FURTHER MECHANICS / MISCELLANEOUS MECHANICS

Dimensional Analysis

Mark scheme 10 questions · 48 marks

EXAM BOARDS AQA 7367/3M · OCR A Y543 · OCR MEI Mech Minor Y431 · OCR MEI Mech Major Y421

Questions

Question	Marks by part	Marks	Page
Question 1	(a) 1 (b) 4	5	4
Question 2	(a) 1 (b) 4	5	5
Question 3	(a) 2 (b) 2 (c) 2	6	6
Question 4	(a) 1 (b) 3	4	7
Question 5	(a) 1 (b) 1 (c) 4	6	8
Question 6		6	9
Question 7		2	10
Question 8	(a) 2 (b) 2	4	11
Question 9	(a) 2 (b) 4	6	12
Question 10		4	13
	Total	**48**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last mark of a part whose answer is printed in the question: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1(a)	$[F] = \mathrm{MLT}^{-2}$	**B1**	Not for the units $\mathrm{kg\,m\,s^{-2}}$ or N.
		[1]	
1(b)	$\mathrm{L} = \mathrm{M}^{\alpha}(\mathrm{LT}^{-1})^{\beta}(\mathrm{MLT}^{-2})^{\gamma}$	**M1**	Equates the dimensions of both sides, using their $[F]$ from (a) for $[R]$. Condone one slip in a dimension. Powers need not be expanded.
	M: $\alpha + \gamma = 0$ L: $\beta + \gamma = 1$ T: $-\beta - 2\gamma = 0$	**A1**	All three correct, oe.
	$\Rightarrow \alpha = \ldots,\ \beta = \ldots,\ \gamma = \ldots$	**dM1**	Solves their three equations by any means to find all three powers.
	$\alpha = 1,\ \beta = 2,\ \gamma = -1$	**A1**	
		[4]	
Total for Question 1		**5**	

Question	Scheme	Marks	Notes
2(a)	$[G] = \mathrm{M}^{-1}\mathrm{L}^{3}\mathrm{T}^{-2}$	**B1**	Symbols in any order. Not for the units $\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$.
		[1]	
2(b)	$\mathrm{T} = (\mathrm{M}^{-1}\mathrm{L}^{3}\mathrm{T}^{-2})^{\alpha}\mathrm{M}^{\beta}\mathrm{L}^{\gamma}$	**M1**	Forms a dimensional equation using their $[G]$ from (a) and the dimensions of T, M and r, each raised to its power. Condone one slip in a dimension.
	M: $-\alpha + \beta = 0$ L: $3\alpha + \gamma = 0$ T: $-2\alpha = 1$	**A1**	All three correct, oe.
	$\alpha = \ldots,\ \beta = \ldots,\ \gamma = \ldots$	**dM1**	Solves their three equations from equating powers for all three values. Condone sign slips.
	$\alpha = -\frac{1}{2},\ \beta = -\frac{1}{2},\ \gamma = \frac{3}{2}$	**A1**	All three correct, oe.
		[4]	
Total for Question 2		**5**	

Question	Scheme	Marks	Notes
3(a)	$[\Gamma] = [g][r^2] = \mathrm{LT}^{-2} \times \mathrm{L}^2$	**M1**	Uses $\Gamma = gr^2$ with $[g] = \mathrm{LT}^{-2}$ and $[r] = \mathrm{L}$. Condone a slip in combining powers.
	$[\Gamma] = \mathrm{L}^3\mathrm{T}^{-2}$	**A1**	
		[2]	
3(b)	$[v^2] = [u^2] = \mathrm{L}^2\mathrm{T}^{-2}$ $\left[2\Gamma\left(\frac{1}{R} - \frac{1}{r}\right)\right] = \text{“}\mathrm{L}^3\mathrm{T}^{-2}\text{”} \times \mathrm{L}^{-1}$	**M1**	Finds the dimensions of the speed terms and of the Γ term, using their $[\Gamma]$ of the form $\mathrm{L}^\alpha\mathrm{T}^\beta$ and L^{-1} for the bracket.
	$= \mathrm{L}^2\mathrm{T}^{-2}$ All terms $\mathrm{L}^2\mathrm{T}^{-2}$, so consistent	**A1 AG**	cso, from a correct $[\Gamma]$. $[\Gamma]$ must not come from assuming this equation consistent. All three terms shown as $\mathrm{L}^2\mathrm{T}^{-2}$, with a conclusion in words. Comment on the factor 2 not required.
		[2]	
3(c)	$[W] = \mathrm{M} \times \text{“}\mathrm{L}^3\mathrm{T}^{-2}\text{”} \times \mathrm{L}^{-1}$	**M1**	Multiplies $[m] = \mathrm{M}$, their $[\Gamma]$ of the form $\mathrm{L}^\alpha\mathrm{T}^\beta$ and L^{-1} for the bracket. Condone a slip in combining powers.
	$[W] = \mathrm{ML}^2\mathrm{T}^{-2}$; $\mathrm{kg\,m^2\,s^{-2}}$ or J	**A1**	Both the dimensions and the units; accept J, N m or $\mathrm{kg\,m^2\,s^{-2}}$.
		[2]	
	Total for Question 3	**6**	

Question	Scheme	Marks	Notes
4(a)	$[p] = \mathrm{ML}^{-1}\mathrm{T}^{-2}$	**B1**	Not for the units Pa or $\mathrm{kg\,m^{-1}\,s^{-2}}$.
		[1]	
4(b)	$\mathrm{ML}^{-1}\mathrm{T}^{-2} = (\mathrm{ML}^{-3})^{\alpha}(\mathrm{L})^{\beta}(\mathrm{LT}^{-2})$ $= \mathrm{M}^{\alpha}\mathrm{L}^{-3\alpha+\beta+1}\mathrm{T}^{-2}$	**M1**	Equates their $[p]$ to a product of powers of dimensions for ρ, h and g, with k dimensionless. Condone one slip in a dimension.
	M: $\alpha = 1$	**A1**	
	L: $-3\alpha + \beta + 1 = -1 \Rightarrow \beta = 1$	**A1**	
		[3]	
	Total for Question 4	**4**	

Question	Scheme	Marks	Notes
5(a)	$[\rho] = \mathrm{ML}^{-3}$	**B1**	Accept $\mathrm{M/L}^3$ oe. Not for the units $\mathrm{kg\,m}^{-3}$.
		[1]	
5(b)	Not possible: $\frac{1}{2}$ is dimensionless	**E1**	States it is not possible because the constant is dimensionless, or a pure number. Not for "no" alone.
		[1]	
5(c)	$\mathrm{MLT}^{-2} = (\mathrm{ML}^{-3})^{\alpha}(\mathrm{L}^2)^{\beta}(\mathrm{LT}^{-1})^{\gamma}$	**M1**	Forms a dimensional equation using their $[\rho]$ from (a) and the dimensions of F, A and v. Condone one slip in a dimension, and $\frac{1}{2}$ left in.
	M: $\alpha = 1$ T: $-\gamma = -2$ L: $-3\alpha + 2\beta + \gamma = 1$	**A1**	All three equations correct, oe.
	$-3(1) + 2\beta + 2 = 1 \Rightarrow \beta = \ldots$	**dM1**	Solves their three equations, substituting their α and γ to find β.
	$\alpha = 1$, $\beta = 1$, $\gamma = 2$	**A1**	
		[4]	
Total for Question 5		**6**	

Question	Scheme	Marks	Notes
6	$\left[k_1 mv\frac{\mathrm{d}v}{\mathrm{d}x}\right] = \mathrm{M}\times\mathrm{LT}^{-1}\times\frac{\mathrm{LT}^{-1}}{\mathrm{L}}$	**M1**	Replaces the quantities in at least one term by their dimensions and combines them. Condone one slip.
	$= \mathrm{MLT}^{-2}$	**A1**	Accept $v\frac{\mathrm{d}v}{\mathrm{d}x}$ treated as an acceleration, LT^{-2}.
	$[k_2\mu mg] = \mathrm{M}\times\mathrm{LT}^{-2} = \mathrm{MLT}^{-2}$	**A1**	k_2 and μ dimensionless, stated or implied by their omission.
	$[Q] = \mathrm{ML}^2\mathrm{T}^{-2}$	**B1**	
	$\left[k_3\frac{Q}{x}\right] = \frac{\mathrm{ML}^2\mathrm{T}^{-2}}{\mathrm{L}} = \mathrm{MLT}^{-2}$	**A1**	
	$[F] = \mathrm{MLT}^{-2}$, the same as every term $\Rightarrow$ dimensionally consistent	**E1**	States $[F] = \mathrm{MLT}^{-2}$ and concludes consistent, with all three terms correctly shown as MLT^{-2}.
	Total for Question 6	**6**	

Question	Scheme	Marks	Notes
7	Exponent dimensionless: $\left[\frac{kt}{m}\right] = 1$ $\Rightarrow \frac{[k]\mathrm{T}}{\mathrm{M}} = 1 \Rightarrow [k] = \ldots$	**M1**	Sets the dimensions of $\frac{kt}{m}$ equal to 1 with $[t]$ and $[m]$ substituted, or equates $\left[\frac{mg}{k}\right]$ with $[v]$ in M, L, T.
	$[k] = \mathrm{MT}^{-1}$	**A1**	Any equivalent form. Not for the units $\mathrm{kg\,s^{-1}}$.
	Total for Question 7	**2**	

Question	Scheme	Marks	Notes
8(a)	$[T]^2 = \frac{[a]^3}{[\mu]} \Rightarrow \mathrm{T}^2 = \frac{\mathrm{L}^3}{[\mu]}$	**M1**	Substitutes $[T] = \mathrm{T}$ and $[a] = \mathrm{L}$, with $4\pi^2$ treated as dimensionless, into the equation. Condone slips in rearranging.
	$[\mu] = \mathrm{L}^3\mathrm{T}^{-2}$	**A1**	oe, such as $\mathrm{L}^3/\mathrm{T}^2$. Not for the units $\mathrm{m}^3\,\mathrm{s}^{-2}$.
		[2]	
8(b)	$[H] = \mathrm{M}\sqrt{\text{“}\mathrm{L}^3\mathrm{T}^{-2}\text{”}\,\mathrm{L}}$ $= \mathrm{M}\left(\mathrm{L}^4\mathrm{T}^{-2}\right)^{\frac{1}{2}}$	**M1**	Uses $[m] = \mathrm{M}$, $[a] = \mathrm{L}$, $1 - e^2$ dimensionless and their $[\mu]$ in terms of L and T, and halves the powers under the root.
	$[H] = \mathrm{ML}^2\mathrm{T}^{-1}$	**A1**	oe. Not for the units $\mathrm{kg\,m}^2\,\mathrm{s}^{-1}$.
		[2]	
	Total for Question 8	**4**	

Question	Scheme	Marks	Notes
9(a)	$[k][v]^2 = [F]$ $\Rightarrow [k](\mathrm{LT}^{-1})^2 = \mathrm{MLT}^{-2}$	**M1**	Equates the dimensions of kv^2 to those of force, with $[F]$ in terms of M, L and T. Condone a slip in squaring $[v]$.
	$[k] = \mathrm{ML}^{-1}$	**A1**	oe, such as $\mathrm{ML}^{-1}\mathrm{T}^0$. Not for the units $\mathrm{kg\,m}^{-1}$.
		[2]	
9(b)	$[k^2 m^\alpha u^\beta g^\gamma] = \mathrm{L}$ $(\text{“}\mathrm{ML}^{-1}\text{”})^2\,\mathrm{M}^\alpha(\mathrm{LT}^{-1})^\beta(\mathrm{LT}^{-2})^\gamma = \mathrm{L}$	**M1**	Sets the dimensions of the third term equal to L, or to those of another term, using their $[k]$ in M, L, T. Condone one slip in $[u]$ or $[g]$.
	M: $2 + \alpha = 0$ L: $-2 + \beta + \gamma = 1$ T: $-\beta - 2\gamma = 0$	**A1**	All three equations correct, oe.
	$\beta = -2\gamma \Rightarrow -2 - 2\gamma + \gamma = 1$ $\Rightarrow \gamma = \ldots$	**dM1**	Solves their three equations to find all of α, β and γ, by any method.
	$\alpha = -2,\ \beta = 6,\ \gamma = -3$	**A1**	
		[4]	
	Total for Question 9	**6**	

Question	Scheme	Marks	Notes
10	$(\mathrm{LT}^{-1})^{\delta} = \mathrm{L}^2\mathrm{T}^{-2}$ $(\mathrm{LT}^{-1})^{\gamma} = \sqrt{\mathrm{L}^2\mathrm{T}^{-2}} = \mathrm{LT}^{-1}$	**M1**	Equates the dimensions of the terms added inside the square root, or of $u^{\gamma}\sin\theta$ with the root, taking $\sin\theta$ as dimensionless. Condone one dimension slip.
	$\delta = 2,\ \gamma = 1$	**A1**	
	$\mathrm{L} = \frac{(\mathrm{LT}^{-1})^{\alpha}}{(\mathrm{LT}^{-2})^{\beta}}\,\mathrm{LT}^{-1}$ $\Rightarrow \alpha - \beta + 1 = 1,\ -\alpha + 2\beta - 1 = 0$ $\Rightarrow \alpha = \ldots,\ \beta = \ldots$	**M1**	Uses their bracket dimensions as a power product of L and T, equates powers of L and T with $[R]$ and solves for α and β. Condone sign slips.
	$\alpha = 1,\ \beta = 1$	**A1**	
	Total for Question 10	**4**	