FURTHER MECHANICS / MOMENTUM & COLLISIONS

Collisions in 2 Dimensions

Mark scheme 12 questions · 126 marks

EXAM BOARDS Edexcel FM1 9FM0/3C · AQA 7367/3M · OCR A Y543 · OCR MEI Mech Major Y421 CIE 9231 Paper 3

Questions

Question	Marks by part	Marks	Page
Question 1	(a) 3 (b) 3	6	4
Question 2	(a) 5 (b) 4 (c) 2	11	5
Question 3		5	6
Question 4	(a) 4 (b) 5	9	7
Question 5	(a) 6 (b) 4	10	8
Question 6	(a) 8 (b) 1	9	9
Question 7	(a) 5 (b) 6	11	10
Question 8	(a) 2 (b) 4 (c) 2 (d) 2 (e) 1	11	11
Question 9	(a) 7 (b) 4	11	13
Question 10	(a) 5 (b) 3 (c) 2 (d) 4	14	14
Question 11	(a) 4 (b) 4 (c) 6	14	16
Question 12	(a) 4 (b) 4 (c) 6 (d) 1	15	18
	Total	**126**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last mark of a part whose answer is printed in the question: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [***n***] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	$\mathbf{v} = 10 \times \frac{3\mathbf{i}+4\mathbf{j}}{5} = 6\mathbf{i} + 8\mathbf{j}$	**B1**	Or the **j**-component 8 alone; soi.
	$e = \frac{\text{“8”}}{12}$	**M1**	Divides their **j**-component after impact, from a velocity along $3\mathbf{i} + 4\mathbf{j}$, by 12. Condone sign slips. Not for a ratio of speeds.
	$e = \frac{2}{3}$	**A1**	Exact, or awrt 0.667.
		[3]	
1(b)	Compares components parallel to the side-board: before 9, after “6”	**M1 A1ft**	**M1** Identifies the **i**-components as parallel to the board and compares them before and after impact. **A1ft** 9 and 6, ft their **i**-component from (a).
	$9 \neq 6$, so there is an impulse parallel to the board $\Rightarrow$ the side-board is not smooth	**E1**	Concludes not smooth because the parallel component changes, so the board exerts an impulse along it. Needs differing components; not for “not smooth” alone.
		[3]	
	Total for Question 1	**6**	

Question	Scheme	Marks	Notes
2(a)	Parallel to BC: $10\cos\alpha = \ldots$ Perpendicular to BC: $10\sin\alpha = \ldots$	**M1**	Resolves the velocity parallel and perpendicular to BC using $\tan\alpha = \frac{3}{4}$. Condone sin and cos interchanged.
	8 parallel, 6 perpendicular	**A1**	
	After impact: parallel "8", perpendicular "6"e	**M1**	Multiplies their perpendicular component by e and keeps their parallel component, from resolving the $10\,\mathrm{m\,s^{-1}}$; or uses $\tan\beta = e\tan\alpha$, implying the M1 A1 for resolving.
	$\tan\beta = \frac{\text{"6"}e}{\text{"8"}} = \frac{1}{2}$	**dM1**	Sets perpendicular over parallel component after impact equal to $\frac{1}{2}$, or $e\tan\alpha = \frac{1}{2}$.
	$\frac{6e}{8} = \frac{1}{2} \Rightarrow e = \frac{2}{3}$	**A1 AG**	Correct equation in e solved without error to $\frac{2}{3}$. A0 for substituting $e = \frac{2}{3}$ to check.
		[5]	
2(b)	Parallel to AB: $6 \times \frac{2}{3}$ Perpendicular to AB: $\frac{3}{4} \times$ "8"	**M1**	Multiplies the component parallel to BC by $\frac{3}{4}$ and keeps the component perpendicular to BC from after the first impact.
	4 parallel, 6 perpendicular to AB	**A1**	
	Speed $= \sqrt{\text{"4"}^2 + \text{"6"}^2} = \ldots$	**M1**	Uses Pythagoras with their two components after the impact with AB.
	$2\sqrt{13}$ $[\approx 7.21]\,\mathrm{m\,s^{-1}}$	**A1**	$\sqrt{52}$ oe, or awrt 7.21.
		[4]	
2(c)	Include friction or air resistance	**E1**	States a refinement relaxing a modelling assumption, such as friction from the table or cushions, air resistance, or the puck's size or spin.
	Model the puck's size or spin	**E1**	States a second refinement, different from the first, on the same basis. Either order.
		[2]	
	Total for Question 2	**11**	

Question	Scheme	Marks	Notes
3	$\frac{1}{2}mv^2 = 0.64 \times \frac{1}{2}mu^2 \Rightarrow v = \frac{4}{5}u$	**B1**	$v = 0.8u$ oe, or $\frac{v}{u} = \frac{4}{5}$; may be implied by later working.
	$u\cos\alpha = v\cos\theta$, $\theta = 90° - \alpha$ $\Rightarrow u\cos\alpha =$ "$\frac{4}{5}u$" $\sin\alpha \Rightarrow \tan\alpha = \ldots$	**M1**	Equates components parallel to the barrier and uses $\alpha + \theta = 90°$ to reach an equation in α, with their v a multiple of u.
	$\tan\alpha = \frac{5}{4}$	**A1**	
	$e = \dfrac{v\sin\theta}{u\sin\alpha} = \dfrac{v\cos\alpha}{u\sin\alpha} = \ldots$	**M1**	Forms e as the perpendicular component after over the perpendicular component before, in terms of α or θ. Not for parallel components or speeds.
	$e = \dfrac{\frac{4}{5}}{\frac{5}{4}} = \dfrac{16}{25}$	**A1**	$\frac{16}{25}$ or 0.64, cso. Not from the kinetic energy fraction.
	Total for Question 3	**5**	

Question	Scheme	Marks	Notes
4(a)	$\mathbf{v}-\mathbf{u}=(-2-13)\mathbf{i}+(-11-9)\mathbf{j}$	**M1**	Subtracts the two velocities, in either order, or resolves both along $4\mathbf{i}-3\mathbf{j}$. Condone one sign slip.
	$=-15\mathbf{i}-20\mathbf{j}=-5(3\mathbf{i}+4\mathbf{j})$	**A1**	$\pm(15\mathbf{i}+20\mathbf{j})$, need not be written as a multiple of $3\mathbf{i}+4\mathbf{j}$; or both components 25.
	Impulse is perpendicular to the smooth wall: "$(3\mathbf{i}+4\mathbf{j})$" $\cdot(4\mathbf{i}-3\mathbf{j})=\ldots$	**M1**	States the impulse or change in velocity is perpendicular to the wall and tests it against $4\mathbf{i}-3\mathbf{j}$ by scalar product or gradients, or argues their equal components lie along the wall.
	$(3\mathbf{i}+4\mathbf{j})\cdot(4\mathbf{i}-3\mathbf{j})=12-12=0$ $\Rightarrow AB \parallel (4\mathbf{i}-3\mathbf{j})$	**A1 AG**	Correct zero scalar product, or equal components 25 and 25, from correct work, concluding AB is parallel to $4\mathbf{i}-3\mathbf{j}$.
		[4]	
4(b)	$\mathbf{u}\cdot\frac{1}{5}(3\mathbf{i}+4\mathbf{j})=\frac{1}{5}(39+36)=15$	**M1 A1**	**M1** Resolves either velocity perpendicular to the wall, by scalar product with a vector perpendicular to the wall or otherwise. Condone sign slips. **A1** Or 75 if $3\mathbf{i}+4\mathbf{j}$ is not normalised.
	$\mathbf{v}\cdot\frac{1}{5}(3\mathbf{i}+4\mathbf{j})=\frac{1}{5}(-6-44)=-10$	**A1**	±10, or ±50 on the same unnormalised vector.
	$e=\frac{\text{speed of separation perpendicular to the wall}}{\text{speed of approach perpendicular to the wall}}$ $=\frac{\text{"10"}}{\text{"15"}}$	**M1**	Divides their perpendicular speed after impact by their perpendicular speed before, both from the same direction, or an equivalent equation.
	$e=\frac{2}{3}$	**A1**	
		[5]	
	Total for Question 4	**9**	

Question	Scheme	Marks	Notes
5(a)	$(26\mathbf{i}-7\mathbf{j})\cdot\frac{1}{5}(3\mathbf{i}+4\mathbf{j})=\ldots$ $(26\mathbf{i}-7\mathbf{j})\cdot\frac{1}{5}(4\mathbf{i}-3\mathbf{j})=\ldots$	**M1**	Resolves the velocity parallel and perpendicular to the cushion, by scalar products or angles. Condone sign slips and unnormalised directions.
	Parallel 10, perpendicular 25	**A1**	Both, oe such as 50 and 125 with unnormalised directions, or -25 along $-4\mathbf{i}+3\mathbf{j}$.
	Parallel to the cushion: "10", unchanged Perpendicular: "25" reversed and multiplied by $\frac{3}{5}$	**M1**	Multiplies their perpendicular component by $\frac{3}{5}$ and keeps their parallel component. Not if $\frac{3}{5}$ also multiplies the parallel component.
	$\frac{3}{5}\times 25=15$, away from the cushion	**A1**	Magnitude 15 with reversed direction, oe.
	$\mathbf{v}=$ "10" $\left(\frac{3\mathbf{i}+4\mathbf{j}}{5}\right)-$ "15" $\left(\frac{4\mathbf{i}-3\mathbf{j}}{5}\right)$	**dM1**	Combines their components with the directions, scaled consistently, to give $\mathbf{v}$ in $\mathbf{i}$, $\mathbf{j}$ form, or solves simultaneous equations for the components of $\mathbf{v}$.
	$=(6\mathbf{i}+8\mathbf{j})-(12\mathbf{i}-9\mathbf{j})$ $=-6\mathbf{i}+17\mathbf{j}$	**A1 AG**	Correct substituted combination leading without error to the printed $\mathbf{v}$; combined steps accepted.
		[6]	
5(b)	$\mathbf{I}=0.2\big[(-6\mathbf{i}+17\mathbf{j})-(26\mathbf{i}-7\mathbf{j})\big]$	**M1**	Uses $m(\mathbf{v}-\mathbf{u})$ with $m=0.2$ and both velocities. Condone subtraction in the wrong order.
	$=0.2(-32\mathbf{i}+24\mathbf{j})$	**A1**	oe, such as $-6.4\mathbf{i}+4.8\mathbf{j}$. Condone its negative.
	$\lvert\mathbf{I}\rvert=0.2\sqrt{\text{"32"}^2+\text{"24"}^2}$	**M1**	Uses Pythagoras on both components of their impulse vector.
	$=8\,\text{N s}$	**A1**	8 from the normal components, $0.2(25+15)$, also scores 4 marks.
		[4]	
	Total for Question 5	**10**	

Question	Scheme	Marks	Notes
6(a)	After BC: $\frac{u}{\sqrt{2}}$ parallel to BC $\frac{1}{2} \times \frac{u}{\sqrt{2}}$ perpendicular to BC	**M1**	Resolves u parallel and perpendicular to BC, keeps the parallel component and multiplies the perpendicular one by $\frac{1}{2}$.
	$\frac{u}{2\sqrt{2}}$ parallel to AB, $\frac{u}{\sqrt{2}}$ perpendicular to AB	**A1**	Both, oe, each with its correct direction.
	$\tan 60° = \frac{e \times \text{“}\frac{u}{\sqrt{2}}\text{”}}{\text{“}\frac{u}{2\sqrt{2}}\text{”}} \Rightarrow e = \ldots$	**M1**	Uses a trigonometric ratio of $60°$ or $30°$ with their components after AB, e multiplying only the perpendicular one.
	$e = \frac{\sqrt{3}}{2}$	**A1**	oe, or awrt 0.866.
	$v^2 = \text{“}\left(\frac{u}{2\sqrt{2}}\right)^2\text{”}$ $+ \left(\text{“}\frac{\sqrt{3}}{2}\text{”} \times \text{“}\frac{u}{\sqrt{2}}\text{”}\right)^2$	**M1**	Finds the final speed or its square from their final components, by Pythagoras or by trigonometry with $60°$.
	$v^2 = \frac{u^2}{2}$	**A1**	oe, such as $v = \frac{u}{\sqrt{2}}$.
	$\frac{KE_{\text{after}}}{KE_{\text{before}}} = \frac{\frac{1}{2}m \times \text{“}\frac{u^2}{2}\text{”}}{\frac{1}{2}mu^2}$	**dM1**	Compares final with initial KE, as a ratio or otherwise, using their v^2 as a multiple of u^2.
	$= \frac{1}{2}$, so final KE is 50% of initial KE	**A1 AG**	Ratio $\frac{1}{2}$ reached without error from correct components, and a conclusion of 50%. Nothing for working back from 50%.
		[8]	
6(b)	Friction causes further loss of KE, so the percentage remaining would be less than 50%	**E1**	States the percentage would be less than 50%, or that the model overestimates it, giving friction or roughness as the cause.
		[1]	
	Total for Question 6	**9**	

Question	Scheme	Marks	Notes
7(a)	$(7\mathbf{i}-4\mathbf{j})\cdot\frac{\mathbf{i}-2\mathbf{j}}{\sqrt{5}}=\frac{7+8}{\sqrt{5}}=3\sqrt{5}$	**M1 A1**	**M1** Finds the component of either velocity perpendicular to the wall, by a scalar product with a normal, unit or not, or otherwise. Condone sign slips. **A1** $3\sqrt{5}$ oe, or a consistent multiple such as 15 from the normal $\mathbf{i}-2\mathbf{j}$.
	$(2\mathbf{i}+6\mathbf{j})\cdot\frac{\mathbf{i}-2\mathbf{j}}{\sqrt{5}}=-2\sqrt{5}$	**A1**	Magnitude $2\sqrt{5}$, or the same multiple as before, with the direction reversed.
	$e=\frac{\text{speed of separation perpendicular to the wall}}{\text{speed of approach perpendicular to the wall}}$ $=\frac{\text{“}2\sqrt{5}\text{”}}{\text{“}3\sqrt{5}\text{”}}$	**M1**	Divides their perpendicular speed after impact by their perpendicular speed before, both from the same normal.
	$e=\frac{2}{3}$	**A1**	Or awrt 0.667.
		[5]	
7(b)	$(2\mathbf{i}+6\mathbf{j})\cdot\frac{\mathbf{i}-2\mathbf{j}}{\sqrt{5}}=\ldots$ $(2\mathbf{i}+6\mathbf{j})\cdot\frac{2\mathbf{i}+\mathbf{j}}{\sqrt{5}}=\ldots$	**M1**	Finds the components of $2\mathbf{i}+6\mathbf{j}$ parallel and perpendicular to the second wall, by scalar products or by writing it as $a(\mathbf{i}-2\mathbf{j})+b(2\mathbf{i}+\mathbf{j})$. Condone sign slips.
	Parallel $-2\mathbf{i}+4\mathbf{j}$, perpendicular $4\mathbf{i}+2\mathbf{j}$	**A1**	Both, as vectors, or as scalar components $\pm2\sqrt{5}$ and $2\sqrt{5}$ with the unit vectors they belong to.
	$\mathbf{v}=(\text{“}-2\mathbf{i}+4\mathbf{j}\text{”})-\frac{1}{2}(\text{“}4\mathbf{i}+2\mathbf{j}\text{”})=\ldots$	**M1**	Multiplies their perpendicular component by $\frac{1}{2}$ and keeps their parallel component, combining them into the velocity after impact. Condone sign slips.
	$\mathbf{v}=-4\mathbf{i}+3\mathbf{j}$	**A1**	
	$\cos\alpha=\frac{(2\mathbf{i}+6\mathbf{j})\cdot(\text{“}-4\mathbf{i}+3\mathbf{j}\text{”})}{\sqrt{40}\times\text{“}5\text{”}}=\ldots$	**M1**	Uses the scalar product of $2\mathbf{i}+6\mathbf{j}$ and their velocity after impact over the product of their magnitudes, or the difference of their directions from a common axis.
	$\alpha=72$	**A1**	Final answer rounded to 72.
		[6]	
	Total for Question 7	**11**	

Question	Scheme	Marks	Notes
8(a)	Momentum perpendicular to AB: $2\sin 15° = v\sin\theta - (\sqrt{3}-1)\sin 45°$ $\Rightarrow v\sin\theta = \ldots$	**M1**	Forms a momentum equation perpendicular to AB with all three components, masses cancelled or kept throughout. Condone sign slips.
	$v\sin\theta = \sqrt{6}-\sqrt{2} = 2\times(\sqrt{3}-1)\sin 45°$ Red: twice the perpendicular speed over twice the distance, so equal times	**A1**	Shows exactly that the red ball's perpendicular component is twice the white ball's, links this to the distances and concludes equal times.
		[2]	
8(b)	$2\cos 15° = (\sqrt{3}-1)\cos 45° + v\cos\theta$	**M1**	Forms a momentum equation parallel to AB with all three components. Condone sign slips.
	$v\cos\theta = \sqrt{2}$	**A1**	oe, or awrt 1.41
	$\tan\theta = \frac{\text{“}\sqrt{6}-\sqrt{2}\text{”}}{\text{“}\sqrt{2}\text{”}}$	**dM1**	Finds θ from their two components, such as their $v\sin\theta$ from (a) over their $v\cos\theta$.
	$\tan\theta = \sqrt{3}-1 \Rightarrow \theta = 36.2°$	**A1 AG**	Both components correct, leading without error to $\theta = 36.2°$. Working back from $36.2°$ earns nothing.
		[4]	
8(c)	$v^2 = \text{“}\sqrt{2}\text{”}^2 + \text{“}\sqrt{6}-\sqrt{2}\text{”}^2$	**M1**	Adds the squares of their two components, or divides one component by the matching trigonometric ratio of θ.
	$v = \sqrt{10-4\sqrt{3}} \approx 1.75$	**A1**	oe, or awrt 1.75
		[2]	
8(d)	Same time t: $s_{\text{red}} = vt$, $s_{\text{white}} = (\sqrt{3}-1)t$ “1.75” > 0.732	**M1**	Compares their v with $\sqrt{3}-1$ using the common time from (a), or compares both distances found in terms of one perpendicular distance.
	The red ball travels the greater distance	**A1ft**	ft their v compared with $\sqrt{3}-1$. Needs a conclusion naming the ball.
		[2]	

Question 8 continued

Question	Scheme	Marks	Notes
8(e)	Include friction between the balls and the table	**E1**	Names friction, rolling resistance, air resistance, spin or the balls' size. Not for "more realistic" alone.
		[1]	
	Total for Question 8	**11**	

Question	Scheme	Marks	Notes
9(a)	Parallel component $v\cos\alpha$ unchanged After AB: perpendicular $\frac{3}{5}v\sin\alpha$	**M1 A1**	**M1** Multiplies the perpendicular component by the coefficient of restitution and keeps the parallel component, at either wall; or uses tan(angle after) $= e\times$ tan(angle before) at each wall.
	After CD: perpendicular $\frac{1}{2}\times$ "$\frac{3}{5}v\sin\alpha$" $= \frac{3}{10}v\sin\alpha$	**A1**	
	$\tan\left(\frac{\pi}{4}-\alpha\right) = \frac{\text{"}\frac{3}{10}v\sin\alpha\text{"}}{\text{"}v\cos\alpha\text{"}}$	**M1**	Forms the tangent of the angle with CD as their perpendicular component over their parallel component after the second impact.
	$\frac{1-\tan\alpha}{1+\tan\alpha} = \frac{3}{10}\tan\alpha$ $\Rightarrow 10(1-\tan\alpha) = 3\tan\alpha(1+\tan\alpha)$	**dM1**	Expands $\tan\left(\frac{\pi}{4}-\alpha\right)$ by a compound angle formula and clears fractions to a quadratic in $\tan\alpha$. Condone sign slips.
	$3\tan^2\alpha + 13\tan\alpha - 10 = 0$	**A1**	oe
	$(3\tan\alpha - 2)(\tan\alpha + 5) = 0$ $0 < \tan\alpha < 1 \Rightarrow \tan\alpha = \frac{2}{3}$	**A1 AG**	cso. Solves the correct quadratic and rejects -5 with a reason, such as α acute. Substituting $\frac{2}{3}$ alone earns nothing.
		[7]	
9(b)	$\frac{KE_{\text{after}}}{KE_{\text{before}}} = \frac{\frac{1}{2}m\left(v^2\cos^2\alpha + \frac{9}{100}v^2\sin^2\alpha\right)}{\frac{1}{2}mv^2}$	**M1**	Uses the squares of their final components from (a) to compare final and initial KE, by ratio or by difference.
	Fraction lost $= 1-\cos^2\alpha - \frac{9}{100}\sin^2\alpha$ $= \frac{91}{100}\sin^2\alpha$	**A1**	oe, or the equivalent with values of $\sin\alpha$ and $\cos\alpha$ already substituted.
	$\tan\alpha = \frac{2}{3} \Rightarrow \sin^2\alpha = \ldots\ [= \frac{4}{13}]$	**dM1**	Uses $\tan\alpha = \frac{2}{3}$ to find $\sin^2\alpha$, or $\sin\alpha$ and $\cos\alpha$, by a right-angled triangle or an identity.
	$\frac{91}{100}\times\frac{4}{13} = \frac{7}{25} \Rightarrow 28\%$	**A1**	Not for $\frac{7}{25}$ or 0.28 left unconverted.
		[4]	
	Total for Question 9	**11**	

Question	Scheme	Marks	Notes
10(a)	$u\cos(45^\circ+\alpha) = v\cos(45^\circ-\alpha)$	**M1**	Equates the velocity components parallel to the plane before and after impact. Condone sign slips.
	$v = u\tan(45^\circ-\alpha)$	**A1**	oe
	$I = m\left(v\sin(45^\circ-\alpha) + u\sin(45^\circ+\alpha)\right)$	**M1**	Forms impulse as change in momentum perpendicular to the plane, or as the magnitude of $m\mathbf{v} - m\mathbf{u}$. Condone sign slips.
	$I = m\big(\text{“}u\tan(45^\circ-\alpha)\text{”}\sin(45^\circ-\alpha) + u\cos(45^\circ-\alpha)\big)$ $= mu\,\frac{\sin^2(45^\circ-\alpha)+\cos^2(45^\circ-\alpha)}{\cos(45^\circ-\alpha)}$	**dM1**	Substitutes their v in terms of u and α and uses trigonometric identities to reach a single trigonometric term.
	$= mu\sec(45^\circ-\alpha)$ Same magnitude on the plane (Newton's third law)	**A1 AG**	Correct substituted impulse leading without error to the printed result, with Newton's third law or "equal and opposite" stated.
		[5]	
10(b)	$e = \frac{v\sin(45^\circ-\alpha)}{u\sin(45^\circ+\alpha)}$	**M1**	Applies restitution to the velocity components perpendicular to the plane, with e on the approach component.
	$v^2\sin^2(45^\circ-\alpha) + v^2\cos^2(45^\circ-\alpha)$ $= e^2u^2\sin^2(45^\circ+\alpha) + u^2\cos^2(45^\circ+\alpha)$	**dM1**	Squares and adds the restitution equation and the parallel equation from (a), or forms v^2 as the sum of the squared components after impact.
	$v^2 = u^2\left(\cos^2(45^\circ+\alpha) + e^2\sin^2(45^\circ+\alpha)\right)$	**A1 AG**	Printed result reached without error from correct equations.
		[3]	
10(c)	loss of KE $= \frac{1}{2}m(u^2-v^2)$, with v^2 from (b)	**M1**	Forms $\frac{1}{2}mu^2 - \frac{1}{2}mv^2$ and substitutes the printed v^2.
	$= \frac{1}{2}mu^2(1-e^2)\sin^2(45^\circ+\alpha)$	**A1 AG**	Correct substituted expression leading without error to the printed result.
		[2]	
10(d)	$e = \frac{\text{“}u\tan(45^\circ-\alpha)\text{”}\sin(45^\circ-\alpha)}{u\sin(45^\circ+\alpha)} = \ldots$	**M1**	Eliminates u and v between the parallel and restitution equations to give e in terms of α alone.
	$e = \tan^2(45^\circ-\alpha)$	**A1**	oe, such as $\frac{\tan(45^\circ-\alpha)}{\tan(45^\circ+\alpha)}$.

Question 10 continued

Question	Scheme	Marks	Notes
	$\sin^2(45° + \alpha) = \cos^2(45° - \alpha) = \frac{1}{1+e}$ loss of KE $= \frac{1}{2}mu^2(1 - e^2) \times$ "$\frac{1}{1+e}$"	**dM1**	Uses a trigonometric identity with their e to write $\sin^2(45° + \alpha)$ in terms of e, and substitutes into the result of (c).
	loss of KE $= \frac{1}{2}mu^2(1 - e)$	**A1**	oe, in m, u and e only.
		[4]	
	Total for Question 10	**14**	

Question	Scheme	Marks	Notes
11(a)	After AB: parallel to AB $u\cos\alpha$, perpendicular to AB $eu\sin\alpha$	**M1 A1**	**M1** Multiplies the component perpendicular to AB by e and keeps the parallel one, or uses $\tan\theta = e\tan\alpha$ for the rebound angle θ. Condone sign slips. **A1** Both components correct, oe in terms of the rebound angle.
	$\tan\beta = \frac{u\cos\alpha}{eu\sin\alpha}$	**M1**	Forms $\tan\beta$ as their component perpendicular to BC over their component parallel to BC, oe via $\beta = 90°-$ their rebound angle.
	$= \frac{1}{e\tan\alpha} \Rightarrow \tan\alpha\tan\beta = \frac{1}{e}$	**A1 AG**	Correct ratio from correct components, leading without error to the printed result.
		[4]	
11(b)	After BC: $\begin{pmatrix} -eu\cos\alpha \\ eu\sin\alpha \end{pmatrix}$ $\Rightarrow \tan\gamma = \frac{eu\cos\alpha}{eu\sin\alpha}$	**M1**	Applies restitution at BC to their components after the first impact and forms $\tan\gamma$ as perpendicular over parallel to BC, or uses $\tan\gamma = e\tan\beta$ with (a).
	$\tan\gamma = \cot\alpha$	**A1**	oe, such as $\frac{1}{\tan\alpha}$.
	$\gamma = 90° - \alpha$	**A1**	oe, such as $\frac{\pi}{2} - \alpha$.
	Final direction is parallel to the initial direction, in the opposite sense	**E1**	States both parallel and opposite in sense; "reversed" or "antiparallel" accepted. Not for "parallel" alone.
		[4]	
11(c)	Final speed $= eu \Rightarrow \frac{KE_{\text{after}}}{KE_{\text{before}}} = e^2 = \frac{9}{25}$	**M1**	Finds the final speed from their components after both impacts and equates the ratio of final to initial KE to $\frac{9}{25}$.
	$e = \frac{3}{5}$	**A1**	oe; $-\frac{3}{5}$ rejected or not given.
	First: loss of KE $= \frac{1}{2}mu^2$ $\quad - \frac{1}{2}m\left(u^2\cos^2\alpha + e^2u^2\sin^2\alpha\right)$ Second: loss of KE $= \frac{1}{2}m\left(u^2\cos^2\alpha + e^2u^2\sin^2\alpha\right)$ $\quad - \frac{1}{2}me^2u^2$	**M1**	Forms both losses, or one loss and the total, as KE before minus KE after, from their speeds squared. Condone sign slips; m may be omitted.
	$\frac{1}{2}mu^2(1-e^2)\sin^2\alpha$ and $\frac{1}{2}mu^2(1-e^2)\cos^2\alpha$	**A1**	Both, as multiples of $\sin^2\alpha$ and $\cos^2\alpha$, oe with $e = \frac{3}{5}$, such as $\frac{8}{25}mu^2\sin^2\alpha$; or one loss and the total $\frac{16}{25}\cdot\frac{1}{2}mu^2$.

Question 11 continued

Question	Scheme	Marks	Notes
	$\frac{1}{2}mu^2(1-e^2)\sin^2\alpha = 4\times\frac{1}{2}mu^2(1-e^2)\cos^2\alpha$ $\Rightarrow \tan^2\alpha = \ldots$	**dM1**	Sets their first loss equal to four times their second, or to $\frac{4}{5}$ of the total (the second to $\frac{1}{5}$), and rearranges to a value of $\tan\alpha$, $\tan^2\alpha$ or α.
	$\tan\alpha = 2$	**A1**	-2 rejected or not given.
		[6]	
	Total for Question 11	**14**	

Question	Scheme	Marks	Notes
12(a)	Parallel to BC: $V\cos\beta = U\cos\alpha$	**M1**	Equates the velocity components parallel to BC before and after impact. Condone trigonometric slips.
	Perpendicular to BC: $V\sin\beta = e_1 U\sin\alpha$	**M1**	Uses restitution on the components perpendicular to BC, with e_1 on the approach component. Condone trigonometric slips.
	$\dfrac{V\sin\beta}{V\cos\beta} = \dfrac{e_1 U\sin\alpha}{U\cos\alpha}$	**dM1**	Divides one equation by the other to eliminate U and V.
	$\tan\beta = e_1\tan\alpha$	**A1 AG**	Both component equations correct, leading without error to the printed result.
		[4]	
12(b)	Parallel to CD: $W\cos\gamma = V\sin\beta$	**M1**	Equates the velocity components parallel to CD before and after impact, or applies (a)'s result at CD with angle $90^\circ - \beta$ (implies M1 M1 dM1). Condone trigonometric slips.
	Perpendicular to CD: $W\sin\gamma = e_2 V\cos\beta$	**M1**	Uses restitution on the components perpendicular to CD, with e_2 on the approach component. Condone trigonometric slips.
	$\tan\gamma = \dfrac{e_2 V\cos\beta}{V\sin\beta}$ $\Rightarrow e_2 = \tan\beta\tan\gamma$	**dM1**	Divides to eliminate V and W, reaching e_2 in terms of β and γ.
	$e_2 = (e_1\tan\alpha)\tan\gamma$ $\Rightarrow e_1\tan\alpha\tan\gamma = e_2$	**A1 AG**	Correct relation with the result of part (a) substituted, leading without error to the printed result.
		[4]	
12(c)	$\angle PAB = 90^\circ - \alpha$, $\angle QAB = \gamma$ $\angle PAQ = \angle QAB - \angle PAB$	**M1**	Expresses $\angle PAQ$ in terms of α and γ using angle facts at A, or $\tan\alpha$ and $\tan\gamma$ as length ratios, or by any valid geometry. Condone one angle slip.
	$\angle PAQ = \alpha + \gamma - 90^\circ$	**A1**	oe, or $\tan\alpha = \frac{AB}{BP}$ and $\tan\gamma = \frac{AD}{DQ}$.

Question 12 continued

Question	Scheme	Marks	Notes
	$\angle PAQ > 0 \Rightarrow \alpha > 90° - \gamma$ $\Rightarrow \tan\alpha > \tan(90° - \gamma)$	**M1**	Uses $\angle PAQ > 0$ for an inequality in α and γ and applies a trigonometric function, or uses $BP < AD$ and $DQ < AB$ in their length ratios.
	$\tan\alpha > \dfrac{1}{\tan\gamma} \Rightarrow \tan\alpha\tan\gamma > 1$	**A1**	With a reason, such as acute angles or tan increasing.
	$e_2 = e_1\tan\alpha\tan\gamma$, $\tan\alpha\tan\gamma > 1$	**dM1**	Uses part (b) with their inequality of the form $\tan\alpha\tan\gamma > k$ to compare e_2 with e_1.
	$e_2 > e_1$	**A1 AG**	Minimum closing statement $e_2 > e_1$ as the condition for the ball to return to A. A0 if the previous A1 was not earned.
		[6]	
12(d)	$\tan\alpha\tan\gamma = 1 \Rightarrow \gamma = 90° - \alpha$ Path from Q is parallel to AP	**E1**	Deduces $\alpha + \gamma = 90°$ oe from part (b) and concludes the path from Q is parallel to AP; AD need not be named.
		[1]	
	Total for Question 12	**15**	