Questions

1. **(a)** State the dimensions of force. **[1]**

(b) The braking distance of a vehicle can be modelled by the formula

$$d = m^a u^b R^c$$

where,

m is the mass of the vehicle in kg
u is the initial speed in $\mathrm{m\,s^{-1}}$
R is the constant braking force in N

Use dimensional analysis to find the values of a, b and c. **[4]**

2. A small satellite moves in a circular orbit of radius r about the centre of a much more massive planet of mass M. It is suggested that the period T of the orbit depends only on G, M and r, and that

$$T = kG^{\alpha}M^{\beta}r^{\gamma}$$

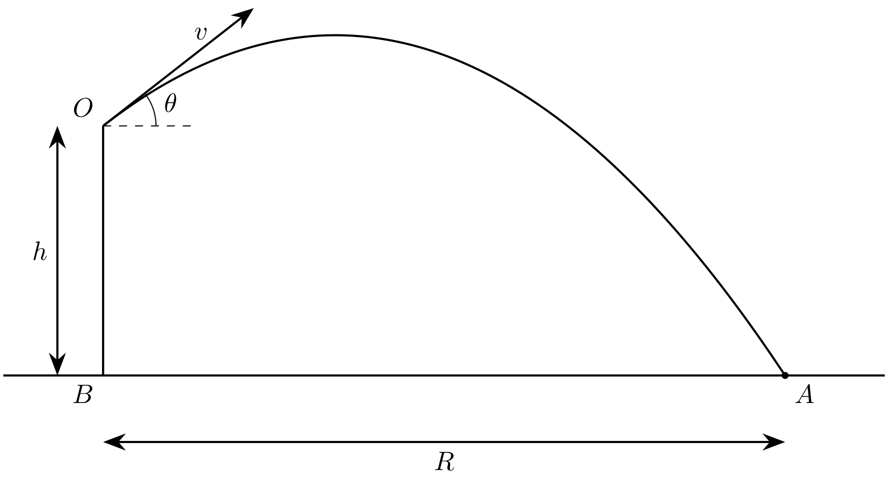
where k is a dimensionless constant.

It is given that

$$G = 6.67 \times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$$

(c) Write down the dimensions of G. **[1]**

(d) By using dimensional analysis, determine the values of α, β and γ. **[3]**



3. A particle is projected with speed v from a point O at the top of a cliff of height h above horizontal ground. The angle of projection is θ above the horizontal. The particle lands at the point A on the ground, and B is the foot of the cliff, so that $BA = R$, as shown in the diagram.

You are given that

$$R = \frac{v^\alpha \cos\theta}{g^\beta}\left(v^\gamma \sin\theta + \sqrt{v^\delta \sin^2\theta + 2gh}\right)$$

Use dimensional analysis to find α, β, γ and δ. **[4]**

4. The increase in pressure p pascals between the surface and the bottom of a liquid of depth h metres and density $\rho\,\mathrm{kg\,m^{-3}}$ is modelled by

$$p = k\rho^{\alpha}h^{\beta}g$$

where $g\,\mathrm{m\,s^{-2}}$ is the acceleration due to gravity and k is a dimensionless constant.

(a) State the dimensions of p. **[1]**

(b) Use dimensional analysis to find the value of α and the value of β. **[3]**

5. The force, F, exerted on a flat plate by a water jet is thought to depend on the density of the water, ρ, the cross-sectional area of the jet, A, and the speed of the water, v. You are given that ρ is measured in $\mathrm{kg\,m^{-3}}$.

(a) Find the dimensions of density ρ. **[2]**

A model of the force suggests the relationship

$$F = \frac{1}{2}\rho^{\alpha}A^{\beta}v^{\gamma}$$

(b) Explain whether or not it is possible to use dimensional analysis to verify that the constant $\frac{1}{2}$ is correct. **[1]**

(c) Use dimensional analysis to find the values of α, β and γ. **[4]**

6. A motor pulls a crate of mass m along a rough horizontal floor. The equation below is suggested to model the pulling force exerted by the motor, F, while the crate moves in a straight line.

$$F = k_1 m v \frac{\mathrm{d}v}{\mathrm{d}x} + k_2 \mu m g + k_3 \frac{Q}{x}$$

in which,

- v is the speed of the crate when it has travelled distance x
- μ is the coefficient of friction between the crate and the floor
- g is the acceleration due to gravity
- Q is the total thermal energy transferred to the surroundings after the crate has moved distance x
- k_1, k_2 and k_3 are dimensionless constants

Determine whether the equation is dimensionally consistent. **[6]**

7. The speed of a particle falling vertically through a resistive medium is modelled by the formula

$$v = \frac{mg}{k}\left(1 - e^{-kt/m}\right)$$

where,

v is the speed of the particle
m is the mass of the particle
g is the acceleration due to gravity
t is the time since the particle was released
k is a constant

Use dimensional analysis to determine the dimensions of k. **[4]**

8. A comet of mass m measured in kg follows an elliptical orbit around a planet. The semi-major axis of the orbit is a measured in metres and the orbital period is T measured in seconds. The eccentricity e of the orbit is dimensionless.

A constant μ associated with the planet is defined by

$$T^2 = \frac{4\pi^2 a^3}{\mu}$$

and the angular momentum of the comet is given by

$$H = m\sqrt{\mu a(1 - e^2)}$$

By using dimensional analysis, find the dimensions of:

(a) μ **[3]**

(b) H **[3]**

9. A small ball of mass m kg is projected vertically upwards with speed $u\,\mathrm{m\,s^{-1}}$. During the motion, when its speed is $v\,\mathrm{m\,s^{-1}}$, it experiences an air resistance of magnitude kv^2 N, where k is a constant.

(a) Determine the dimensions of k. **[3]**

The greatest height reached by the ball is H m. A formula approximating H is

$$H = \frac{u^2}{2g} - \frac{ku^4}{4mg^2} + \frac{1}{6}k^2 m^\alpha u^\beta g^\gamma$$

where $g\,\mathrm{m\,s^{-2}}$ is the acceleration due to gravity.

(b) Use dimensional analysis to find α, β and γ. **[4]**

10. A small probe is launched vertically upwards from the surface of a spherical moon. The radius of the moon is R metres, and the probe is at a distance r metres from the centre of the moon. The gravitational field strength g at distance r is modelled by

$$g = \frac{\Gamma}{r^2}$$

where Γ is a constant associated with the moon.

The probe has initial speed u at the surface and speed v when it reaches distance r. These speeds are related by

$$v^2 = u^2 - 2\Gamma\left(\frac{1}{R} - \frac{1}{r}\right)$$

The work done against gravity in moving a probe of mass m from the surface to distance r is

$$W = m\Gamma\left(\frac{1}{R} - \frac{1}{r}\right)$$

(a) Find the dimensions of Γ. **[2]**

(b) Use dimensional analysis to show that the equation for v^2 is dimensionally consistent. **[2]**

(c) Find the dimensions and SI units of W. **[2]**