FURTHER MECHANICS / MISCELLANEOUS MECHANICS

Rigid Bodies, Sliding & Toppling

Questions by topic 15 questions · 180 marks

EXAM BOARDS Edexcel FM2 9FM0/4C · AQA 7367/3M · OCR A Y543 · OCR MEI Mech Minor Y431 OCR MEI Mech Major Y421 · CIE 9231 Paper 3

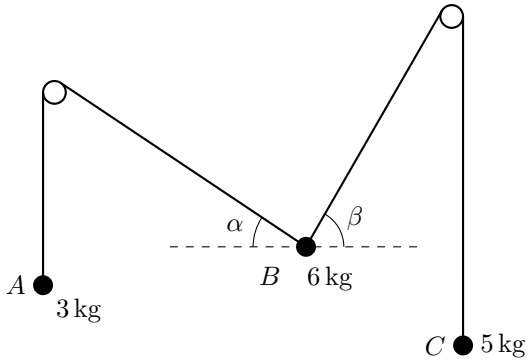
Instructions

Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question gives a value. Where a question leaves g unspecified and a final answer depends on it, the solutions also give that answer for $g = 10\,\mathrm{m\,s^{-2}}$.

CIE (9231 Paper 3): papers take $g = 10\,\mathrm{m\,s^{-2}}$; use the $g = 10$ final answers where the solutions give them.

Questions

Question 1	*6 marks*	2
Question 2	*9 marks*	3
Question 3	*17 marks*	4
Question 4	*13 marks*	5
Question 5	*9 marks*	6
Question 6	*10 marks*	7
Question 7	*8 marks*	8
Question 8	*12 marks*	9
Question 9	*12 marks*	10
Question 10	*14 marks*	11
Question 11	*16 marks*	12
Question 12	*15 marks*	13
Question 13	*14 marks*	14
Question 14	*14 marks*	15
Question 15	*11 marks*	16



1. One end of a light inextensible string is attached to a particle A of mass 3 kg. The other end of the string is attached to a particle B of mass 6 kg.
A second light inextensible string has one end attached to B and its other end attached to a particle C of mass 5 kg. Each string passes over a small smooth fixed pulley, and both strings are taut.
The system is in equilibrium with all three particles at rest in the same vertical plane, and with A and C hanging vertically below their pulleys. As shown in the diagram above, the portion of string between B and the pulley above A makes an angle α with the horizontal, and the portion of string between B and the pulley above C makes an angle β with the horizontal.

(a) Draw a closed figure to represent the three forces acting on B. **[1]**

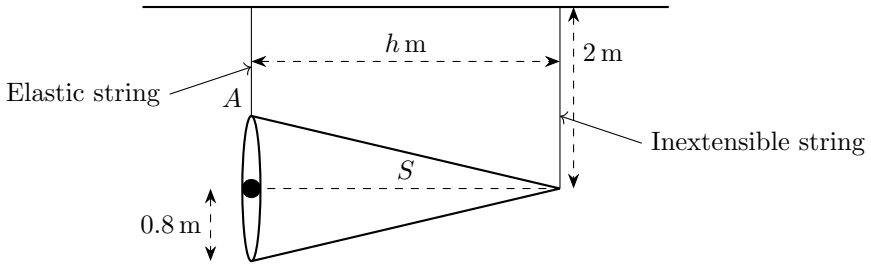
(b) Hence, or otherwise, find the following, giving each answer in degrees to 1 decimal place.

- The value of α
- The value of β

[5]

2. A shape S is formed by attaching a particle of mass $2m$ kg to the centre of the base of a uniform solid cone of mass $4m$ kg.
The height of the cone is h m and the radius of its base is 0.8 m.

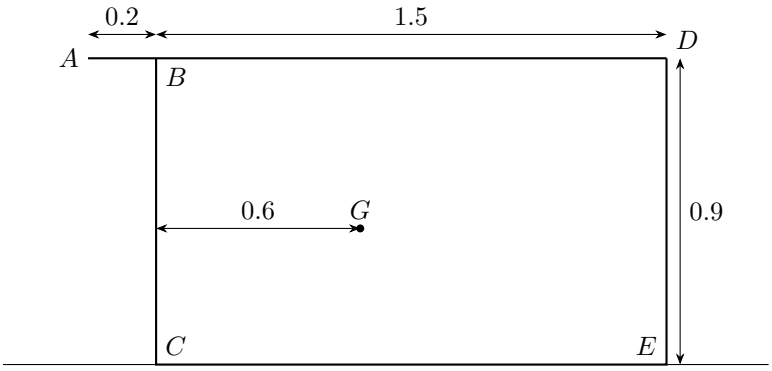
(a) Explain why the centre of mass of S lies on the axis of the cone. **[1]**



Two light strings are attached to S. A light inextensible string of length 2 m is attached at the vertex of the cone, and a light elastic string of modulus of elasticity $15mg$ N is attached at the point A on the edge of the base.
The other ends of the strings are attached to a horizontal ceiling so that both strings are vertical.
Shape S hangs in equilibrium with the axis of the cone horizontal and A vertically above the centre of the base, as shown in the diagram above.

(b) Find, in terms of m and g, the tension in each string. **[5]**

(c) Determine the natural length of the elastic string. **[3]**

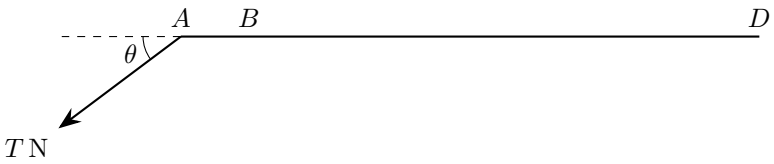


3. A side view of a free-standing cupboard on a horizontal floor is shown in the diagram above. The cupboard consists of a base CE, vertical ends BC and DE, and a horizontal top AD which overhangs the end BC. The dimensions of the cupboard, in metres, are shown in the diagram. The cupboard and its contents have a total weight of 350 N and centre of mass at G.

(a) Calculate the magnitude of the vertical force required for the cupboard to be on the point of tipping in each of the following cases.

(i) The force acts downwards at A. **[3]**

(ii) The force acts upwards at D. **[3]**



A force of magnitude T N is now applied at A, acting downwards and away from the cupboard at an angle θ to the horizontal, as shown in the diagram above, where $\tan\theta = \frac{3}{4}$.

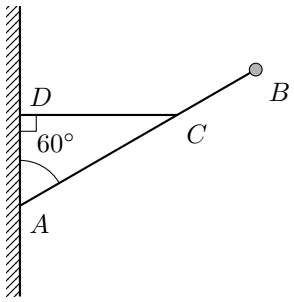
(b) By considering the horizontal and vertical components of the force at A, show that the anticlockwise moment of this force about C is $\frac{21T}{25}$ N m. **[4]**

With this force acting, the cupboard is in equilibrium and is on the point of tipping but not on the point of sliding.

(c) Find the value of T, and find the range of possible values of the coefficient of friction between the base of the cupboard and the floor. **[7]**

4. A flagpole, AB, is 3 m long. The flagpole is modelled as a non-uniform rod so that, at a distance x metres from A, the mass per unit length of the flagpole, m kg m^{-1}, is given by $m = 20 - 4x$.

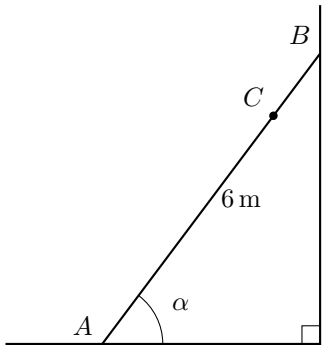
(a) Find the mass of the flagpole, and show that its centre of mass is at a distance of $\frac{9}{7}$ m from A. **[5]**



The end A of the flagpole is fixed to a point on a vertical wall. The flagpole lies in a vertical plane perpendicular to the wall and makes an angle of 60° with the upward vertical through A. A cable has one end attached to the point C on the flagpole, where $AC = 2$ m, and the other end attached to the point D on the wall, vertically above A. The cable is horizontal. A small ball of mass 5 kg is attached to the flagpole at B. The flagpole and ball rest in equilibrium, as shown in the diagram above.
The cable is modelled as a light inextensible string and the ball is modelled as a particle.

(b) Using the model, find the tension in the cable and the magnitude of the force exerted on the flagpole by the wall at A. **[7]**

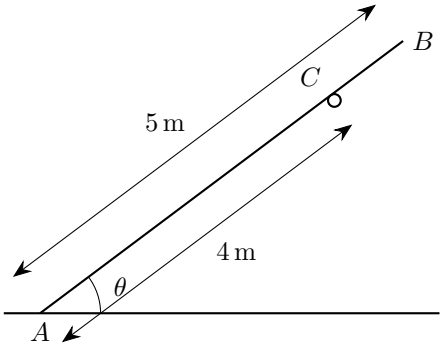
(c) In reality the ball has a small but non-zero radius, and it is attached so that its centre lies on AB produced, a short distance beyond B. State, with a reason, whether the true tension in the cable is likely to be greater or less than your answer to part (b). **[1]**



5. A uniform ladder AB, of mass 20 kg and length 6 m, rests with end A on rough horizontal ground. The end B of the ladder rests against a rough vertical wall.
The ladder is inclined at an angle α to the ground, where $\tan\alpha = \frac{4}{3}$, as shown in the diagram above.
The coefficient of friction between the ladder and the ground is $\frac{1}{2}$.
The coefficient of friction between the ladder and the wall is $\frac{1}{3}$.
A man of mass 60 kg starts at A and slowly climbs the ladder. The ladder is on the point of slipping when the man reaches the point C on the ladder.
The ladder is modelled as a uniform rod which lies in a vertical plane perpendicular to the wall, and the man is modelled as a particle.

(a) Find the magnitude of the normal reaction between the ladder and the wall at B when the man is at C. **[5]**

(b) Find the distance AC. **[4]**

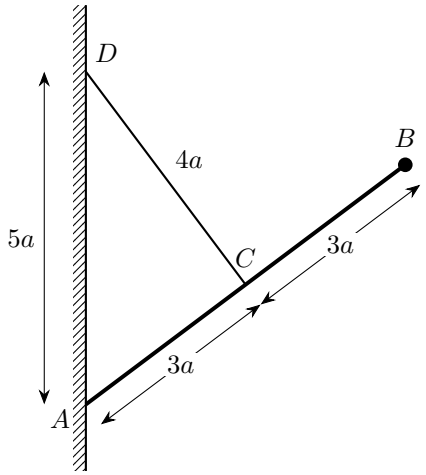


6. A uniform rod AB has mass 6 kg and length 5 m. The end A of the rod is in contact with rough horizontal ground.
The rod rests in equilibrium on a smooth horizontal peg at the point C of the rod, where $AC = 4$ m, and is inclined at an angle θ to the ground, where $\tan\theta = \frac{3}{4}$, as shown in the diagram above.
The rod lies in a vertical plane perpendicular to the peg.

(a) Find the magnitude of the normal contact force between the peg and the rod. **[3]**

A particle of mass m kg is now attached to the rod at B, and the rod rests in the same position on the peg. The coefficient of friction between the rod and the ground is $\frac{3}{4}$.

(b) Find the greatest value of m for which the rod remains in equilibrium. **[7]**



7. A uniform rod AB, of length $6a$ and weight W, has its end A in contact with a rough vertical wall, with B above the level of A. A light inextensible string of length $4a$ has one end attached to the midpoint C of the rod and the other end attached to a point D on the wall, vertically above A, where $AD = 5a$, as shown in the diagram above. A particle of weight kW is attached to the rod at B. The rod and the string lie in a vertical plane perpendicular to the wall.
The coefficient of friction between the rod and the wall is $\frac{1}{12}$. The rod is in limiting equilibrium and is about to slip up the wall.

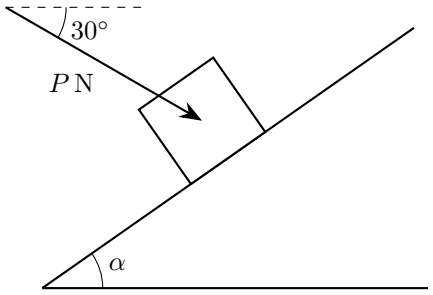
(a) Find the value of k. **[6]**

(b) Find, in terms of W, the magnitude of the total force exerted by the wall on the rod. **[2]**

8. A block B of mass m kg rests on a rough slope whose angle of inclination α to the horizontal can be adjusted. The coefficient of friction between B and the slope is $\frac{1}{2}$.

(a) The slope is set so that $\tan\alpha = \frac{3}{4}$, and B is projected up a line of greatest slope with speed $4\,\mathrm{m\,s^{-1}}$. Find the distance that B travels before it comes to instantaneous rest. **[4]**

(b) Show that B does not remain at rest, and find the magnitude of its acceleration as it slides back down the slope. **[3]**



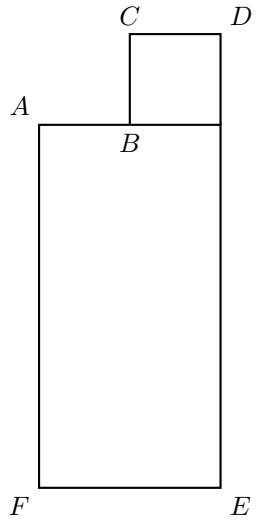
The angle of the slope is now changed. A force of magnitude P N is applied to B in the vertical plane containing a line of greatest slope through B, directed towards the slope at 30° below the horizontal, as shown in the diagram above. P is first set at a value for which B is at rest, and is then gradually increased.

(c) Show that, for B to slide up the slope,

$$P\,(2\cos(\alpha + 30^\circ) - \sin(\alpha + 30^\circ)) > mg\,(2\sin\alpha + \cos\alpha)$$ **[3]**

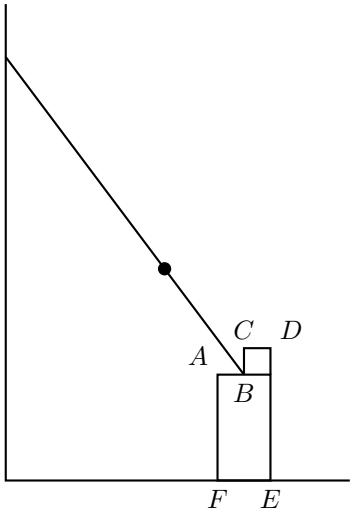
(d) Find, in degrees, the least value of α for which B cannot be made to slide up the slope, however large P becomes. **[2]**

9. Two blocks have rectangular cross sections.
One block has mass 8 kg and its cross section is a rectangle of width 20 cm and height 40 cm.
The other block has mass 2 kg and its cross section is a square with sides of length 10 cm.
The blocks are fixed together to form the composite body shown in the diagram below, in which $FE = 20$ cm, $AF = 40$ cm and $BC = CD = 10$ cm.



(a) Find the distance of the centre of mass of the composite body from AF. **[2]**

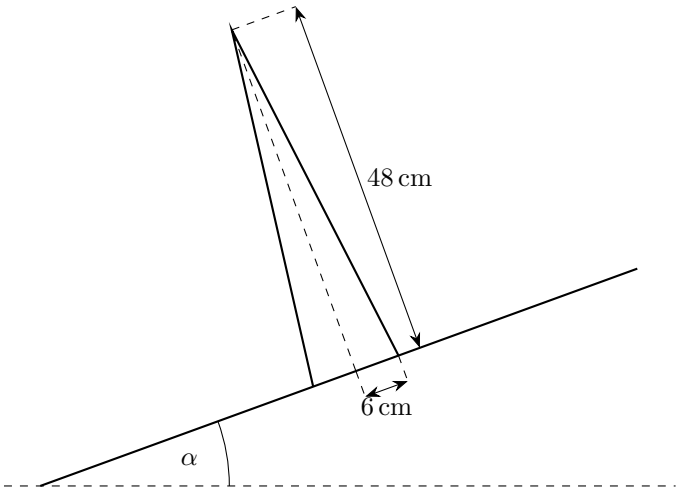
(b) A uniform rod has mass 8 kg and length 1.5 m.
One end of the rod rests against a smooth vertical wall, and the other end rests on the composite body at the point B.
The composite body stands with FE on a rough horizontal surface, and the coefficient of friction between the composite body and the surface is 0.2.
The rod lies in a vertical plane perpendicular to the wall, and the angle between the rod and AB is α, where $\tan\alpha = \frac{4}{3}$.
A particle of mass 6 kg is fixed to the rod at a distance of x m from B.
The rod, particle and composite body are shown in the diagram below.



(i) Write down the magnitude of the vertical reaction force acting on the rod at B, in terms of g. **[1]**

(ii) Show that the magnitude of the horizontal reaction force acting on the rod at B is $3g(1 + x)$. **[3]**

(iii) Find the greatest value of x for which the composite body neither slides nor topples. Fully justify your answer. **[6]**



10. A uniform solid right circular cone L has base radius 6 cm, height 48 cm and mass 6 kg. L is placed with its base on a rough plane inclined at an angle α to the horizontal, as shown in the diagram above.

(a) On a copy of the diagram, mark the forces acting on L. **[1]**

The coefficient of friction between L and the plane is 0.6. Initially $\alpha = 20°$.

(b) Show that L rests in equilibrium on the plane. **[4]**

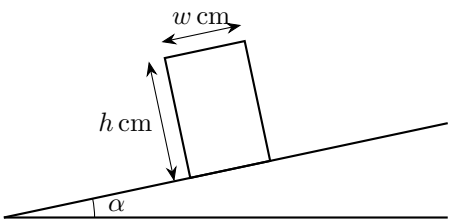
A couple is applied to L, acting in the vertical plane of the diagram. It is given that L will topple if the couple is applied in an anticlockwise sense, but L will not topple if the couple is applied in a clockwise sense.

(c) Find the range of possible values of the magnitude of the couple. **[4]**

The couple is now removed and the plane is slowly tilted so that α increases from 20°.

(d) Determine whether L topples first without sliding or slides first without toppling, and find the value of α at which L first ceases to be in equilibrium. **[5]**

11. A block of mass m kg is modelled as a uniform rectangular prism whose cross section has width w cm and height h cm. The block rests on a rough plane inclined at an angle α to the horizontal, with the cross section containing a line of greatest slope, as shown in the diagram below.



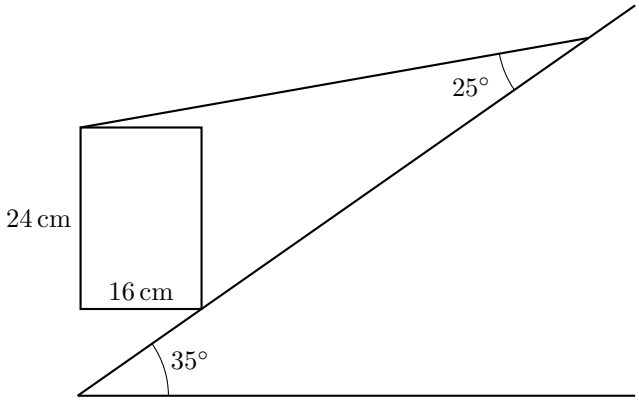
At first the block is in equilibrium, and then α is gradually increased.

(a) Assuming that the block does not slide first, show that it is on the point of toppling when $\tan\alpha = \frac{w}{h}$. **[2]**

It is now given that $w = 16$ and that the coefficient of friction between the block and the plane is 0.8. At first, the block is in equilibrium. As α is increased, equilibrium is broken by the block toppling, rather than sliding.

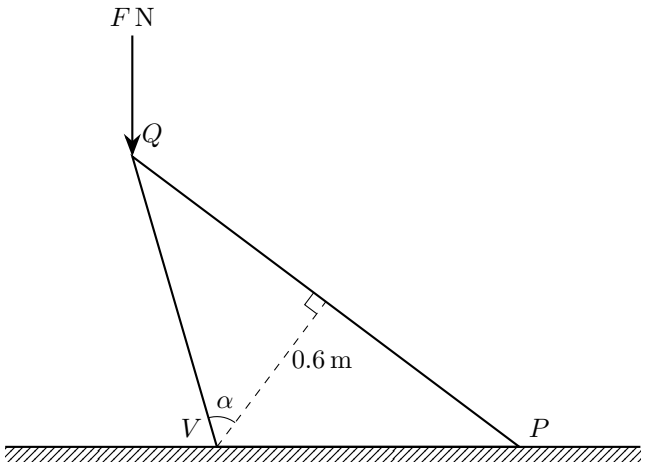
(b) Determine the range of possible values of h. **[4]**

For the remainder of the question you are given that $h = 24$, $m = 12$ and $\alpha = 35°$.
One end of a light inextensible string is attached to the top downslope edge of the block. The other end of the string is attached to a point further up the inclined plane, and the string passes above the block. The string makes an angle of 25° with the inclined plane, so that the block is held in equilibrium with its base horizontal and only its lower upslope edge in contact with the plane, as shown in the diagram below. Both the string and the centre of mass of the block lie in the same vertical plane as a line of greatest slope.



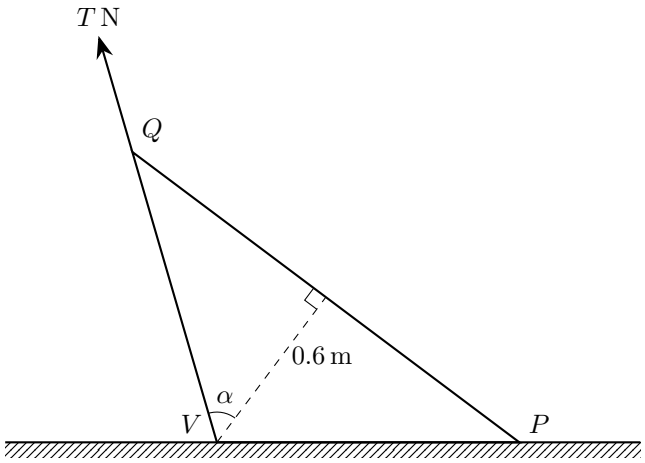
(c) Determine the tension in the string. **[4]**

(d) Determine the angle between the inclined plane and the total contact force that the inclined plane exerts on the block. **[6]**



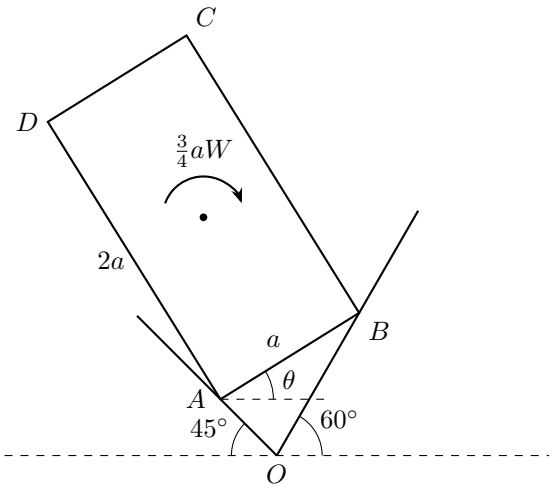
12. A uniform solid cone has height 0.6 m, semi-vertical angle α, where $\tan\alpha = \frac{4}{3}$, and weight 280 N.
The cone lies with its curved surface on a horizontal plane. The point P on the circumference of its base is in contact with the plane, V is the vertex of the cone and PQ is a diameter of its base.
A vertical force of magnitude F N, acting downwards, is applied to the cone at Q, as shown in the first diagram above.

(a) Find the least value of F for which the cone topples about V. **[4]**



The force of magnitude F N is removed. A force of magnitude T N is now applied to the cone at Q, acting along VQ produced in the direction from V to Q, as shown in the second diagram above. The value of T is gradually increased from zero.
The coefficient of friction between the cone and the plane is 0.9.

(b) Determine whether the cone first slides or first topples about P, and find the value of T at which this happens. **[11]**



13. The diagram above shows a uniform rectangular lamina $ABCD$, of weight W, in which $AB = a$ and $AD = 2a$. The lamina is in equilibrium in a vertical plane that also contains the point O. The vertex A rests on a smooth plane inclined at an angle of 45° to the horizontal, and the vertex B rests on a smooth plane inclined at an angle of 60° to the horizontal.
OA is a line of greatest slope of the plane inclined at 45° to the horizontal and OB is a line of greatest slope of the plane inclined at 60° to the horizontal.
The side AB is inclined at an acute angle θ to the horizontal, with B higher than A, and the lamina is kept in equilibrium in this position by a clockwise couple of magnitude $\frac{3}{4}aW$.

(a) By resolving horizontally and vertically, find, in terms of W, the magnitude of the normal contact force between the plane and the lamina at B, giving your answer in simplified surd form. **[6]**

(b) By taking moments about A, show that θ satisfies the equation

$$2\left(5-\sqrt{3}\right)\sin\theta - 2\left(2-\sqrt{3}\right)\cos\theta = 3$$

[5]

(c) Hence find θ, giving your answer in degrees correct to 1 decimal place. **[3]**

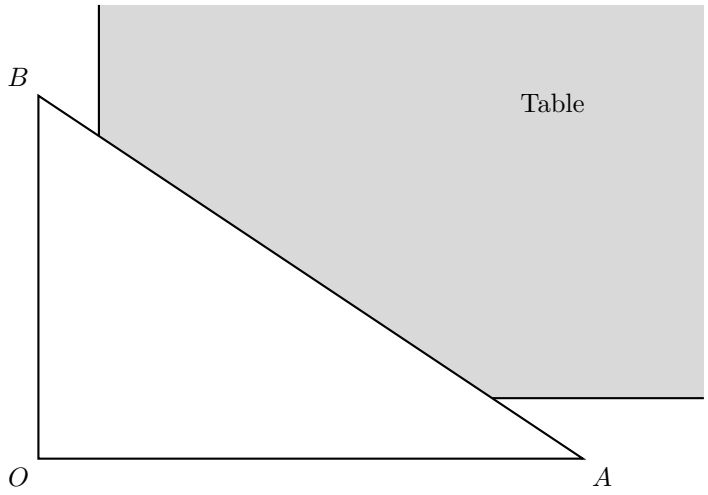
14. A uniform triangular lamina OAB of mass M has vertices at the origin $O(0,0)$, $A(18a,0)$ and $B(0,12a)$, where a is a positive constant. A small object P of mass m is attached to the lamina at the point (x,y). The centre of mass of the system consisting of the lamina and P is at the point $(\bar{x},\bar{y})$. P is modelled as a particle.

(a) Show that

$$\bar{x} = \frac{6Ma + mx}{M + m}$$ **[1]**

(b) Find a corresponding expression for $\bar{y}$. **[1]**

The lamina, with P no longer attached, is placed on a horizontal rectangular table, with OA and OB parallel to the edges of the table, and partly overhanging two edges of the table, as shown in the diagram below. The corner of the table is at the point $(2a, 2a)$.



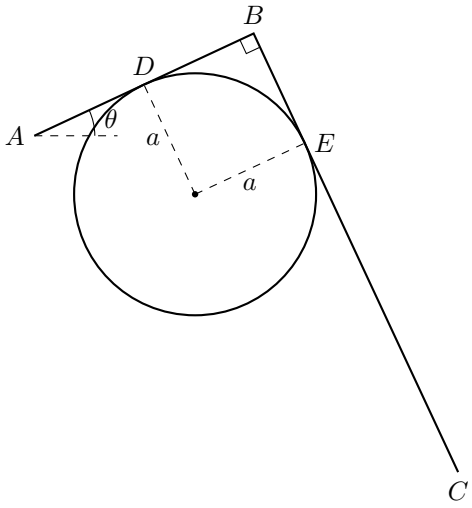
When P is placed on the lamina at O, the lamina topples over one of the edges of the table.

(c) Show that $m > M$. **[4]**

The lamina is now put back on the table in the same position as before. P is placed at the point $(12a, 4a)$ on the smooth upper surface of the lamina, and is projected towards O. At a subsequent instant during the motion, P is at the point $(12ak, 4ak)$, where $0 < k < 1$.

(d) Given that $m > 2M$, and assuming that the lamina has not yet toppled, find, in terms of M and m, the value of k for which the centre of mass of the system lies on the line of the table edge parallel to OB. **[3]**

(e) Given now that $m > M$, show that the lamina topples before P reaches O, and determine which table edge it topples over, justifying your answer. **[5]**



15. A rough cylinder of radius a is fixed with its axis horizontal. A rigid frame consists of two uniform rods AB and BC, rigidly joined at B with AB perpendicular to BC. The rod AB has length $2a$ and weight W, and the rod BC has length $4a$ and weight $2W$.
The frame rests on the cylinder in a vertical plane perpendicular to the axis of the cylinder, with AB at an angle θ to the horizontal, as shown in the diagram above. The rods touch the cylinder at D and E, where D is the midpoint of AB and E is the point of BC with $BE = a$.
The coefficient of friction between each rod and the cylinder is μ, and the frame is about to slip in a clockwise direction. The normal reaction between AB and the cylinder is R and the normal reaction between BC and the cylinder is N.

(a) Find the ratio $R : N$ in terms of μ. **[7]**

(b) Given that $\mu = \frac{2}{3}$, find the value of $\tan\theta$ and find R in terms of W, giving your answer in exact form. **[4]**