FURTHER MECHANICS / MOMENTUM & COLLISIONS

Oblique Collisions of Smooth Spheres

Mark scheme 10 questions · 155 marks

EXAM BOARDS Edexcel FM1 9FM0/3C · OCR A Y543 · OCR MEI Mech Major Y421 · CIE 9231 Paper 3

Questions

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	Total	**155**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last mark of a part whose answer is printed in the question: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	**j**-component of A unchanged: $-2u$	**B1**	soi, such as by its use in the scalar product.
	$u(4\mathbf{i}-2\mathbf{j})\cdot(v\mathbf{i}-\text{“}2u\text{”}\mathbf{j})=0$ $\Rightarrow 4uv+4u^2=0\Rightarrow v=\ldots$	**M1**	Sets the scalar product of the velocities of A before and after to zero, or the product of their gradients to -1, with their **j**-component, and solves. Condone sign slips.
	$\mathbf{v}_A=u(-\mathbf{i}-2\mathbf{j})$	**A1**	oe, such as speed $\sqrt{5}u$ with a correct direction.
		[3]	
1(b)	$2m(4u)+3m(-u)=2m(\text{“}-u\text{”})+3mv_B$ $\Rightarrow v_B=\ldots$	**M1**	Forms momentum conservation along the line of centres with all four terms, using their **i**-component of $\mathbf{v}_A$ from (a). Condone sign slips.
	$v_B=\frac{7u}{3}$	**A1**	oe, such as $\mathbf{v}_B=u\left(\frac{7}{3}\mathbf{i}-3\mathbf{j}\right)$.
	$e=\dfrac{\text{“}\frac{7u}{3}\text{”}+\text{“}u\text{”}}{4u+u}=\ldots$	**M1**	Uses speed of separation over speed of approach with their **i**-components only. Condone sign slips; not for whole speeds or **j**-components.
	$e=\frac{2}{3}$	**A1**	
	$KE_{\text{before}}=\frac{1}{2}(2m)\left((4u)^2+(2u)^2\right)$ $\quad+\frac{1}{2}(3m)\left(u^2+(3u)^2\right)$ $KE_{\text{after}}=\frac{1}{2}(2m)\left(u^2+(2u)^2\right)$ $\quad+\frac{1}{2}(3m)\left(\text{“}\frac{7u}{3}\text{”}^2+(3u)^2\right)$	**M1**	Finds KE_{before} and KE_{after}, or the loss of KE, from both components of each velocity, or the loss from the **i**-components alone, with their v_B. Condone arithmetic slips.
	$KE_{\text{before}}=35mu^2$, $KE_{\text{after}}=\frac{80}{3}mu^2$	**A1**	Both correct, or $35mu^2$ and loss of KE $\frac{25}{3}mu^2$.
	$\dfrac{35mu^2-\frac{80}{3}mu^2}{35mu^2}\times 100\%=23.8\%$	**A1**	awrt 23.8%.
		[7]	
Total for Question 1		**10**	

Question	Scheme	Marks	Notes
2	A: parallel 4, perpendicular 3 B: parallel 2, perpendicular 2 Perpendicular components after: 3 and 2	**B1**	Both velocities resolved, and perpendicular components 3 and 2 stated or used after the collision.
	$2(4) - 4(2) = -2v_A + 4v_B$	**M1 A1**	**M1** Uses conservation of momentum along the line of centres with their parallel components, four terms, masses paired correctly. Condone sign slips. **A1** Correct for their directions after the collision.
	$v_A + v_B = e(4 + 2)$	**M1 A1**	**M1** Uses speed of separation equal to e times speed of approach along the line of centres, with their components. Condone sign slips. **A1** Correct for their directions after the collision.
	$\Rightarrow v_A = \ldots\ [= 4e],\ v_B = \ldots\ [= 2e]$	**dM1**	Solves their momentum and restitution equations for both speeds in terms of e.
	$(-\text{“}4e\text{”})(\text{“}2e\text{”}) + 3 \times 2 = 0 \Rightarrow e = \ldots$	**M1**	Uses perpendicularity with their components after the collision in terms of e, by a zero scalar product or gradients with product -1, oe.
	$e = \frac{\sqrt{3}}{2}$	**A1**	A0 if the negative root is also given as an answer.
	$I = 2\left(\text{“}2\sqrt{3}\text{”} - (-4)\right)$	**M1**	Uses mass times change of A's velocity along the line of centres with their v_A, or B's change of momentum. Condone sign slips.
	$I = 8 + 4\sqrt{3}$ (N s)	**A1**	Exact, oe such as $4(2 + \sqrt{3})$.
	Total for Question 2	**10**	

Question	Scheme	Marks	Notes
3(a)	Perpendicular components unchanged: $3\mathbf{j}$ for A, $-2\mathbf{j}$ for B	**B1**	May be implied by their use in the scalar product.
	$3(4)+1(-3)=3v_A+v_B$	**M1 A1**	**M1** Forms a momentum equation along the line of centres with all four terms and correct masses. Condone sign slips. **A1** Correct equation, oe.
	$v_B-v_A=e(4+3)$	**M1 A1**	**M1** Uses restitution along the line of centres with e multiplying the speed of approach. Condone sign slips. **A1** Correct equation, with signs consistent with their momentum equation.
	$\Rightarrow v_A=\ldots\ \left[=\frac{9-7e}{4}\right]$, $v_B=\ldots\ \left[=\frac{9+21e}{4}\right]$	**dM1**	Solves their momentum and restitution equations for both v_A and v_B in terms of e.
	$(v_A\mathbf{i}+3\mathbf{j})\cdot(v_B\mathbf{i}-2\mathbf{j})=0$ $\Rightarrow$ "$\frac{9-7e}{4}$" $\times$ "$\frac{9+21e}{4}$" $-6=0$ $\Rightarrow 49e^2-42e+5=0$	**M1**	Sets the scalar product of their velocities after impact to zero, or product of gradients -1, with their perpendicular components, reaching a quadratic in e.
	$e=\frac{1}{7}$ or $e=\frac{5}{7}$	**A1**	Both values, or awrt 0.143 and 0.714.
		[8]	
3(b)	$KE_{\text{before}}=\frac{1}{2}(3)(4^2+3^2)+\frac{1}{2}(1)(3^2+2^2)=44$	**B1**	44, or 28.5 from the parallel components where the loss is found that way; implied by a correct loss of KE for either e.
	$e=\frac{1}{7}$: $KE_{\text{after}}=\frac{1}{2}(3)(\text{"2"}^2+3^2)$ $+\frac{1}{2}(1)(\text{"3"}^2+2^2)$ $[=26]$	**M1**	Finds KE_{after} for at least one e, with their parallel speeds from (a) and perpendicular components 3 and 2, or finds the loss from parallel components alone.
	loss of KE $=44-26=18\,\text{J}$	**A1**	Must be identified with $e=\frac{1}{7}$.
	$e=\frac{5}{7}$: $KE_{\text{after}}=35\Rightarrow$ loss of KE $=9\,\text{J}$	**A1**	Must be identified with $e=\frac{5}{7}$.
		[4]	
	Total for Question 3	**12**	

Question	Scheme	Marks	Notes
4(a)	Perpendicular components unchanged: $2u$ for A, $-2u$ for B (along $\mathbf{j}$)	**B1**	Both, soi by the **j**-components of the final velocities.
	$2m(4u) + m(-2u) = 2mv_A + mv_B$	**M1 A1**	**M1** Applies conservation of momentum along **i** with all four terms, each mass with its own sphere's **i**-component. Condone sign slips. **A1** Correct equation, oe, with any consistent directions for the speeds after.
	$v_B - v_A = e(4u + 2u)$	**M1 A1**	**M1** Uses restitution with **i**-components only and e multiplying the speed of approach. Condone sign slips. **A1** Correct equation, oe, with directions consistent with their momentum equation.
	$\Rightarrow v_A = \ldots,\ v_B = \ldots$	**dM1**	Solves their two equations for both speeds along **i** in terms of u and e.
	$\mathbf{v}_A = 2u(1-e)\mathbf{i} + 2u\mathbf{j}$	**A1**	oe, as a vector or column vector, here and for $\mathbf{v}_B$.
	$\mathbf{v}_B = 2u(1+2e)\mathbf{i} - 2u\mathbf{j}$	**A1**	
		[8]	
4(b)	"$2u(1-e)$" $\times$ "$2u(1+2e)$" $+ (2u)(-2u) = 0$	**M1**	Sets the scalar product of their velocities from (a), with **i**-components in terms of e, to zero; or product of gradients -1.
	$e(1-2e) = 0 \Rightarrow e = 0$ or $e = \frac{1}{2}$	**A1**	Both values.
	$KE_{\text{before}} = \frac{1}{2}(2m)(16u^2 + 4u^2)$ $\quad + \frac{1}{2}m(4u^2 + 4u^2)$ $\quad = 24mu^2$	**B1**	$24mu^2$, or $18mu^2$ from the parallel components where the loss is found that way; implied by a correct loss of KE for either e.
	$e = 0$: $KE_{\text{after}} = \frac{1}{2}(2m)(4u^2 + 4u^2)$ $\quad + \frac{1}{2}m(4u^2 + 4u^2)$	**M1**	Finds KE_{after} for one of their values of e, squaring both components of each velocity from their (a), or finds the loss of KE from the parallel components alone.
	loss of KE $= 24mu^2 - 12mu^2 = 12mu^2$	**A1**	Each loss must be matched to its value of e.

Question 4 continued

Question	Scheme	Marks	Notes
	$e = \frac{1}{2}$: $KE_{\text{after}} = \frac{1}{2}(2m)(u^2 + 4u^2)$ $+ \frac{1}{2}m(16u^2 + 4u^2)$ $\Rightarrow$ loss of KE $= 24mu^2 - 15mu^2 = 9mu^2$	**A1**	
		[6]	
	Total for Question 4	**14**	

Question	Scheme	Marks	Notes
5(a)	Perpendicular components unchanged: A: $10\sin\alpha = 6$, B: $6\sqrt{2}\sin\frac{\pi}{4} = 6$	**B1**	Needs at least B's perpendicular component, 6, kept after impact; may be implied by $\tan\beta = \frac{6}{v_B}$.
	$\tan\beta = \frac{6}{v_B} = 2 \Rightarrow v_B = 3$	**B1**	
	$8m_A - 6m_B = -m_A v_A + m_B v_B$	**M1 A1**	**M1** Forms momentum along the line of centres with four terms, using the parallel components $10\cos\alpha$ and $6\sqrt{2}\cos\frac{\pi}{4}$. Any consistent directions; condone sign slips. **A1** Correct equation, oe, with v_B or 3.
	$v_A + v_B = 14e$	**M1 A1**	**M1** Uses Newton's law along the line of centres, with e multiplying the approach speed from the parallel components. Condone sign slips. **A1** Correct equation, oe, consistent with their momentum directions.
	$8m_A - 6m_B = -m_A(14e - \text{“3”}) + \text{“3”}m_B$ $\Rightarrow \frac{m_B}{m_A} = \ldots$	**dM1**	Eliminates v_A between their two equations, with their numerical v_B, and rearranges to $\frac{m_B}{m_A}$ in terms of e, or e in terms of $\frac{m_B}{m_A}$.
	$\frac{m_B}{m_A} = \frac{5+14e}{9}$	**A1**	oe, such as $e = \frac{9k-5}{14}$ with $k = \frac{m_B}{m_A}$.
	$e = 0$ and $e = 1 \Rightarrow \frac{m_B}{m_A} = \ldots$	**M1**	Substitutes $e = 0$ and $e = 1$ into their relation of the form $\frac{m_B}{m_A} = p + qe$, or applies $0 \le e \le 1$ to it rearranged for e.
	$\frac{5}{9} \le \frac{m_B}{m_A} \le \frac{19}{9}$	**A1**	Inclusive inequalities, oe such as $\left[\frac{5}{9}, \frac{19}{9}\right]$, or awrt 0.556 and 2.11. A0 for strict inequalities.
	$\frac{19}{9}$ (upper endpoint): $e = 1$, perfectly elastic	**E1**	Identifies the upper endpoint, $\frac{19}{9}$, as the perfectly elastic case.
		[11]	
5(b)	$\frac{4}{3} = \frac{5+14e}{9} \Rightarrow e = \ldots$	**M1**	Substitutes $m_A = 3$, $m_B = 4$ into their relation from (a), or their momentum and restitution equations, and solves for e; or uses momentum with them and their v_B to find v_A.
	$e = \frac{1}{2}$	**A1**	Or a correct equation in v_A on the momentum route.

Question 5 continued

Question	Scheme	Marks	Notes
	$v_A = 14\left(\frac{1}{2}\right) - 3 = 4$	**A1**	
	loss of KE $= \frac{1}{2}(3)\left(8^2 - \text{“}4\text{”}^2\right)$ $+ \frac{1}{2}(4)\left(6^2 - 3^2\right)$	**M1**	Finds $KE_{\text{before}} - KE_{\text{after}}$ for both spheres with their numerical v_A, using full speeds or parallel components throughout. Condone one slip.
	$= 126\,\text{J}$	**A1**	
		[5]	
	Total for Question 5	**16**	

Question	Scheme	Marks	Notes
6(a)	Resolves along $3\mathbf{i}+4\mathbf{j}$ and $-4\mathbf{i}+3\mathbf{j}$: P: $\frac{0.8(3)+4.4(4)}{5}$, $\frac{0.8(-4)+4.4(3)}{5}$ Q: $\frac{1.2(3)-3.4(4)}{5}$, $\frac{1.2(-4)-3.4(3)}{5}$	**M1**	Resolves at least one velocity along and perpendicular to the line of centres, by scalar products with unit vectors or by angles. Condone sign slips or a missing $\frac{1}{5}$.
	P: 4 along, 2 perpendicular Q: -2 along, -3 perpendicular	**A1**	All four correct, oe with signs reversed for their chosen positive directions.
	Perpendicular components unchanged: P 2, Q -3	**B1**	Stated, or used for P in the perpendicularity condition.
	$0.4(4)+0.5(-2)=-0.4v_P+0.5v_Q$	**M1 A1**	**M1** Equates momentum along the line of centres before and after, with all four terms, each mass with its own sphere's components. Condone sign slips. **A1** Correct equation oe, for any consistent choice of directions.
	$4(-v_P)+2(2)=0$	**M1**	Sets the scalar product of P's velocities before and after to zero, or an equivalent perpendicularity condition, keeping P's perpendicular component. Condone sign slips.
	$\Rightarrow v_P=1$	**A1**	oe for their direction of v_P, such as -1 with P's original direction positive.
	$0.6=-0.4+0.5v_Q \Rightarrow v_Q=2$	**A1**	
	$e=\frac{\text{“1”}+\text{“2”}}{4+2}$	**M1**	Uses speed of separation over speed of approach along the line of centres, with their speeds after and the components before. Condone sign slips.
	$e=\frac{1}{2}$	**A1**	
		[10]	
6(b)	$\mathbf{v}_Q=\frac{\text{“2”}}{5}(3\mathbf{i}+4\mathbf{j})+\frac{3}{5}(4\mathbf{i}-3\mathbf{j})$	**M1**	Adds their v_Q times a unit vector along the line of centres and Q's perpendicular component times a perpendicular unit vector. Condone sign slips.
	$=(3.6\mathbf{i}-0.2\mathbf{j})\,\text{m s}^{-1}$	**A1**	

Question 6 continued

Question	Scheme	Marks	Notes
	$\cos\theta = \frac{1.2(\text{“}3.6\text{”})+(-3.4)(\text{“}-0.2\text{”})}{\sqrt{13}\sqrt{\text{“}13\text{”}}}$	**M1**	Uses $\mathbf{u}_Q \cdot \mathbf{v}_Q$ over $\lvert\mathbf{u}_Q\rvert\lvert\mathbf{v}_Q\rvert$ with their $\mathbf{v}_Q$ in **i**, **j** form, or the difference of the two directions found by trigonometry.
	$\cos\theta = \frac{5}{13} \Rightarrow \theta = 67.4°$	**A1**	awrt 67.4°.
		[4]	
6(c)	The line of centres, and so the impulse, is horizontal	**E1**	States that the line of centres, or the impulse, is horizontal or in the plane of **i** and **j**. Not for restating equal radii alone.
		[1]	
	Total for Question 6	**15**	

Question	Scheme	Marks	Notes
7(a)	Resolves along $\frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j})$ and $\frac{1}{\sqrt{2}}(-\mathbf{i}+\mathbf{j})$: P: $\frac{6+2}{\sqrt{2}}$, $\frac{-6+2}{\sqrt{2}}$; Q: $\frac{-2+2}{\sqrt{2}}$, $\frac{2+2}{\sqrt{2}}$	**M1**	Resolves at least one velocity along and perpendicular to the line of centres, by scalar product with unit vectors or by angles. Condone sign slips.
	P: $4\sqrt{2}$, $-2\sqrt{2}$; Q: 0, $2\sqrt{2}$	**A1**	All four, oe; perpendicular signs may follow their chosen direction.
	Smooth, so no impulse perpendicular to the line of centres: perpendicular components unchanged, $2\sqrt{2}$ each	**B1**	Needs both the statement and the reason that smooth spheres give no impulse (or force) perpendicular to the line of centres.
	$0.4(4\sqrt{2})+0.6(0)=0.4v_P+0.6v_Q$	**M1 A1**	**M1** Applies conservation of momentum along the line of centres with all terms, using their parallel components. Condone sign slips.
	$v_Q-v_P=e(4\sqrt{2}-0)$	**M1 A1**	**M1** Uses Newton's law along the line of centres with their parallel components, e on the approach side. Condone sign slips. **A1** Signs consistent with their momentum equation.
	$8\sqrt{2}=2v_P+3(v_P+4\sqrt{2}\,e)$ $\Rightarrow v_P=\ldots,\ v_Q=\ldots$	**dM1**	Eliminates one speed from their momentum and restitution equations and finds both speeds in terms of e, or solves momentum with the KE equation for both speeds.
	$v_P=\frac{4\sqrt{2}}{5}(2-3e)$, $v_Q=\frac{8\sqrt{2}}{5}(1+e)$	**A1**	Both, oe; or $\frac{2\sqrt{2}}{5}$ and $\frac{12\sqrt{2}}{5}$ on the KE route.
	$\frac{1}{2}(0.4)(4\sqrt{2})^2-\frac{1}{2}(0.4)v_P^2$ $-\frac{1}{2}(0.6)v_Q^2=2.88$	**M1 A1**	**M1** Equates the loss of KE to 2.88, using the parallel components throughout or the full speeds before and after. Condone sign slips.
	Substitutes their v_P, v_Q: $15e^2+10=\frac{55}{4}\Rightarrow e=\ldots$	**dM1**	Substitutes their speeds in terms of e into their KE equation and solves for e by any means, or finds e from restitution with their speeds, rejecting the root with $e<0$.
	$e=\frac{1}{2}$	**A1**	Any negative root rejected or not given.

Question 7 continued

Question	Scheme	Marks	Notes
	$\mathbf{v}_P = \text{“}\frac{2\sqrt{2}}{5}\text{”} \times \frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j})$ $+ 2\sqrt{2} \times \frac{1}{\sqrt{2}}(\mathbf{i}-\mathbf{j})$	**M1**	Combines their numerical parallel speed and the unchanged perpendicular component, using unit vectors or angles of 45°.
	$\mathbf{v}_P = (2.4\mathbf{i} - 1.6\mathbf{j})\,\text{m s}^{-1}$	**A1**	oe, including speed and direction.
	$\mathbf{v}_Q = (0.4\mathbf{i} + 4.4\mathbf{j})\,\text{m s}^{-1}$	**A1**	oe, including speed and direction.
		[16]	
7(b)	$\cos\theta = \frac{6(\text{“}2.4\text{”})+2(\text{“}-1.6\text{”})}{\sqrt{40}\sqrt{\text{“}2.4\text{”}^2+\text{“}1.6\text{”}^2}}$	**M1**	Uses the scalar product of $\mathbf{u}_P$ and their $\mathbf{v}_P$ over their magnitudes, or the difference of the two directions' angles to $\mathbf{i}$.
	$\theta = 52.1°$	**A1**	
		[2]	
	Total for Question 7	**18**	

Question	Scheme	Marks	Notes
8(a)	$\tan\theta = \frac{5r-4r}{\sqrt{3}r} \Rightarrow \theta = \ldots$ A: $\sqrt{3}U\cos 30°$, B: $U\sin 30°$	**M1**	Finds the angle from the separations r and $\sqrt{3}r$ (or $2r$) by any trigonometry, or resolves both velocities along the line of centres, which may use the printed 30°.
	$\theta = 30°$ $\frac{3U}{2} + \frac{U}{2} = 2U$	**A1 AG**	Correct ratio giving 30°, and components $\frac{3U}{2}$ and $\frac{U}{2}$ added to give $2U$, without error.
		[2]	
8(b)	Perpendicular components unchanged: $\frac{\sqrt{3}U}{2}$	**B1**	For B at least; implied by its use in resolving B's velocity.
	$2m\left(\frac{3U}{2}\right) - 3m\left(\frac{U}{2}\right) = 3mv_B - 2mv_A$	**M1 A1**	**M1** Applies conservation of momentum along the line of centres with four terms and correctly paired masses. Condone sign slips. **A1** Correct equation, oe for any consistent directions.
	$v_A + v_B = \frac{2}{3}(2U)$	**M1 A1**	**M1** Uses restitution along the line of centres, with e multiplying the speed of approach. Condone sign slips. **A1** Correct equation, with directions consistent with the momentum equation.
	$\Rightarrow v_B = \ldots$	**dM1**	Solves their momentum and restitution equations simultaneously for B's component along the line of centres.
	$v_B = \frac{5U}{6}$	**A1**	
	"$\frac{5U}{6}$" $\sin 30° -$ "$\frac{\sqrt{3}U}{2}$" $\sin 60°$	**M1**	Resolves vertically both their components of B's velocity, along and perpendicular to the line of centres, and combines them. Condone sign slips.
	$= -\frac{U}{3}$, so $\frac{U}{3}$ downwards	**A1**	
	$4r =$ "$\frac{U}{3}$"$t + \frac{1}{2}gt^2$	**M1**	Uses constant acceleration vertically with their vertical component, $\pm g$ and a vertical displacement, to form an equation in t.
	$3gt^2 + 2Ut - 24r = 0$	**A1**	Correct equation in t, with a fall of $4r$, oe.

Question 8 continued

Question	Scheme	Marks	Notes
	$t = \frac{-2U \pm \sqrt{(2U)^2+4(3g)(24r)}}{6g}$	**dM1**	Solves their three-term quadratic in t by the formula or completing the square.
	$t = \frac{\sqrt{U^2+72gr}-U}{3g}$	**A1**	Positive root only, any equivalent exact form.
		[13]	
	Total for Question 8	**15**	

Question	Scheme	Marks	Notes
9(a)	$\overrightarrow{PQ} = 7\mathbf{i} + 26\mathbf{j} + t\big(-5\mathbf{j} - (2\mathbf{i} + 6\mathbf{j})\big)$	**M1**	Forms the displacement between the centres at time t, by relative velocity or both position vectors. Condone sign slips.
	$= (7 - 2t)\mathbf{i} + (26 - 11t)\mathbf{j}$	**A1**	oe, in either direction.
	$(7 - 2t)^2 + (26 - 11t)^2 = 5^2$	**M1**	Sets the magnitude of their displacement, with components linear in t, equal to the sum of the radii.
	$5t^2 - 24t + 28 = 0$	**A1**	oe, such as $125t^2 - 600t + 700 = 0$.
	$(t - 2)(5t - 14) = 0 \Rightarrow t = 2,\ \frac{14}{5}$ Real roots, so the spheres collide First contact at $t = 2\,\text{s}$	**A1**	Needs real roots or a positive discriminant stated to imply collision, and $t = 2$ chosen as the smaller root.
		[5]	
9(b)	At $t = 2$: $\overrightarrow{PQ} = 3\mathbf{i} + 4\mathbf{j}$, the line of centres	**B1**	Any multiple of $3\mathbf{i} + 4\mathbf{j}$, or its angle with $\mathbf{i}$.
	Resolves along $\frac{1}{5}(3\mathbf{i} + 4\mathbf{j})$ and $\frac{1}{5}(-4\mathbf{i} + 3\mathbf{j})$: $P:\ \frac{1}{5}\big(2(3) + 6(4)\big),\ \frac{1}{5}\big(2(-4) + 6(3)\big)$ $Q:\ \frac{1}{5}\big(0(3) - 5(4)\big),\ \frac{1}{5}\big(0(-4) - 5(3)\big)$	**M1**	Resolves at least one velocity along and perpendicular to their line of centres, by scalar products or angles. Condone sign slips.
	P: parallel 6, perpendicular 2 Q: parallel -4, perpendicular -3	**A1**	All four, with signs consistent with their directions.
	Perpendicular components unchanged: P 2, Q -3	**B1**	Stated, or used in finding v_Q or $\mathbf{v}_P$.
	$3(6) + 2(-4) = 3v_P + 2v_Q$	**M1 A1**	**M1** Applies conservation of momentum along the line of centres, with four terms in their parallel components. Condone sign slips. **A1** oe.
	$v_Q - v_P = e(6 + 4)$	**M1 A1**	**M1** Uses restitution along the line of centres with their parallel components. Condone sign slips, not e on the wrong side. **A1** oe.
	$v_Q^2 + 3^2 = \big(3\sqrt{5}\big)^2 \Rightarrow v_Q = \ldots\ [= 6]$	**M1**	Combines the unchanged perpendicular component of Q, their 3, with the speed $3\sqrt{5}$ to find its parallel component.

Question 9 continued

Question	Scheme	Marks	Notes
	$v_Q = 2 + 6e = \text{“6”} \Rightarrow e = \ldots$	**dM1**	Uses their v_Q with their momentum and restitution equations to solve for e, in either order.
	$e = \frac{2}{3}$	**A1**	$\frac{2}{3}$ or awrt 0.667, with no other value.
	$v_P = \text{“6”} - 10\left(\text{“}\frac{2}{3}\text{”}\right)\ [= -\frac{2}{3}]$ $\mathbf{v}_P = \text{“}-\frac{2}{3}\text{”} \times \frac{1}{5}(3\mathbf{i} + 4\mathbf{j})$ $\quad + \frac{2}{5}(-4\mathbf{i} + 3\mathbf{j})$	**M1**	Combines their parallel component of P with its perpendicular component 2 using the two unit vectors, or uses vector momentum with their velocity of Q.
	$\mathbf{v}_P = \left(-2\mathbf{i} + \frac{2}{3}\mathbf{j}\right)\ \text{m s}^{-1}$	**A1**	
		[13]	
9(c)	$\mathbf{I} = 3\big(\mathbf{0} - \text{“}(-2\mathbf{i} + \frac{2}{3}\mathbf{j})\text{”}\big)$	**M1**	Uses impulse as change in momentum of P, with mass 3 and their vector velocity of P. Condone sign slips.
	$\mathbf{I} = (6\mathbf{i} - 2\mathbf{j})\ \text{N s}$	**A1**	
	$\tan\theta = \dfrac{\text{“2”}}{\text{“6”}} \Rightarrow \theta = \ldots$	**M1**	Uses the ratio of their components, or a scalar product with $\mathbf{i}$.
	$\theta = 18.4^\circ$ south of east	**A1**	awrt 18.4°, oe. A0 without south of east.
		[4]	
	Total for Question 9	**22**	

Question	Scheme	Marks	Notes
10(a)(i)	Smooth, so no impulse perpendicular to the line of centres; impulse acts along the line of centres	**E1**	Links smoothness to no perpendicular impulse or force. Not for "along the line of centres" alone.
		[1]	
10(a)(ii)	$\mathbf{v} \cdot \frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j})$ and $\mathbf{v} \cdot \frac{1}{\sqrt{2}}(-\mathbf{i}+\mathbf{j})$ for each sphere	**M1**	Resolves at least one velocity along and perpendicular to the line of centres, by scalar products or otherwise. Condone a missing $\frac{1}{\sqrt{2}}$.
	A: $\frac{5u}{\sqrt{2}}$, $-\frac{3u}{\sqrt{2}}$; B: $\frac{u}{\sqrt{2}}$, $\frac{3u}{\sqrt{2}}$	**A1**	All four, oe such as multiples of $\mathbf{i}+\mathbf{j}$ and $-\mathbf{i}+\mathbf{j}$.
	Perpendicular components unchanged: $-\frac{3u}{\sqrt{2}}$, $\frac{3u}{\sqrt{2}}$	**B1**	Stated, or used in their final velocities.
	$m\left(\frac{5u}{\sqrt{2}}\right) + \lambda m\left(\frac{u}{\sqrt{2}}\right) = -mv_A + \lambda m v_B$	**M1 A1**	**M1** Equates momentum along the line of centres before and after, using their parallel components. Condone sign slips.
	$v_A + v_B = e\left(\frac{5u}{\sqrt{2}} - \frac{u}{\sqrt{2}}\right)$	**M1 A1**	**M1** Uses Newton's law along the line of centres with their parallel components, e multiplying the speed of approach. Condone sign slips.
	$-v_A + \lambda\left(2\sqrt{2}\,eu - v_A\right) = \frac{(5+\lambda)u}{\sqrt{2}}$ $\Rightarrow v_A = \ldots,\ v_B = \ldots$	**dM1**	Eliminates one speed between their two equations to find the other in terms of u, λ and e. Condone algebraic slips.
	$v_A = \frac{u(4\lambda e-\lambda-5)}{\sqrt{2}(1+\lambda)}$	**A1**	oe, with sign consistent with their chosen direction.
	$v_B = \frac{u(5+\lambda+4e)}{\sqrt{2}(1+\lambda)}$	**A1**	oe, with sign consistent with their chosen direction.
	$\mathbf{v}_A = -\frac{\text{"}v_A\text{"}}{\sqrt{2}}(\mathbf{i}+\mathbf{j}) + \frac{3u}{2}(\mathbf{i}-\mathbf{j})$	**M1**	Combines their parallel speed and the perpendicular component with unit vectors along and perpendicular to the line of centres, for at least one sphere. Condone sign slips.
	$\mathbf{v}_A = \frac{u}{1+\lambda}\big[(4+2\lambda-2\lambda e)\mathbf{i} + (1-\lambda-2\lambda e)\mathbf{j}\big]$	**A1**	oe in terms of $\mathbf{i}$ and $\mathbf{j}$.
	$\mathbf{v}_B = \frac{u}{1+\lambda}\big[(1-\lambda+2e)\mathbf{i} + (4+2\lambda+2e)\mathbf{j}\big]$	**A1**	oe in terms of $\mathbf{i}$ and $\mathbf{j}$.
		[13]	

Question 10 continued

Question	Scheme	Marks	Notes
10(b)	loss of KE $= \frac{1}{2}m\left(\frac{5u}{\sqrt{2}}\right)^2 + \frac{1}{2}\lambda m\left(\frac{u}{\sqrt{2}}\right)^2$ $- \frac{1}{2}m\text{“}v_A\text{”}^2 - \frac{1}{2}\lambda m\text{“}v_B\text{”}^2$	**M1 A1**	**M1** Forms $KE_{\text{before}} - KE_{\text{after}}$ from their parallel components, or from their full velocities from (a)(ii). Condone sign slips.
	loss of KE $= \frac{mu^2}{4(1+\lambda)}\left[(25+\lambda)(1+\lambda)\right.$ $\left.-(5+\lambda)^2 - 16\lambda e^2\right]$	**dM1**	Substitutes their v_A and v_B, expands and collects over a common denominator. Condone algebraic slips.
	loss of KE $= \frac{4\lambda mu^2(1-e^2)}{1+\lambda}$	**A1**	oe.
	$\frac{4\lambda mu^2(1-e^2)}{1+\lambda} = mu^2 \Rightarrow e^2 = \frac{3\lambda-1}{4\lambda}$	**A1**	oe.
	$-\text{“}v_A\text{”} < 0 \Rightarrow 4\lambda e - \lambda - 5 > 0$	**M1**	Sets their component of A's velocity along $\mathbf{i}+\mathbf{j}$ negative, from v_A or $\mathbf{v}_A \cdot (\mathbf{i}+\mathbf{j})$, giving an inequality in e and λ.
	$e > \frac{\lambda+5}{4\lambda}$	**A1**	oe.
	$\text{“}\frac{3\lambda-1}{4\lambda}\text{”} > \frac{(\lambda+5)^2}{16\lambda^2}$ $\Rightarrow 11\lambda^2 - 14\lambda - 25 > 0$	**dM1**	Eliminates e using their e^2, such as by squaring their inequality, to reach an inequality in λ alone. Squaring need not be justified.
	$(\lambda+1)(11\lambda-25) > 0 \Rightarrow \lambda > \frac{25}{11}$	**A1**	Strict inequality, $\lambda > \frac{25}{11}$ or awrt 2.27, with $\lambda < -1$ rejected or not given.
		[9]	
	Total for Question 10	**23**	