks	Notes
3	$\mathbf{v} = \begin{pmatrix} -9\sin 30° \\ 9\cos 30° \end{pmatrix} = \begin{pmatrix} -\frac{9}{2} \\ \frac{9\sqrt{3}}{2} \end{pmatrix}$	**B1**	Both components, oe such as -4.5 and awrt 7.79, or momenta 6 and 3.6, or velocities 15 and 9, at 120° for the cosine rule.
	Using impulse = change in momentum: $\mathbf{J} = m\mathbf{v} - m\mathbf{u}$ $\mathbf{J} = 0.4\text{“}\begin{pmatrix} -\frac{9}{2} \\ \frac{9\sqrt{3}}{2} \end{pmatrix}\text{”} - 0.4\begin{pmatrix} 15 \\ 0 \end{pmatrix} = \ldots$	**M1**	Forms $m\mathbf{v} - m\mathbf{u}$ in components with $m = 0.4$ and their $\mathbf{v}$ of speed 9, or the cosine rule on the momenta or the velocities. Condone sign slips.
	$\mathbf{J} = \begin{pmatrix} -\frac{39}{5} \\ \frac{9\sqrt{3}}{5} \end{pmatrix}$	**A1**	oe, such as -7.8 and awrt 3.12, or $J^2 = 70.56$; or for $\mathbf{v} - \mathbf{u}$, -19.5 and awrt 7.79, or $\lvert\mathbf{v} - \mathbf{u}\rvert^2 = 441$.
	$J = \sqrt{\text{“}\left(-\frac{39}{5}\right)\text{”}^2 + \text{“}\left(\frac{9\sqrt{3}}{5}\right)\text{”}^2} = \ldots$	**M1**	Applies Pythagoras to both components of their impulse (or $\mathbf{v} - \mathbf{u}$), or takes the square root from the cosine rule, multiplying by 0.4 where needed.
	$J = 8.4$	**A1**	Or $\frac{42}{5}$.
	Total for Question 3	**5**	

Question	Scheme	Marks	Notes
4(a)	$\mathbf{u} = 2(4\mathbf{i} - 3\mathbf{j})$ $[= 8\mathbf{i} - 6\mathbf{j}]$ $\mathbf{I} = 3\mathbf{i} + 4\mathbf{j}$ Using impulse = change in momentum: $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ $3\mathbf{i} + 4\mathbf{j} = 0.5\mathbf{v} - 0.5(8\mathbf{i} - 6\mathbf{j})$ $\Rightarrow \mathbf{v} = \ldots$	**M1**	Uses $m\mathbf{v} - m\mathbf{u}$ with mass 0.5, and $\mathbf{u}$ and $\mathbf{I}$ each along its given direction. Condone a scaling slip and sign slips.
	$\mathbf{v} = 14\mathbf{i} + 2\mathbf{j}$	**A1**	
	$\lvert\mathbf{v}\rvert = \sqrt{\text{“14”}^2 + \text{“2”}^2}$	**M1**	Finds the magnitude of their $\mathbf{v}$, a vector of the form $a\mathbf{i} + b\mathbf{j}$.
	$\lvert\mathbf{v}\rvert = 10\sqrt{2}$ $[\approx 14.1]\,\text{m s}^{-1}$	**A1**	$10\sqrt{2}$, $\sqrt{200}$ or awrt 14.1.
		[4]	
4(b)	Using the scalar product: $\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert}$ $= \frac{(8)(\text{“14”}) + (-6)(\text{“2”})}{10 \times \text{“}10\sqrt{2}\text{”}} \Rightarrow \theta = \ldots$	**M1**	Uses the scalar product of their $\mathbf{u}$ (or $4\mathbf{i} - 3\mathbf{j}$) and their $\mathbf{v}$ over the product of their magnitudes, or adds the angles each makes with $\mathbf{i}$.
	$\theta = 45$	**A1**	
		[2]	
Total for Question 4		**6**	

Question	Scheme	Marks	Notes
5(a)	$\mathbf{a} = \frac{8\mathbf{i}-6\mathbf{j}}{0.8} = 10\mathbf{i} - 7.5\mathbf{j}$	**B1**	Or the impulse $\mathbf{F}t = 3.2\mathbf{i} - 2.4\mathbf{j}$ on an impulse-momentum route, or the change in velocity $4\mathbf{i} - 3\mathbf{j}$; soi.
	$\mathbf{u} = 3\mathbf{i} + 4\mathbf{j}$ $\mathbf{v} = (3\mathbf{i} + 4\mathbf{j}) + 0.4(\text{“}10\mathbf{i} - 7.5\mathbf{j}\text{”})$	**M1**	Uses $\mathbf{v} = \mathbf{u} + \mathbf{a}t$, or impulse = change in momentum, with an initial velocity vector in the direction $3\mathbf{i} + 4\mathbf{j}$. Condone a scaling slip and sign slips.
	$\mathbf{v} = 7\mathbf{i} + \mathbf{j}$	**A1**	
	$\lvert\mathbf{v}\rvert = \sqrt{\text{“}7\text{”}^2 + \text{“}1\text{”}^2}$	**M1**	Finds the magnitude of their two-component final velocity by Pythagoras.
	$\lvert\mathbf{v}\rvert = 5\sqrt{2}\ [\approx 7.07]\,\text{m}\,\text{s}^{-1}$	**A1**	$5\sqrt{2}$, $\sqrt{50}$ or awrt 7.07.
		[5]	
5(b)	Using the scalar product: $\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert} = \frac{3(\text{“}7\text{”})+4(\text{“}1\text{”})}{5\,(\text{“}5\sqrt{2}\text{”})}$	**M1**	Divides the scalar product of the initial velocity and their final velocity by the product of their magnitudes, or subtracts the two directions' angles with $\mathbf{i}$.
	$\theta = 45°$	**A1**	Or $\frac{\pi}{4}$, awrt 0.785.
		[2]	
	Total for Question 5	**7**	

Question	Scheme	Marks	Notes
6	$\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$ Using impulse = change in momentum: $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ $\mathbf{I} = \frac{4}{5}\left[(3\mathbf{i} + 4\mathbf{j}) - \lambda(\mathbf{i} + 2\mathbf{j})\right]$	**M1**	Uses $m\mathbf{v} - m\mathbf{u}$ with $\mathbf{u} = \lambda(\mathbf{i} + 2\mathbf{j})$ and $\mathbf{v}$ a multiple of $3\mathbf{i} + 4\mathbf{j}$. Either order of subtraction; condone a slip in the mass.
	$\mathbf{I} = \frac{4}{5}\left[(3 - \lambda)\mathbf{i} + (4 - 2\lambda)\mathbf{j}\right]$	**A1**	oe, either sign.
	$\left(\frac{4}{5}\right)^2\left[(3 - \lambda)^2 + (4 - 2\lambda)^2\right]$ $= \left(\frac{8\sqrt{2}}{5}\right)^2$	**M1**	Equates the magnitude of their impulse, using both components, to $\frac{8\sqrt{2}}{5}$, or its square. Condone a slip with the factor $\frac{4}{5}$.
	$5\lambda^2 - 22\lambda + 17 = 0$	**A1**	Correct three-term quadratic, oe.
	$(5\lambda - 17)(\lambda - 1) = 0 \Rightarrow \lambda = \ldots$	**dM1**	Solves their three-term quadratic by factorising with the product of the constants equal to the constant term, the formula or completing the square. Calculator allowed.
	$\lambda = \frac{17}{5}$ or $\lambda = 1$	**A1**	
	Total for Question 6	**6**	

Question	Scheme	Marks	Notes
7	Using impulse = change in momentum: $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ $\lambda(2\mathbf{i} + \mathbf{j} + 2\mathbf{k}) = \mathbf{v} - (3\mathbf{i} + \mathbf{j} + \mathbf{k})$ $\Rightarrow \mathbf{v} = \ldots$	**M1**	Uses impulse as final minus initial momentum, with mass 1, to find $\mathbf{v}$ in terms of λ. Condone sign slips.
	$\mathbf{v} = (3 + 2\lambda)\mathbf{i} + (1 + \lambda)\mathbf{j} + (1 + 2\lambda)\mathbf{k}$	**A1**	
	$\frac{1}{2}(1)\lvert\mathbf{v}\rvert^2 = 19$ $\Rightarrow (\text{“}3 + 2\lambda\text{”})^2 + (\text{“}1 + \lambda\text{”})^2 + (\text{“}1 + 2\lambda\text{”})^2 = 38$	**M1**	Sets $\frac{1}{2}(1)$ times the sum of the squares of all three of their components of $\mathbf{v}$, each in terms of λ, equal to 19. Condone a missing $\frac{1}{2}$.
	$9\lambda^2 + 18\lambda - 27 = 0$	**A1**	Correct three-term quadratic, oe.
	$\lambda^2 + 2\lambda - 3 = 0 \Rightarrow (\lambda + 3)(\lambda - 1) = 0$ $\Rightarrow \lambda = \ldots$	**dM1**	Solves their three-term quadratic by factorising with the product of the constants equal to the constant term, the formula or completing the square; calculator allowed.
	$\lambda = 1$ or $\lambda = -3$	**A1**	Both values, and no others.
	Total for Question 7	**6**	

Question	Scheme	Marks	Notes
8	Initial KE $= \frac{1}{2}(1)(1^2+3^2+(-1)^2) = \frac{11}{2}$	**B1**	Accept 5.5.
	$\frac{1}{2}(1)\lvert\mathbf{v}\rvert^2 =$ "$\frac{11}{2}$" $+ 6.5 \Rightarrow \lvert\mathbf{v}\rvert^2 = \ldots$	**M1**	Equates $\frac{1}{2}(1)\lvert\mathbf{v}\rvert^2$, or $\frac{1}{2}(1)(6\lambda^2)$, to their initial KE plus 6.5. Condone a missing $\frac{1}{2}$ on one side only.
	$\lvert\mathbf{v}\rvert^2 = 24$	**A1**	soi, such as from $6\lambda^2 = 24$.
	$\mathbf{v} = \lambda(\mathbf{i}+2\mathbf{j}+\mathbf{k})$ $\Rightarrow 6\lambda^2 =$ "24" $\Rightarrow \lambda = \ldots$	**M1**	Writes $\mathbf{v}$ as a multiple of $\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and equates its squared magnitude to their $\lvert\mathbf{v}\rvert^2$, or uses a unit vector scaled by their speed.
	$\lambda = 2 \Rightarrow \mathbf{v} = 2\mathbf{i}+4\mathbf{j}+2\mathbf{k}$	**A1**	Not if $\lambda = -2$ is used, alone or with $\lambda = 2$.
	Using impulse = change in momentum: $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ $=$ "$(2\mathbf{i}+4\mathbf{j}+2\mathbf{k})$" $-(\mathbf{i}+3\mathbf{j}-\mathbf{k})$	**M1**	Subtracts the initial velocity from their final velocity, in either order, with the mass of 1 used or implied.
	$\mathbf{I} = (\mathbf{i}+\mathbf{j}+3\mathbf{k})\,\mathrm{N\,s}$	**A1**	cao. Accept column vector form.
	Total for Question 8	**7**	