

FURTHER MECHANICS / KINEMATICS & DYNAMICS

Projectile Motion

Questions by topic

9 questions · 93 marks

Instructions

Unless a question states otherwise, take $g = 9.8 \text{ m s}^{-2}$ wherever a numerical value of g is required. Solutions also give final answers for $g = 10 \text{ m s}^{-2}$ where these differ.

Questions

Question 1	8 marks	2
Question 2	6 marks	3
Question 3	9 marks	4
Question 4	9 marks	5
Question 5	14 marks	6
Question 6	12 marks	7
Question 7	11 marks	8
Question 8	7 marks	9
Question 9	17 marks	10

1. A particle P is projected from a point O at the edge of a cliff, 40 m above horizontal ground. Its initial speed is $u \text{ m s}^{-1}$ and its angle of projection is 30° above the horizontal, directed away from the cliff.
The particle moves freely under gravity until it strikes the ground.

Immediately before impact, the direction of motion of P is perpendicular to its initial direction of motion.

Find the initial speed u and the time taken to reach the ground, giving both answers in exact form.

[8]

- 2.** A particle P is projected from a point O , which is 30 m above horizontal ground, with speed $u \text{ m s}^{-1}$ at an angle θ above the horizontal, where $0 < \theta < 90^\circ$.

The particle moves freely under gravity and lands on the ground at a horizontal distance of 40 m from O .

The velocity of P immediately before it lands is perpendicular to its initial velocity.

Find u and θ , giving the angle to one decimal place.

[6]

- 3.** A particle P is projected from a point O at time $t = 0$ with speed $u \text{ m s}^{-1}$ at an angle θ above the horizontal, where $0 < \theta < 90^\circ$.
The particle moves freely under gravity, with $g = 10 \text{ m s}^{-2}$.
After 6 seconds, its speed is $\sqrt{5}u \text{ m s}^{-1}$.

(a) Show that

$$u^2 + 30u \sin \theta - 900 = 0 \quad [3]$$

- (b) At time $t = 6 \text{ s}$, the particle is at a point A below the horizontal level of O .
The displacement vector $\overrightarrow{OA}$ is perpendicular to the initial velocity of P .
Find the exact value of u , the value of θ , and the exact distance OA .

[6]

4. At time $t = 0$, a particle P is projected from a point O on horizontal ground with speed 26 m s^{-1} at an angle α above the horizontal, where $\sin \alpha = \frac{12}{13}$.
A point A on the ground lies 40 m from O in the direction of the horizontal motion of P .

At time $t = T \text{ s}$, where $T > 0$, a second particle Q is projected vertically upwards from A with speed 22 m s^{-1} .

The particles collide while Q is travelling upwards.

Before the collision, both particles move freely under gravity.

Neglect air resistance throughout.

(a) Find the time at which the particles collide and their height above the ground at that instant. [2]

(b) Find the exact value of T . [3]

(c) Immediately after the collision, the speed of P is $v \text{ m s}^{-1}$ and its direction of motion is above the horizontal.

Its horizontal velocity component is unchanged by the collision, and it subsequently moves freely under gravity.

Find its greatest height above the ground in terms of v . Hence find the least possible value of v for P to reach a height of 40 m above the ground. [4]

5. A particle P is projected from a point A which is vertically above a fixed point O on horizontal ground, with $OA = h$.

Its initial velocity is $u(4\mathbf{i} + 3\mathbf{j})$, where $u > 0$ and $h > 0$.

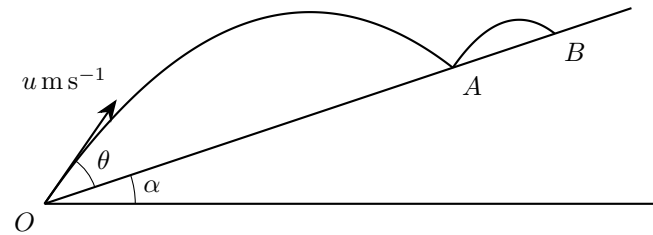
The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are horizontal and vertically upwards respectively.

The particle moves freely under gravity, with constant acceleration $-g\mathbf{j}$.

At time $t = 0$, the particle leaves A .

Its position vector relative to O at time t is $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$.

- (a) Starting from an expression for $\ddot{\mathbf{r}}$, use integration to obtain $\mathbf{r}$ in terms of t , u , h and g . [4]
- (b) Eliminate t to obtain the Cartesian equation of the path of P . [3]
- (c) Hence find the maximum height of P above the ground, in terms of u , h and g . [3]
- (d) When P first strikes the ground, its velocity is perpendicular to its initial velocity. Determine h in terms of u and g . [4]



6. A small ball P is projected from a point O at the foot of a smooth plane inclined at an angle α to the horizontal, where $\tan \alpha = \frac{1}{3}$. Its initial speed is $u \text{ m s}^{-1}$, and its initial velocity makes an angle θ above the plane, where $\tan \theta = \frac{3}{4}$.

The ball first strikes the plane at A , where $OA = 15\sqrt{10} \text{ m}$.

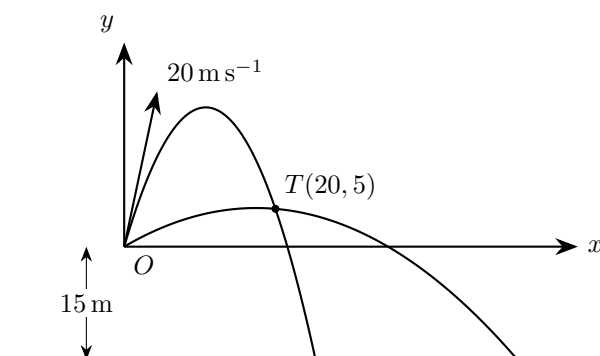
Its motion takes place in the vertical plane containing a line of greatest slope of the inclined plane, and air resistance is negligible.

- (a) Find u and the components of the ball's velocity parallel and perpendicular to the plane immediately before impact at A . State the direction of each component. [8]

The ball rebounds from the plane at A and next strikes it at B , further up the plane, where

$$AB = \frac{15\sqrt{10}}{4} \text{ m}$$

- (b) Find the coefficient of restitution between the ball and the plane, explaining why any other algebraic solution must be rejected. [4]



7. A launcher projects balls at speed 20 m s^{-1} from a point O , which is 15 m above horizontal ground. Horizontal and vertically upwards displacements from O are denoted by x m and y m respectively. A small target T has coordinates $(20, 5)$, as shown in the diagram above.

At time $t = 0$, a ball B is projected at an angle θ above the positive horizontal direction, where $0 \leq \theta < 90^\circ$. Model the ball as a particle moving freely under gravity until it first reaches the ground.

- (a) By first writing x and y in terms of t , find the two possible values of θ for which the ball passes through T . Give the angles to the nearest 0.1° . [5]
- (b) For a fixed horizontal displacement $x > 0$, use a quadratic in $\tan \theta$ to determine the greatest height y that the ball can reach. Hence state the equation of the upper boundary of the reachable region to the right of the y -axis. [3]
- (c) Hence find the greatest possible horizontal distance from O to the point where the ball first reaches the ground, giving your answer exactly. Find also the launch angle that achieves this distance, to the nearest 0.1° . [3]

8. A plane is inclined at an unknown angle α above the horizontal, where $0 < \alpha < 90^\circ$. Points O and A lie on a line of greatest slope of the plane, with A uphill from O and $OA = 18$ m.

A particle is projected from O with speed $u \text{ ms}^{-1}$ at an angle θ above the horizontal, where $\tan \theta = 3$ and $0 < \theta < 90^\circ$.

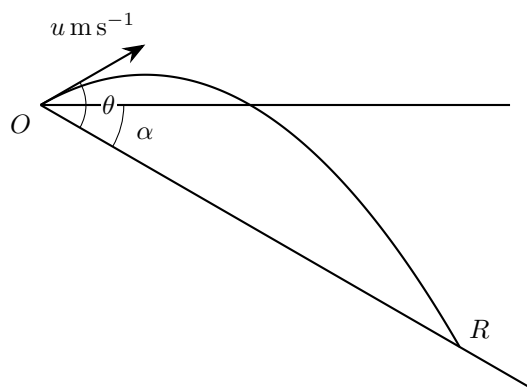
Its initial horizontal velocity is directed towards A .

The particle moves freely under gravity in the vertical plane containing O and A .

The particle strikes the inclined plane at A , and its velocity immediately before impact is perpendicular to the inclined plane.

Find all possible pairs of values of α and u , giving each value to 3 significant figures.

[7]



9. A small ball P is projected from a point O on a plane which slopes downwards at an angle α to the horizontal, where $0 < \alpha < \frac{\pi}{2}$.

Its initial speed is $u \text{ m s}^{-1}$, and its initial velocity makes an angle θ above the downhill direction of the plane.

The ball moves in a vertical plane containing a line of greatest slope and next meets the plane at R , where $OR = d \text{ m}$.

Assume that air resistance is negligible and that $0 < \theta < \frac{\pi}{2} + \alpha$.

- (a) Derive an expression for the time of flight in terms of u , θ , α and g .
Hence show that the speed required to reach R satisfies

$$u^2 = \frac{gd \cos^2 \alpha}{2 \sin \theta \cos(\theta - \alpha)} \quad [8]$$

- (b) The point R is fixed. Using a trigonometric identity, determine the minimum possible value of u , and the value of θ for which this minimum occurs. Give your answers in terms of d , g and α . [4]
- (c) Now take $\alpha = 30^\circ$, $d = 20$ and $g = 10 \text{ m s}^{-2}$. The ball is projected with speed $5\sqrt{6} \text{ m s}^{-1}$. Find the two possible values of θ and the corresponding times of flight, giving the times exactly. [5]