FURTHER MECHANICS / MOMENTUM & COLLISIONS

Collisions in 1 Dimension

Mark scheme 12 questions · 138 marks

EXAM BOARDS Edexcel FM1 9FM0/3C · AQA 7367/3M · OCR A Y543 · OCR MEI Mech Minor Y431 OCR MEI Mech Major Y421 · CIE 9231 Paper 3

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	Total	**138**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last mark of a part whose answer is printed in the question: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [***n***] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1	$1(7) + 2(-2) = 1(\pm 1) + 2v_B$	**M1 A1**	**M1** Uses conservation of momentum with all four terms, each mass paired with its velocity, and A's velocity after as 1 or -1. Condone sign slips. **A1** Correct equation for either case, signs consistent with their positive direction.
	$v_A = 1 \Rightarrow v_B = 1$: speed $1\,\mathrm{m\,s^{-1}}$	**A1**	
	$v_A = -1 \Rightarrow v_B = 2$: speed $2\,\mathrm{m\,s^{-1}}$	**A1**	Withhold if a further speed is given as possible.
	Total for Question 1	**4**	

Question	Scheme	Marks	Notes
2(a)	$0.5(7) + 1.0(2) = 0.5v_A + 1.0(5)$	**M1 A1**	**M1** Forms a momentum equation with all four mass-velocity terms. Condone sign slips.
	$v_A = 1$	**A1**	
	$5 - \text{“1”} = e(7 - 2) \Rightarrow e = \ldots$	**M1**	Uses speed of separation over speed of approach with their v_A from a momentum equation and the given speeds. Condone sign slips.
	$4 = 5e \Rightarrow e = 0.8$	**A1 AG**	Correct restitution equation with $v_A = 1$, leading without error to $e = 0.8$ or $\frac{4}{5}$.
		[5]	
2(b)	Include friction or air resistance, rather than a smooth track.	**E1**	Names a specific refinement, such as including friction or other resistance. Not for “more accurate” or similar alone.
		[1]	
	Total for Question 2	**6**	

Question	Scheme	Marks	Notes
3(a)	$I = 2m(-v) - 2m(4u)$	**M1**	Uses impulse as change in momentum of A, mass $2m$, from $4u$ to its velocity after. Condone sign slips.
	$2m(v + 4u) = 12mu$	**A1**	Correct equation in v and u, with either sign.
	$v = 2u$	**A1**	
		[3]	
3(b)	B moves in A's original direction: $2m(4u) - 3mku = -2m(\text{“}2u\text{”}) + 3m(\text{“}2u\text{”})$	**M1 A1ft**	**M1** Equates total momentum before and after, all four terms, with their speed of A from (a) reversed, for either direction of B. Condone sign slips. **A1ft** Correct equation, oe; ft their speed of A from (a).
	$k = 2$	**A1**	Or $k = 6$, if that case comes first.
	B moves opposite to A's original direction: $2m(4u) - 3mku = -2m(\text{“}2u\text{”}) - 3m(\text{“}2u\text{”})$	**M1**	Forms the momentum equation for the other direction of B after the collision.
	$k = 6$	**A1**	Or $k = 2$, for the other case.
		[5]	
	Total for Question 3	**8**	

Question	Scheme	Marks	Notes
4(a)	$2(4)+6(0)=2v_A+6v_B$	**M1 A1**	**M1** Equates total momentum before and after, each mass with its own velocity. Condone sign slips. **A1** Correct equation, oe.
	$v_B-v_A=e(4-0)$	**M1 A1**	**M1** Uses restitution with e multiplying the speed of approach and a difference of the final velocities. Condone sign slips. **A1** Correct equation, oe, consistent with the signs in the momentum equation.
	$4v_B=4+4e \Rightarrow v_B=\ldots$	**dM1**	Eliminates one velocity from their two equations to find the other in terms of e.
	$v_B=1+e$	**A1**	
	$(1+e)-v_A=4e \Rightarrow v_A=1-3e$	**A1 AG**	Correct equations solved without error to $v_A=1-3e$. A0 if $1-3e$ is assumed, such as substituted to find v_B, rather than derived.
		[7]	
4(b)(i)	$8=8v \Rightarrow$ speed $1\,\text{m}\,\text{s}^{-1}$	**B1**	
		[1]	
4(b)(ii)	$1-3e<0 \Rightarrow e>\ldots$	**M1**	Sets $1-3e<0$, or $v_A<0$ in their positive direction, and solves for e.
	$\frac{1}{3}<e\leq 1$	**A1**	Both bounds, strict at $\frac{1}{3}$ and including 1, as one range or as two inequalities given together. A0 for $e>\frac{1}{3}$ alone.
		[2]	
4(b)(iii)	$KE_{\text{before}}=\frac{1}{2}(2)(4^2)=16$	**B1**	
	$KE_{\text{after}}=\frac{1}{2}(2)(1-3e)^2+\frac{1}{2}(6)\text{“}(1+e)\text{”}^2$	**M1**	Sums $\frac{1}{2}mv^2$ for both spheres, using $1-3e$ and their v_B from (a) in terms of e.
	$=4+12e^2$	**A1**	
	$16-\text{“}(4+12e^2)\text{”}=9 \Rightarrow e=\ldots$	**dM1**	Sets their KE_{before} minus their KE_{after} equal to 9 and solves for e.

Question 4 continued

Question	Scheme	Marks	Notes
	$e = \frac{1}{2}$	**A1**	A0 if $e = -\frac{1}{2}$ is left as a possible value.
		[5]	
	Total for Question 4	**15**	

Question	Scheme	Marks	Notes
5(a)	$1(2)+3(0)=1v_A+3v_B$	**M1 A1**	**M1** Uses conservation of momentum with masses paired with velocities and all nonzero terms present. Condone sign slips.
	$v_B-v_A=e(2-0)$	**M1 A1**	**M1** Uses restitution, separation in terms of v_A and v_B equated to e times the approach speed. Condone sign slips.
	$2=(v_B-2e)+3v_B \Rightarrow v_B=\ldots$	**dM1**	Solves the two equations simultaneously, eliminating one velocity to reach the other in terms of e.
	$v_B=\frac{1+e}{2}$	**A1**	oe
	$v_A=\frac{1+e}{2}-2e=\frac{1}{2}(1-3e)$	**A1 AG**	Printed result reached without error from correct equations; either velocity may be found first.
		[7]	
5(b)	$KE_{\text{after}}=0.52\times\frac{1}{2}(1)(2^2)=1.04$	**B1**	Or loss of KE $=0.48\times 2=0.96$, used with $KE_{\text{before}}=2$.
	$KE_{\text{after}}=\frac{1}{2}(1)\left(\frac{1-3e}{2}\right)^2+\frac{1}{2}(3)\left(\text{“}\frac{1+e}{2}\text{”}\right)^2$	**M1**	Sums $\frac{1}{2}mv^2$ for both spheres with $\frac{1}{2}(1-3e)$ and their v_B from (a), each in terms of e. Condone slips in squaring.
	$KE_{\text{after}}=\frac{1}{2}+\frac{3}{2}e^2$	**A1**	oe
	$\text{“}\frac{1}{2}+\frac{3}{2}e^2\text{”}=\text{“1.04”}\Rightarrow e=\ldots$	**dM1**	Equates their KE_{after} in terms of e to their value (or the loss to theirs) and solves for e.
	$e^2=\frac{9}{25}\Rightarrow e=\frac{3}{5}$, rejecting $e=-\frac{3}{5}$	**A1**	$e=\frac{3}{5}$ oe only. A0 if $e=-\frac{3}{5}$ is not rejected.
		[5]	
5(c)	Equal radii put the centres at the same height, so the line of centres is horizontal: the impulse acts along the line of motion and the impact is direct	**E1**	Links equal radii to the line of centres, or the impulse, lying along the line of motion. Not for “direct” or “not oblique” alone.
		[1]	
5(d)	$5v_D-5(1)=\frac{10}{7}(u-1)\Rightarrow v_D=\ldots$	**M1**	Equates the impulse to the change in momentum of D, mass 5 and initial velocity 1. Condone sign slips.

Question 5 continued

Question	Scheme	Marks	Notes
	$v_D = \frac{2u+5}{7}$	**A1**	oe, such as $1 + \frac{2}{7}(u-1)$.
	$2v_C - 2u = -\frac{10}{7}(u-1) \Rightarrow v_C = \ldots$	**M1**	Equates the reversed impulse to the change in momentum of C, or uses conservation of momentum with their v_D in terms of u. Condone sign slips.
	$v_C = \frac{2u+5}{7} = v_D$, so the velocities are equal	**A1 AG**	Correct equation leading without error to $v_C = \frac{2u+5}{7}$, then $v_C = v_D$ or a conclusion in words.
		[4]	
5(e)	loss of KE $= \frac{1}{2}(2)u^2 + \frac{1}{2}(5)(1^2)$ $- \frac{1}{2}(7)\left(\text{“}\frac{2u+5}{7}\text{”}\right)^2$	**M1**	Forms $KE_{\text{before}} - KE_{\text{after}}$, or both for $1 - \frac{KE_{\text{after}}}{KE_{\text{before}}}$, with both spheres in each and their common velocity in terms of u. Condone slips in squaring.
	loss of KE $= \frac{5(u-1)^2}{7}$	**A1**	oe, such as $\frac{10u^2-20u+10}{14}$; or correct KE_{after} and KE_{before}.
	$\frac{\text{“}\frac{5(u-1)^2}{7}\text{”}}{u^2+\frac{5}{2}}$ $\left[= \frac{10(u-1)^2}{7(2u^2+5)}\right]$	**dM1**	Divides their loss by KE_{before} including both spheres, or subtracts their ratio from 1.
	$\frac{10\left(1-\frac{1}{u}\right)^2}{7\left(2+\frac{5}{u^2}\right)} \to \ldots$ as $u \to \infty$	**dM1**	Takes the limit of their fraction as $u \to \infty$, by dividing through by u^2 or comparing leading terms.
	$\frac{5}{7}$	**A1**	$\frac{5}{7}$, or awrt 0.714.
		[5]	
	Total for Question 5	**22**	

Question	Scheme	Marks	Notes
6	$15.8t - 4.9t^2 = 20 - 4.2t - 4.9t^2$	**M1**	Equates the heights of P and Q, each from $s = ut + \frac{1}{2}at^2$, allowing for the 20 m start; relative motion accepted. Condone sign slips.
	$t = 1$, height $= 15.8 - 4.9 = 10.9\,\text{m}$	**A1**	Both $t = 1$ and 10.9 m.
	$u_P = 15.8 - 9.8 = 6$, $u_Q = -4.2 - 9.8 = -14$ $6(6) + 3(-14) = 6v_P + 3v_Q$	**M1 A1**	**M1** Equates total momentum before and after, all four terms, with their velocities at impact. Condone sign slips. **A1** $u_P = 6$, $u_Q = -14$ and the equation correct, oe, such as $2v_P + v_Q = -2$.
	$v_Q - v_P = \frac{1}{2}\big(6 - (-14)\big)$	**M1 A1**	**M1** Uses speed of separation $= \frac{1}{2}\times$ speed of approach, with their velocities at impact. Condone sign slips, but not e on the wrong side. **A1** oe, such as $v_Q - v_P = 10$.
	Solves $2v_P + v_Q =$ "-2", $v_Q - v_P =$ "10" $\Rightarrow v_P = \ldots$	**dM1**	Solves their momentum and restitution equations simultaneously for v_P, by any means.
	$v_P = -4$: P moves downwards at $4\,\text{m}\,\text{s}^{-1}$	**A1**	Speed 4 downwards, or $v_P = -4$ with upwards positive.
	"10.9" $-$ "4"$\tau - 4.9\tau^2 = 0 \Rightarrow \tau = \ldots$	**M1**	Uses a constant-acceleration formula from the collision with their height, their v_P and g, reaching $\tau = \ldots$; a calculator may solve the quadratic. Condone sign slips.
	$\tau = \frac{\sqrt{5741} - 20}{49} = 1.14\,\text{s}$	**A1**	awrt 1.14, or the exact value; the negative root rejected.
	Total for Question 6	**10**	

Question	Scheme	Marks	Notes
7(a)	$10mu = 5mv - 5mu$	**M1 A1**	**M1** Uses impulse as change in momentum of Q, with mass $5m$ and speeds u and v. Condone sign slips. **A1** Correct equation with consistent signs.
	$v = 3u$	**A1**	
		[3]	
7(b)	$2mku + 5mu = 2m(\pm\text{“}3u\text{”}) + 5m(\text{“}3u\text{”})$	**M1**	Equates total momentum before and after in either case, with their v from (a), of the form cu, for both trolleys. Condone sign slips.
	P continues: $2k + 5 = 21 \Rightarrow k = 8$	**A1**	
	P rebounds: $2k + 5 = 9 \Rightarrow k = 2$	**A1**	
	$3u - (-3u) = e(2u - u) \Rightarrow e = \ldots$	**M1**	Uses speed of separation over speed of approach for their rebound case, or compares total KE before and after it. Condone sign slips.
	$e = 6 > 1$, so P cannot rebound: $k = 8$ only	**A1**	Rejects $k = 2$ as $e = 6 > 1$, or as KE is gained, and concludes $k = 8$ only.
		[5]	
Total for Question 7		**8**	

Question	Scheme	Marks	Notes
8(a)	$a = \frac{25}{49} \times 9.8 = 5$ $u_P^2 = 5^2 + 2(-5)(0.9) \Rightarrow u_P = \ldots$	**M1**	Uses $v^2 = u^2 + 2as$ or work-energy over 0.9 m with deceleration μg, for P from A to B. Condone sign slips.
	$u_P = 4$	**A1**	
	$0 = v^2 + 2(-5)(0.9) \Rightarrow v = \ldots$	**M1**	Uses $v^2 = u^2 + 2as$ or work-energy over 0.9 m with deceleration μg, for P returning to rest at A. Condone sign slips.
	Speed of P after the collision $= 3$	**A1**	
	$0.4(\text{“}4\text{”}) + 3(0) = 0.4(-\text{“}3\text{”}) + 3v_Q$	**M1**	Equates total momentum before and after, all four terms, with their speeds of P. Condone sign slips.
	$v_Q = \frac{14}{15}$	**A1**	oe, or awrt 0.933.
	$e = \frac{\text{speed of separation}}{\text{speed of approach}}$ $\text{“}\frac{14}{15}\text{”} - (-\text{“}3\text{”}) = e(\text{“}4\text{”} - 0) \Rightarrow e = \ldots$	**M1**	Uses speed of separation $= e\times$ speed of approach with their velocities. Condone sign slips, but not e on the wrong side.
	$e = \frac{59}{60}$	**A1**	oe, or awrt 0.983.
		[8]	
8(b)	$I = mv - mu$ $I = 0.4(-\text{“}3\text{”}) - 0.4(\text{“}4\text{”})$	**M1**	Finds the change in momentum of P with their speeds and its velocity reversed, or of Q with their v_Q. Condone the order of subtraction.
	2.8 N s to the left	**A1**	Magnitude with direction (to the left, towards A, or opposite to P's initial motion), or -2.8 with the positive direction stated.
		[2]	
8(c)	Rebound speed $\text{“}\frac{14}{15}\text{”}$ $0 = \left(\frac{14}{15}\right)^2 + 2(-\mu g)(0.6)$	**M1 A1**	**M1** Uses $v^2 = u^2 + 2as$ or work-energy over their distance on AB, with deceleration μg and initial speed their v_Q from (a). Condone sign slips. **A1** Correct equation with $u = \frac{14}{15}$ and $s = 0.6$.
	$\mu = \frac{2}{27}$	**A1**	oe, or awrt 0.0741.
		[3]	
	Total for Question 8	**13**	

Question	Scheme	Marks	Notes
9(a)	$m(9u) + 7m(u) = mv_P + 7mv_Q$	**M1 A1**	**M1** Equates total momentum before and after with all four terms, each mass with its velocity. Condone sign slips. **A1** oe, with v_P taken in either direction.
	$v_Q - v_P = e(9u - u)$	**M1 A1**	**M1** Forms a restitution equation with e multiplying a speed of approach from $9u$ and u, and a separation in v_P, v_Q. Condone sign slips. **A1** oe, consistent in direction with their momentum equation.
	$v_P + 7(v_P + 8eu) = 16u \Rightarrow v_P = \ldots$	**dM1**	Solves their momentum and restitution equations simultaneously for v_P in terms of e and u.
	$v_P = (2 - 7e)u$	**A1**	oe, such as a speed $(7e - 2)u$ in the reversed direction.
	"$(2 - 7e)u$" < 0	**M1**	Uses the reversal as an inequality on their v_P of the form $(a + be)u$, $v_P < 0$, or speed > 0 if taken in the reversed direction.
	$\frac{2}{7} < e \leq 1$	**A1**	Both bounds, strict at $\frac{2}{7}$ and including 1, as one range or as two inequalities given together.
		[8]	
9(b)	$v_P = -\frac{13u}{4}$, $v_Q = \frac{11u}{4}$	**B1ft**	ft their v_P from (a) and their v_Q from (a) or found here, with $e = \frac{3}{4}$. Speeds with directions accepted.
	$KE_{\text{before}} - KE_{\text{after}}$ $= \frac{1}{2}m(9u)^2 + \frac{1}{2}(7m)u^2$ $- \left[\frac{1}{2}m\left(\frac{13u}{4}\right)^2 + \frac{1}{2}(7m)\left(\frac{11u}{4}\right)^2\right]$	**M1 A1**	**M1** Subtracts total KE after from total KE before, each with both particles' $\frac{1}{2}mv^2$ terms and their velocities. Condone slips in squaring. **A1** Both totals correct, unsimplified, or $44mu^2$ and $\frac{127}{4}mu^2$ seen.
	loss of KE $= \frac{49}{4}mu^2$	**A1**	oe, such as $12.25mu^2$.
		[4]	
9(c)	Compares P's rebound speed with Q's speed: $f \times$ "$\frac{13u}{4}$" with "$\frac{11u}{4}$"	**M1**	Multiplies their speed of P after the collision by f and compares it with their speed of Q, by an inequality or an equation.
	$\frac{13fu}{4} > \frac{11u}{4} \Rightarrow 13f > 11$	**A1**	Correct strict inequality, oe.

Question 9 continued

Question	Scheme	Marks	Notes
	$\frac{11}{13} < f \leq 1$	**A1**	Both bounds, strict at $\frac{11}{13}$ and including 1, as one range or as two inequalities given together.
		[3]	
	Total for Question 9	**15**	

Question	Scheme	Marks	Notes
10(a)	$3m(4u) + m(-u) = 3mv_A + mv_B$	**M1 A1**	**M1** Forms a momentum equation with all four terms, each mass with its velocity. Condone sign slips. **A1** Correct equation in any consistent direction.
	$v_B - v_A = e\big(4u - (-u)\big)$	**M1 A1**	**M1** Forms a restitution equation with e multiplying the speed of approach. Condone sign slips. **A1** Correct equation, consistent in direction with their momentum equation.
	$3v_A + (v_A + 5eu) = 11u \Rightarrow v_A = \ldots$ $\Rightarrow v_B = v_A + 5eu = \ldots$	**dM1**	Eliminates one velocity from their two equations and solves for v_B, directly or via v_A.
	$v_B = \frac{u}{4}(11 + 15e) > 0$, so towards the wall	**A1 AG**	From correct equations without error, with $v_B > 0$ or towards the wall stated.
		[6]	
10(b)	$v_B - v_A = 5eu > 0$, so B reaches the wall first	**B1**	Needs a reason, such as $v_B > v_A$ from their part (a) values.
	$v_A = \frac{u}{4}(11 - 5e) > 0$ as $e \leq 1$ so A still moves towards the wall	**M1**	Shows their v_A in terms of e is positive using $e \leq 1$; v_A may be found in (a).
	B rebounds away from the wall while A moves towards it $\Rightarrow$ second collision	**A1 AG**	Concludes in words that they collide again, as B rebounds away from the wall, at any speed, while A still moves towards it.
		[3]	
10(c)	$I = m\left(-\frac{V}{3}\right) - mV$, $V = \frac{u}{4}(11 + 15e)$	**M1**	Uses $I = mv - mu$ for B with speeds V and $\frac{1}{3}V$ taken in opposite directions.
	$\lvert I\rvert = \frac{4mV}{3}$	**A1**	Either sign. Implied by a correct final answer.
	$\lvert I\rvert = \frac{mu}{3}(11 + 15e)$	**A1**	Positive, oe such as $\frac{11mu+15emu}{3}$.
		[3]	
10(d)	loss of KE $= \frac{1}{2}mV^2 - \frac{1}{2}m\left(\frac{V}{3}\right)^2$	**M1**	Subtracts $\frac{1}{2}mv^2$ for B at speed $\frac{1}{3}V$ from that at speed V.
	$= \frac{4}{9}mV^2$	**A1**	Correct expression in V or in m, u, e; need not be simplified.

Question 10 continued

Question	Scheme	Marks	Notes
	loss of KE $= \frac{mu^2(11+15e)^2}{36}$	**A1**	Or the expanded $\frac{mu^2}{36}(225e^2+330e+121)$.
		[3]	
	Total for Question 10	**15**	

Question	Scheme	Marks	Notes
11(a)	$m(5u) = mv_P + 4mv_Q$	**M1 A1**	**M1** Forms a momentum equation with three terms, each a mass times a velocity. Condone sign slips.
	$v_Q - v_P = e(5u - 0)$	**M1 A1**	**M1** Forms a restitution equation with e multiplying the speed of approach. Condone sign slips. **A1** Signs consistent with their momentum equation.
	$5u = v_P + 4(v_P + 5eu) \Rightarrow v_P = \ldots$	**dM1**	Eliminates one velocity from their two equations and solves for the velocity of P. Condone sign slips.
	$v_P = (1 - 4e)u < 0$ as $e > \frac{2}{3}$ $\Rightarrow$ speed of $P = (4e - 1)u$	**A1 AG**	cso, from correct equations. Needs $e > \frac{2}{3}$, or $4e - 1 > 0$, used to give the speed as $(4e - 1)u$.
		[6]	
11(b)	$v_Q = (1 + e)u$	**B1**	Seen in (a) or (b).
	Speed of P after the wall: $f(4e - 1)u$	**B1**	$f(4e - 1)u$ oe; may be seen as a velocity.
	Second collision if P is faster than Q: $f(4e - 1)u > (1 + e)u$	**M1**	Forms an inequality with their speed of P after the wall greater than their speed of Q. Condone $\geq$.
	$f > \frac{1+e}{4e-1}$	**A1**	oe, strict inequality.
	$\frac{1+e}{4e-1} < f \leq 1$	**A1**	Both bounds, with $f = 1$ included, as one range or as two inequalities given together. No check that the lower bound is below 1 is needed.
		[5]	
Total for Question 11		**11**	

Question	Scheme	Marks	Notes
12(a)	$m(6u) + 3m(u) = mv_P + 3mv_Q$	**M1 A1**	**M1** Forms a momentum equation with all four terms, each mass with its own velocity. Condone sign slips.
	$v_Q - v_P = e(6u - u)$	**M1 A1**	**M1** Uses restitution with e multiplying the speed of approach, velocities after the collision on the other side. Condone sign slips. **A1** Correct, with signs consistent with their momentum equation.
	$4v_Q - 9u = 5eu \Rightarrow v_P = \ldots$	**dM1**	Solves their two equations simultaneously for the velocity of P in terms of u and e.
	$v_P = \frac{3(3-5e)u}{4}$, speed $\frac{3u(5e-3)}{4}$	**A1**	Speed oe, positive for $e > \frac{3}{5}$. A0 for a negative expression given as the speed.
		[6]	
12(b)	$3 - 5e < 0 \Rightarrow e > \frac{3}{5}$	**B1**	May be seen in (a).
	$\frac{1}{3} \times$ "$\frac{(9+5e)u}{4}$" $= \frac{(9+5e)u}{12}$	**B1ft**	ft their speed of Q from (a) or found here, multiplied by $\frac{1}{3}$.
	$\frac{(9+5e)u}{12} >$ "$\frac{3u(5e-3)}{4}$" $\Rightarrow e \ldots$	**M1**	Forms an inequality with the rebound speed of Q exceeding their speed of P, both in terms of e, and solves for e. Condone an equation or $\geq$.
	$e < \frac{9}{10}$	**A1**	
	$\frac{3}{5} < e < \frac{9}{10}$	**A1**	Both bounds, strict at both ends, as one range or as two inequalities given together.
		[5]	
	Total for Question 12	**11**	