FURTHER MECHANICS / MISCELLANEOUS MECHANICS

Rigid Bodies, Sliding & Toppling

Mark scheme 15 questions · 180 marks

EXAM BOARDS Edexcel FM2 9FM0/4C · AQA 7367/3M · OCR A Y543 · OCR MEI Mech Minor Y431 OCR MEI Mech Major Y421 · CIE 9231 Paper 3

Questions

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	Total	**180**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last mark of a part whose answer is printed in the question: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

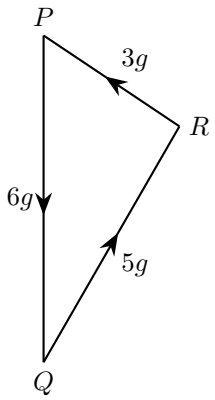
- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)			
	Shape	**B1**	Closed triangle with $6g$ vertical and sides $3g$, $5g$ labelled, arrows following round it. Condone 6, 3, 5 for the sides.
		[1]	
1(b)	$\cos(90° - \alpha) = \dfrac{6^2 + 3^2 - 5^2}{2 \times 6 \times 3}$	**M1**	Uses the cosine rule for the angle between the $6g$ and $3g$ sides, or resolves both ways and eliminates β. Condone sign slips; g may remain.
	$\sin\alpha = \frac{5}{9}$	**A1**	Or $\cos(90° - \alpha) = \frac{5}{9}$, oe.
	$\alpha = 33.7°$	**A1**	awrt 33.7°.
	$\cos(90° - \beta) = \dfrac{6^2 + 5^2 - 3^2}{2 \times 6 \times 5} \Rightarrow \beta = \ldots$	**M1**	Uses the cosine rule for the angle between the $6g$ and $5g$ sides, or the sine rule or resolving with their α, leading to β.
	$\beta = 60.1°$	**A1**	awrt 60.1°.
		[5]	
	Total for Question 1	**6**	

Question	Scheme	Marks	Notes
2(a)	Uniform cone symmetric about its axis, so its centre of mass is on the axis; particle at the centre of the base, also on the axis	**E1**	States that the cone is symmetric about its axis, or its centre of mass is on the axis, and that the particle is on the axis.
		[1]	
2(b)	$6m\bar{x} = 4m \times \frac{h}{4} + 2m \times 0 \Rightarrow \bar{x} = \frac{h}{6}$	**B1**	Centre of mass of S $\frac{h}{6}$ from the base, or the cone's $\frac{h}{4}$ from the base used with the particle's weight separately; either oe from the vertex, soi.
	$T_1 \times h = 6mg \times$ "$\frac{h}{6}$" $\Rightarrow T_1 = \ldots$	**M1**	Takes moments about any point, with the weight and the tensions it needs at their horizontal distances. Condone sign slips.
	$T_1 = mg$	**A1**	Tension mg in the inextensible string.
	$T_1 + T_2 = 6mg \Rightarrow T_2 = \ldots$	**M1**	Resolves vertically with both tensions and the weight, or takes a second moments equation. Condone sign slips.
	$T_2 = 5mg$	**A1**	Tension $5mg$ in the elastic string.
		[5]	
2(c)	Stretched length $2 - 0.8 = 1.2$	**B1**	
	"$5mg$" $= \dfrac{15mg(\text{"}1.2\text{"} - l)}{l} \Rightarrow l = \ldots$	**M1**	Uses $T = \frac{\lambda e}{l}$ with their elastic tension, $\lambda = 15mg$ and extension their stretched length minus l. Condone slips in rearranging.
	$l = 0.9\,\text{m}$	**A1**	
		[3]	
	Total for Question 2	**9**	

Question	Scheme	Marks	Notes
3(a)(i)	$0.2P = 350 \times 0.6$	**M1 A1**	**M1** Forms a moments equation with the whole reaction at C, such as about C with the force at A and the weight, each times a distance. Condone a distance slip.
	$P = 1050\,\text{N}$	**A1**	
		[3]	
3(a)(ii)	$1.5Q = 350 \times 0.6$	**M1 A1**	**M1** Forms a moments equation with the whole reaction at C, such as about C with the force at D and the weight, each times a distance. Condone a distance slip.
	$Q = 140\,\text{N}$	**A1**	
		[3]	
3(b)	$T\cos\theta = \frac{4T}{5}, \quad T\sin\theta = \frac{3T}{5}$	**B1**	Both components, oe such as $0.8T$ and $0.6T$, soi in the moments.
	"$\frac{3T}{5}$" $\times 0.2 +$ "$\frac{4T}{5}$" $\times 0.9$ $= \frac{3T}{25} + \frac{18T}{25}$	**M1 A1**	**M1** Adds the moments about C of their two components, each times a perpendicular distance from C. Condone a distance slip. **A1** Both terms correct, oe such as $0.12T$ and $0.72T$.
	$= \frac{21T}{25}\,\text{N m}$	**A1 AG**	Correct moment sum reaching $\frac{21T}{25}$ without error; $0.84T$ must be shown equal to $\frac{21T}{25}$.
		[4]	
3(c)	$\frac{21T}{25} = 350 \times 0.6$	**M1 A1**	**M1** Forms a moments equation with the reaction and friction at C, such as the moment from (b) equal to the weight's moment about C. Condone a distance slip.
	$T = 250$	**A1**	
	$F = \frac{4}{5} \times 250 = 200\,\text{N}$	**B1**	
	$R = 350 + \frac{3}{5} \times 250 = 500\,\text{N}$	**B1**	
	"200" $< \mu \times$ "500" $\Rightarrow \mu \ldots$	**M1**	Uses $F < \mu R$ with their friction and reaction to find a bound on μ. Condone $\leq$ or $=$.

Question 3 continued

Question	Scheme	Marks	Notes
	$\mu > 0.4$	**A1**	Strict inequality, oe such as $\mu > \frac{2}{5}$. A0 for $\mu \geq 0.4$ or $\mu = 0.4$.
		[7]	
	Total for Question 3	**17**	

Question	Scheme	Marks	Notes
4(a)	$M = \int_0^3 (20 - 4x)\,\mathrm{d}x$ $= \left[20x - 2x^2\right]_0^3$	**M1**	Integrates $20 - 4x$, at least one term correct, and substitutes the limits 0 and 3.
	$M = 42$ (kg)	**A1**	
	$M\bar{x} = \int_0^3 (20x - 4x^2)\,\mathrm{d}x$ $= \left[10x^2 - \frac{4}{3}x^3\right]_0^3$	**M1**	Integrates $x(20 - 4x)$, at least one term correct, and substitutes the limits 0 and 3.
	$M\bar{x} = 54$	**A1**	
	$\bar{x} = \frac{54}{42} = \frac{9}{7}$	**A1 AG**	cso. Correct mass and moment divided, leading without error to $\frac{9}{7}$.
		[5]	
4(b)	$T \times 2\cos 60°$ $= 42g \times \frac{9}{7}\sin 60° + 5g \times 3\sin 60°$	**M1 A1**	**M1** Takes moments about A, or any point with all its relevant forces, each force times a resolved distance. Condone sin/cos confusion. **A1** Correct equation, any equivalent form.
	$T = \frac{69\sqrt{3}\,g}{2} \approx 586\,\mathrm{N}$	**A1**	awrt 586.
	$H = T, \qquad V = 42g + 5g = 47g$	**M1 A1**	**M1** Resolves horizontally and vertically, with all relevant forces in each. **A1** Both correct, H as T or its value.
	$\sqrt{H^2 + V^2} = \sqrt{\text{“586”}^2 + (47g)^2} = \ldots$	**dM1**	Combines their two perpendicular components by Pythagoras.
	$\approx 745\,\mathrm{N}$	**A1**	awrt 745.
		[7]	
4(c)	Greater: the ball's weight acts further from A, so its moment about A is larger.	**E1**	States greater, with a reason from the larger distance or moment of the ball's weight about A. Not for “greater” alone.
		[1]	
	Total for Question 4	**13**	

Question	Scheme	Marks	Notes
5(a)	$F_A = \frac{1}{2}R$ towards the wall, $F_B = \frac{1}{3}S$ upwards	**B1**	Friction limiting at both ends, in these directions, seen or used in the equations.
	$F_A = S$ $R + F_B = 20g + 60g$	**M1 A1**	**M1** Resolves horizontally and vertically, each equation with all relevant forces. Condone sign slips. **A1** Both equations correct, frictions in either form.
	$R = 2S, \quad 2S + \frac{1}{3}S = 80g \Rightarrow S = \ldots$	**dM1**	Substitutes their limiting frictions and solves the two equations together for S.
	$S = \frac{240g}{7} = 336\,\text{N}$	**A1**	$\frac{240g}{7}$ or awrt 336 N.
		[5]	
5(b)	$20g \times 3\cos\alpha + 60g \times x\cos\alpha$ $= S \times 6\sin\alpha + \frac{1}{3}S \times 6\cos\alpha$	**M1 A1**	**M1** Takes moments about any point, with every force that has a nonzero moment. Condone sign slips and sin/cos confusion. **A1** Correct equation, with F_B as $\frac{1}{3}S$ or not.
	$36g + 36gx = 6 \times$ "$\frac{240g}{7}$" $\Rightarrow x = \ldots$	**dM1**	Substitutes their reactions from (a) and solves for AC.
	$AC = \frac{33}{7}$ m $[\approx 4.71\,\text{m}]$	**A1**	$\frac{33}{7}$ or awrt 4.71 m.
		[4]	
	Total for Question 5	**9**	

Question	Scheme	Marks	Notes
6(a)	$4S = 6g \times 2.5\cos\theta$	**M1 A1**	**M1** Takes moments about A, with the peg force perpendicular to the rod at 4 m and the weight at the midpoint. Condone sin/cos confusion. **A1** Correct equation, with $\cos\theta$ or $\frac{4}{5}$.
	$S = 3g \approx 29.4\,\text{N}$	**A1**	$3g$ or awrt 29.4.
		[3]	
6(b)	$4S = 6g \times 2.5\cos\theta + mg \times 5\cos\theta$	**M1**	Takes moments about A, with the peg force perpendicular to the rod and the moments of both weights. Condone sin/cos confusion.
	$S = (3+m)g$	**A1**	
	$F = S\sin\theta$ $R + S\cos\theta = (6+m)g$	**M1**	Resolves horizontally and vertically, each equation with a component of the peg force. Condone sin/cos confusion and sign slips.
	$F = \frac{3(3+m)g}{5}$	**A1**	oe, in terms of m.
	$R = \frac{(18+m)g}{5}$	**A1**	oe, in terms of m.
	"$\frac{3(3+m)g}{5}$" $\leq \frac{3}{4} \times$ "$\frac{(18+m)g}{5}$" $\Rightarrow m \leq \ldots$	**dM1**	Uses $F \leq \frac{3}{4}R$, or $F = \frac{3}{4}R$, with their F and R in terms of m, and solves for m.
	$m = 2$	**A1**	Accept $m \leq 2$.
		[7]	
	Total for Question 6	**10**	

Question	Scheme	Marks	Notes
7(a)	$R = \frac{3T}{5}, \quad F = \frac{1}{12}R$ $\frac{4T}{5} - F = W + kW$	**M1**	Resolves in two perpendicular directions with all relevant forces, using $F = \frac{1}{12}R$ down the wall. Condone sign slips and sine and cosine interchanged.
	$\frac{3T}{4} = W(1+k) \quad \left[\Rightarrow T = \frac{4W(1+k)}{3}\right]$	**A1**	Correct equations, oe.
	$T \times 3a = W \times 3a\sin\theta + kW \times 6a\sin\theta$	**M1**	Takes moments about any point with all relevant forces, each distance resolved. Condone sign slips and sine and cosine interchanged.
	$3aT = \frac{12a}{5}W + \frac{24a}{5}kW$	**A1**	Correct equation, oe.
	$\frac{4W(1+k)}{3} = \frac{4W(1+2k)}{5} \Rightarrow k = \ldots$	**dM1**	Eliminates T (and any other unknown force) between their resolving and moments equations, reaching $k = \ldots$.
	$k = 2$	**A1**	
		[6]	
7(b)	$R = \frac{12W}{5}, \quad F = \frac{W}{5}$ $\sqrt{\left(\frac{12W}{5}\right)^2 + \left(\frac{W}{5}\right)^2}$	**M1**	Finds R and F in terms of W with their k, and combines them by Pythagoras.
	$\frac{\sqrt{145}}{5}W$	**A1**	oe, or awrt $2.41W$.
		[2]	
	Total for Question 7	**8**	

Question	Scheme	Marks	Notes
8(a)	$ma = -\frac{3}{5}mg - \frac{1}{2}\left(\frac{4}{5}mg\right) \Rightarrow a = \ldots$	**M1**	Forms an equation of motion along the slope with ma, the weight component and friction $\frac{1}{2}R$, $R = mg\cos\alpha$. Condone sign slips and sin/cos confusion.
	$a = -g$	**A1**	Deceleration g, or 9.8, either sign.
	$0 = 4^2 + 2\,(\text{“}-g\text{”})\,s \Rightarrow s = \ldots$	**M1**	Uses constant-acceleration formulae with $u = 4$, $v = 0$ and their acceleration to find the distance. Condone sign slips.
	$s = \frac{8}{g} \approx 0.816\,\text{m}$	**A1**	
		[4]	
8(b)	$mg\sin\alpha = \frac{3}{5}mg > \frac{1}{2}R = \frac{2}{5}mg$, so B slides down	**B1**	Compares $\frac{3}{5}mg$ with $\frac{2}{5}mg$, or $\tan\alpha$ with $\frac{1}{2}$, with a conclusion.
	$ma = \frac{3}{5}mg - \frac{2}{5}mg \Rightarrow a = \ldots$	**M1**	Forms an equation of motion with the weight component and their limiting friction in opposite directions. Condone sin/cos confusion.
	$a = \frac{g}{5} = 1.96\,\text{m}\,\text{s}^{-2}$	**A1**	
		[3]	
8(c)	$R = mg\cos\alpha + P\sin(\alpha + 30°)$	**B1**	Any equivalent form of the angle, such as $\cos(60° - \alpha)$.
	$P\cos(\alpha + 30°)$ $> mg\sin\alpha + \frac{1}{2}\,(\text{“}mg\cos\alpha + P\sin(\alpha + 30°)\text{”})$	**M1**	Resolves parallel to the slope with three terms, the components of P and the weight and $\frac{1}{2}$ their R. Equation or inequality accepted; condone sign slips.
	$\Rightarrow P\,(2\cos(\alpha + 30°) - \sin(\alpha + 30°))$ $> mg\,(2\sin\alpha + \cos\alpha)$	**A1 AG**	Correct substituted inequality, or limiting-equilibrium equation with sliding stated as $>$, leading without error to the printed result.
		[3]	
8(d)	$2\cos(\alpha + 30°) - \sin(\alpha + 30°) \leq 0$ $\Rightarrow \tan(\alpha + 30°) = 2 \Rightarrow \alpha = \ldots$	**M1**	Sets the coefficient of P to zero, or ≤ 0, and solves for α. Condone the ratio inverted.

Question 8 continued

Question	Scheme	Marks	Notes
	$\alpha = \tan^{-1} 2 - 30° \approx 33.4°$	**A1**	
		[2]	
	Total for Question 8	**12**	

Question	Scheme	Marks	Notes
9(a)	$10\bar{x} = 8 \times 10 + 2 \times 15$	**M1**	Takes moments about AF with both blocks' masses, their distances and the total mass 10. Condone a slip in one distance.
	$\bar{x} = 11\,\text{cm}$	**A1**	cao, or 0.11 m.
		[2]	
9(b)(i)	$V = 8g + 6g = 14g$	**B1**	
		[1]	
9(b)(ii)	$S \times 1.2 = 8g \times 0.45 + 6g \times 0.6x$	**M1 A1**	**M1** Forms moments for the rod and particle, about B or elsewhere, with all needed terms and perpendicular distances. Condone sign slips and $\sin\alpha$, $\cos\alpha$ interchanged. **A1** Correct equation, oe.
	$S = 3g(1 + x)$ $H = S = 3g(1 + x)$	**A1 AG**	Correct moments equation leading without error to $3g(1 + x)$, with $H = S$ from resolving horizontally stated where S was found.
		[3]	
9(b)(iii)	$R = 10g + 14g = 24g$ $3g(1 + x) \leq 0.2 \times 24g \Rightarrow x \leq \ldots$	**M1**	Resolves for the body, R including both vertical forces, and uses friction $\leq 0.2R$ with friction $3g(1 + x)$. Condone an equation for the inequality.
	$x \leq 0.6$	**A1**	oe; condone $x = 0.6$ from a limiting equation.
	$3g(1 + x) \times 0.4 \leq 10g \times 0.09 + 14g \times 0.1$	**M1**	Takes moments about E, or elsewhere with R at E, with the horizontal force at B and both vertical forces. Condone an equation for the inequality and sign slips.
	$x \leq \frac{11}{12}$	**A1**	oe, or awrt 0.917; condone equality.
	Compares the bounds: $0.6 < \frac{11}{12}$	**M1**	Compares their two bounds on x and takes the smaller as the limiting condition.
	Slides first, so greatest $x = 0.6$	**A1**	cso. $x = 0.6$, with sliding identified as occurring first.
		[6]	
	Total for Question 9	**12**	

Question	Scheme	Marks	Notes
10(a)	Weight $6g$ vertically down through G Normal reaction R perpendicular to the plane Friction F up the plane, along the base	**B1**	All three forces, labelled, in the right directions, with the weight acting through a point on the axis. Any letters accepted.
		[1]	
10(b)	$R = 6g\cos 20°, \quad F = 6g\sin 20°$ $F \leq 0.6R \Rightarrow \tan 20° \leq 0.6$	**M1**	Resolves perpendicular and parallel to the plane and compares the friction needed with $0.6R$, or $\tan 20°$ with 0.6. Condone sign slips.
	$\tan 20° \approx 0.364 < 0.6$, so L does not slide	**A1**	Correct comparison, oe $20.1 < 33.2$, with the conclusion that L does not slide.
	G is 12 cm from the base on the axis $\Rightarrow$ vertical through G meets the plane $12\tan 20°$ from the centre of the base	**M1**	Uses G a quarter of the height from the base in a toppling test, such as this distance or comparing $20°$ with $\tan^{-1}\frac{6}{12}$.
	$12\tan 20° \approx 4.37 < 6$, so L does not topple Neither slides nor topples, so in equilibrium	**A1 AG**	Correct toppling comparison, then equilibrium concluded from no sliding and no toppling. A0 if either conclusion is missing.
		[4]	
10(c)	Moments about the lowest point of the base: $C = 6g\,(0.06\cos 20° - 0.12\sin 20°)$	**M1**	Equates the couple to the moment of the weight about the lower edge, using both components or the weight with its perpendicular distance. Condone sign slips.
	$C > 0.902$	**A1**	awrt 0.902, or 90.2 with N cm units. Inequality not needed for this mark.
	Moments about the highest point of the base: $C = 6g\,(0.06\cos 20° + 0.12\sin 20°)$	**M1**	Equates the couple to the moment of the weight about the upper edge, using both components or the weight with its perpendicular distance. Condone sign slips.
	$0.902 < C \leq 5.73$	**A1**	awrt 0.902 and 5.73, with the signs shown, or $90.2 < C \leq 573$ with N cm units.
		[4]	

Question 10 continued

Question	Scheme	Marks	Notes
10(d)	$6g\sin\alpha = 0.6 \times 6g\cos\alpha \Rightarrow \tan\alpha = 0.6$ $\Rightarrow \alpha = \ldots$	**M1**	Uses limiting friction with α in place of 20° to form an equation in $\tan\alpha$, and solves for α. Condone the ratio inverted.
	$\alpha = 31.0^\circ$	**A1**	awrt 31.0°.
	$12\tan\alpha = 6 \Rightarrow \alpha = \ldots$	**M1**	Sets the vertical through G through the lower edge of the base, or any equivalent toppling condition, and solves for α. Condone the ratio inverted.
	$\alpha = \tan^{-1}\frac{1}{2} = 26.6^\circ$	**A1**	awrt 26.6°.
	$26.6^\circ < 31.0^\circ$: topples first without sliding, at $\alpha = 26.6^\circ$	**A1**	Correct conclusion from comparing both correct angles, with α awrt 26.6°.
		[5]	
	Total for Question 10	**14**	

Question	Scheme	Marks	Notes
11(a)	Weight through the lower downslope edge $\tan\alpha = \frac{\frac{w}{2}}{\frac{h}{2}}$	**M1**	Uses G vertically above the lower downslope edge, or equates moments of the weight's components about it, with offsets $\frac{w}{2}$ and $\frac{h}{2}$. Condone the ratio inverted.
	$\tan\alpha = \frac{w}{h}$	**A1 AG**	Correct toppling condition linked to α, reaching the printed result without error.
		[2]	
11(b)	$F = 0.8R$: $mg\sin\alpha = 0.8\,mg\cos\alpha$	**M1**	Resolves along and perpendicular to the plane with limiting friction $F = 0.8R$ for a sliding condition on α. Condone sin/cos interchange.
	$\tan\alpha = 0.8$	**A1**	oe, such as α awrt $38.7°$.
	Topples first: $\frac{16}{h} <$ "0.8"	**M1**	Shows the toppling value of $\tan\alpha$, or of α, is below their sliding value. Condone $\leq$ or $=$.
	$h > 20$	**A1**	Strict inequality required.
		[4]	
11(c)	String at $35° - 25° = 10°$ to the horizontal	**B1**	soi, or implied by a correct moments equation resolving T another way.
	Moments about the contact edge: $T\cos 10° \times 24 + T\sin 10° \times 16 = 12g \times 8$	**M1 A1**	**M1** Takes moments about the contact edge with the weight and the tension, resolved or with its perpendicular distance. Condone sign slips and sin/cos interchange.
	$T = \frac{96g}{24\cos 10° + 16\sin 10°} \approx 35.6\,\text{N}$	**A1**	awrt 35.6
		[4]	
11(d)	$F + T\cos 25° = 12g\sin 35°$	**M1**	Resolves in one direction, along the plane or another, with the weight, the tension and the contact force. Condone sign slips and sin/cos interchange.
	$F \approx 35.2\,\text{N}$	**A1**	awrt 35.2, or a correct component in their direction.
	$N = 12g\cos 35° + T\sin 25°$	**M1**	Resolves in a second direction with the weight, the tension and the contact force. Condone sign slips and sin/cos interchange.

Question 11 continued

Question	Scheme	Marks	Notes
	$N \approx 111\,\text{N}$	**A1**	awrt 111, or a correct component in their direction.
	$\tan\theta = \frac{\text{“111.4”}}{\text{“35.2”}} \Rightarrow \theta = \ldots$	**dM1**	Finds the angle with the plane from their two components, adjusting for the plane where needed. Condone the ratio inverted.
	$\theta \approx 72.5^\circ$	**A1**	awrt 72.5°
		[6]	
	Total for Question 11	**16**	

Question	Scheme	Marks	Notes
12(a)	$VQ = 1$, $\cos 2\alpha = -\frac{7}{25}$ $\Rightarrow Q$ is 0.28 m horizontally from V	**B1**	Horizontal distance from V to Q, on the side away from P, soi.
	$VG = \frac{3}{4} \times 0.6 = 0.45$ $\Rightarrow 0.45 \cos\alpha = 0.27$ m	**B1**	Horizontal distance from V to G, soi.
	$0.28F = 280 \times 0.27 \Rightarrow F = \ldots$	**M1**	Takes moments about V for F and the weight, each a force times a horizontal distance. Equation or inequality. Condone a slip in one distance.
	$F = 270$	**A1**	
		[4]	
12(b)	$R + 0.96T = 280 \Rightarrow R = 280 - 0.96T$ $S = 0.28T$	**M1 A1**	**M1** Resolves vertically and horizontally, each equation with the relevant component of T. Condone sign slips and sine and cosine interchanged. **A1** Both correct, oe.
	$0.28T = 0.9(280 - 0.96T) \Rightarrow T = \ldots$	**dM1**	Uses $S = 0.9R$ with their R and S, both in terms of T, and solves for T.
	$T = \frac{31500}{143} \approx 220$ (sliding)	**A1**	awrt 220.
	$VP \sin 2\alpha = 0.96$ m	**B1**	Perpendicular distance from P to the line of action of T, or moments of the components of T about P totalling $0.96T$, soi.
	$1 - 0.27 = 0.73$ m	**B1**	Horizontal distance from P to the line of action of the weight, soi.
	$0.96T = 280 \times 0.73$	**M1 A1**	**M1** Takes moments about P with R and S acting at P, equating the moments of T and the weight. Condone a slip in one distance. **A1** Correct equation, oe.
	$T = \frac{2555}{12} \approx 213$ (toppling)	**A1**	awrt 213.
	Compares: "212.9" < "220.3"	**M1**	Compares their values of T for sliding and for toppling about P.
	Topples first about P, at $T \approx 213$	**A1**	Needs both values correct. Conclusion in words with T awrt 213.
		[11]	
	Total for Question 12	**15**	

Question	Scheme	Marks	Notes
13(a)	$R_A \sin 45° = R_B \sin 60°$	**M1 A1**	**M1** Resolves horizontally with components of both normal forces. Condone sin/cos confusion and sign slips.
	$R_A \cos 45° + R_B \cos 60° = W$	**M1 A1**	**M1** Resolves vertically with components of both normal forces and W. Condone sin/cos confusion and sign slips.
	$\frac{\sqrt{3}}{2}R_B + \frac{1}{2}R_B = W \Rightarrow R_B = \ldots$	**dM1**	Eliminates R_A between their two equations to reach R_B in terms of W.
	$R_B = \left(\sqrt{3}-1\right)W$	**A1**	Or $\sqrt{3}W - W$. A0 for an unrationalised form such as $\frac{2W}{\sqrt{3}+1}$.
		[6]	
13(b)	$\frac{1}{2}a\cos\theta - a\sin\theta$	**B1**	Horizontal distance of the centre from A, oe.
	$Wa\left(\frac{1}{2}\cos\theta - \sin\theta\right) + \frac{3}{4}aW$ $= aR_B\cos 60°\cos\theta + aR_B \sin 60°\sin\theta$	**M1 A1**	**M1** Takes moments about A with the weight, the couple and both components of R_B, or R_B with its perpendicular distance. Condone sign and sin/cos slips. **A1** Correct equation, oe.
	$W\left(\frac{1}{2}\cos\theta - \sin\theta\right) + \frac{3}{4}W$ $=$ "$\left(\sqrt{3}-1\right)W$" $\left(\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right)$	**dM1**	Substitutes their R_B in terms of W into their moments equation and collects the $\sin\theta$ and $\cos\theta$ terms.
	$\Rightarrow 2\left(5-\sqrt{3}\right)\sin\theta - 2\left(2-\sqrt{3}\right)\cos\theta = 3$	**A1 AG**	Correct substituted equation with $R_B = \left(\sqrt{3}-1\right)W$, leading without error to the printed equation; combined steps accepted.
		[5]	
13(c)	$R = \sqrt{4\left(5-\sqrt{3}\right)^2 + 4\left(2-\sqrt{3}\right)^2}$ $\tan\alpha = \frac{2-\sqrt{3}}{5-\sqrt{3}} \Rightarrow \alpha = \ldots$	**M1**	Expresses the LHS in a harmonic form, finding R and α from their coefficients. Condone sign slips and the ratio inverted.
	$R = \sqrt{140 - 56\sqrt{3}} \approx 6.558, \quad \alpha \approx 4.687°$	**A1**	awrt 6.56 and 4.69°, or correct values for another harmonic form.

Question 13 continued

Question	Scheme	Marks	Notes
	$\sin(\theta - \alpha) = \frac{3}{6.558} \Rightarrow \theta \approx 31.9°$	**A1**	awrt 31.9°. Withhold if a second value is given as an answer.
		[3]	
	Total for Question 13	**14**	

Question	Scheme	Marks	Notes
14(a)	$(M+m)\bar{x} = 6Ma + mx$ $\Rightarrow \bar{x} = \frac{6Ma+mx}{M+m}$	**B1**	Lamina's centre of mass at $x = 6a$ used in a correct moments equation, leading without error to the printed result.
		[1]	
14(b)	$\bar{y} = \frac{4Ma+my}{M+m}$	**B1**	oe
		[1]	
14(c)	$\bar{x} = \frac{6Ma}{M+m} < 2a \Rightarrow m > \ldots$	**M1**	Uses $\bar{x}$ with $x = 0$ compared with $2a$, as an inequality or the boundary equation, and rearranges for m.
	$m > 2M$	**A1**	
	$\bar{y} = \frac{4Ma}{M+m} < 2a \Rightarrow m > \ldots$	**M1**	Uses $\bar{y}$ with $y = 0$ compared with $2a$, as an inequality or the boundary equation, and rearranges for m.
	$m > M$, and $m > 2M \Rightarrow m > M$, so $m > M$ in either case	**A1 AG**	Both inequalities correct and the right way round, with a conclusion that $m > M$ in either case. Not from one case alone.
		[4]	
14(d)	$\frac{6Ma+12mak}{M+m} = 2a$	**M1 A1**	**M1** Substitutes $x = 12ak$ into $\bar{x}$ from part (a) and sets it equal to $2a$. **A1** Correct equation, oe.
	$k = \frac{m-2M}{6m}$	**A1**	oe, such as $\frac{1}{6} - \frac{M}{3m}$.
		[3]	
14(e)	$\frac{4Ma+4mak}{M+m} = 2a \Rightarrow k = \ldots$	**M1**	Substitutes $y = 4ak$ into $\bar{y}$ from part (b), sets it equal to (or less than) $2a$ and solves for k.
	$k = \frac{m-M}{2m}$	**A1**	oe
	$m > M \Rightarrow 0 < \frac{m-M}{2m} < 1$ so $\bar{y} < 2a$ before P reaches O	**A1**	Shows $0 < k < 1$ using $m > M$, and concludes that the lamina topples before P reaches O.
	Compares with when $\bar{x} = 2a$ $M < m \leq 2M \Rightarrow \bar{x} > \frac{6Ma}{M+m} \geq 2a$ $m > 2M \Rightarrow$ "$\frac{m-M}{2m}$" $-$ "$\frac{m-2M}{6m}$" $= \frac{2m-M}{6m} > 0$	**M1**	Compares their k with when $\bar{x} = 2a$, such as part (d)'s k, or compares $\bar{x}$ at their k with $2a$.

Question 14 continued

Question	Scheme	Marks	Notes
	Topples over the edge parallel to OA $(y = 2a)$	**A1**	cso. Covers both $M < m \leq 2M$ and $m > 2M$, or gives one argument valid for all $m > M$.
		[5]	
	Total for Question 14	**14**	

Question	Scheme	Marks	Notes
15(a)	Friction μR at D towards A, μN at E towards B	**B1**	Both frictions limiting and in these directions, soi by their equations.
	$N - \mu R = 3W\sin\theta$ $R + \mu N = 3W\cos\theta$	**M1 A1**	**M1** Resolves in two perpendicular directions with all relevant forces, weights resolved. Condone sign slips and sin/cos confusion. **A1** Both correct, oe in any two perpendicular directions.
	$\mu Ra + \mu Na + Wa\sin\theta$ $= 2Wa(\cos\theta + \sin\theta)$	**M1 A1**	**M1** Takes moments about any point with all relevant forces, each with a distance of the correct form. Condone sign slips.
	$3\mu(R+N) = (N - \mu R) + 2(R + \mu N)$	**dM1**	Eliminates W and θ from their three equations to reach an equation in R, N and μ only.
	$R : N = (1-\mu) : (4\mu - 2)$	**A1**	oe, such as $\frac{R}{N} = \frac{1-\mu}{2(2\mu-1)}$.
		[7]	
15(b)	$N = 2R$	**B1ft**	ft their ratio from (a) with $\mu = \frac{2}{3}$.
	$\frac{4}{3}R = 3W\sin\theta, \quad \frac{7}{3}R = 3W\cos\theta$ $\Rightarrow \tan\theta = \ldots$	**M1**	Substitutes $\mu = \frac{2}{3}$ and their $N = kR$ into two of their equations from (a) and eliminates R and W.
	$\tan\theta = \frac{4}{7}$	**A1**	
	$R = \frac{9W\sin\theta}{4} = \frac{9\sqrt{65}}{65}W$	**A1**	Any exact equivalent, such as $\frac{9W}{\sqrt{65}}$.
		[4]	
	Total for Question 15	**11**	