FURTHER MECHANICS / KINEMATICS & DYNAMICS

Projectile Motion

Mark scheme 9 questions · 101 marks

EXAM BOARDS OCR MEI Mech Major Y421 · CIE 9231 Paper 3

Questions

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	Total	**101**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last mark of a part whose answer is printed in the question: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

Question	Scheme	Marks	Notes
1	$v_{x0} = u\cos 30° = \frac{\sqrt{3}}{2}u$, $v_{y0} = u\sin 30° = \frac{1}{2}u$	**B1**	Both components.
	Impact 60° below the horizontal: $v_y = -\frac{\sqrt{3}}{2}u\tan 60° = \ldots$	**M1**	Uses perpendicularity, with the horizontal component unchanged, to find v_y in terms of u, by a 60° angle or a zero scalar product. Condone a sign slip.
	$v_y = -\frac{3}{2}u$	**A1**	oe; $\frac{3}{2}u$ downwards accepted.
	$\left(\text{“}-\frac{3}{2}u\text{”}\right)^2 = \left(\frac{1}{2}u\right)^2 + 2(-9.8)(-40)$	**M1 A1**	**M1** Uses $v^2 = v_0^2 + 2as$, or other constant acceleration formulae, vertically with their $v_y = ku$, $\frac{1}{2}u$ and displacement 40 m. Condone sign slips. **A1** Correct equation in u, oe.
	$2u^2 = 784 \Rightarrow u = 14\sqrt{2}$	**A1**	Any exact form, e.g. $\sqrt{392}$.
	$\text{“}-\frac{3}{2}u\text{”} = \frac{1}{2}u - 9.8t$ $\Rightarrow t = \frac{2\times\text{“}14\sqrt{2}\text{”}}{9.8} = \ldots$	**M1**	Uses a vertical constant acceleration formula, with their u or their $v_y = ku$, and solves for t. Condone sign slips.
	$t = \frac{20\sqrt{2}}{7}$ s	**A1**	Any exact form, with any negative root of a quadratic rejected.
	Total for Question 1	**8**	

Question	Scheme	Marks	Notes
2	Vertical velocity at landing $= -u\cos\theta\tan(90^\circ-\theta) = -u\cos\theta\cot\theta$	**M1 A1**	**M1** Uses perpendicularity, by angles, gradients with product -1 or a zero scalar product, to give the landing vertical velocity in terms of u and θ. Condone sign slips. **A1** oe, such as $-\frac{u\cos^2\theta}{\sin\theta}$, with sign consistent with their positive direction.
	$T = \frac{40}{u\cos\theta}$ $-30 = \frac{1}{2}\left(u\sin\theta - \text{“}u\cos\theta\cot\theta\text{”}\right)T$	**M1**	Forms the vertical displacement -30 with $T = \frac{40}{u\cos\theta}$, by $\frac{1}{2}(v_0+v)T$ with their landing vertical velocity, or by $v_0T + \frac{1}{2}aT^2$, when the next A1 also needs u eliminated. Condone sign slips.
	$-30 = 20(\tan\theta - \cot\theta)$ $\Rightarrow 2\tan^2\theta + 3\tan\theta - 2 = 0$	**A1**	Any correct equation in θ alone, oe.
	$(2\tan\theta - 1)(\tan\theta + 2) = 0$ $\Rightarrow \tan\theta = \ldots$	**dM1**	Solves their equation, such as a three-term quadratic in $\tan\theta$ by any method, or via $\tan 2\theta$.
	$\tan\theta = \frac{1}{2}$ [$\tan\theta = -2$ rejected] $\Rightarrow \theta = 26.6^\circ$	**A1**	cao, 26.6° only; $\tan\theta = -2$ rejected or ignored.
	$-\text{“}u\cos\theta\cot\theta\text{”} = u\sin\theta - \frac{40g}{u\cos\theta}$ $\Rightarrow u^2 = 40g\tan\theta \Rightarrow u = \ldots$	**M1**	Uses a vertical equation in g, such as $v = v_0 + at$ with their landing vertical velocity, with $T = \frac{40}{u\cos\theta}$ and their θ, to find u. Condone sign slips.
	$u = 14$	**A1**	
	Total for Question 2	**8**	

Question	Scheme	Marks	Notes
3(a)	$v_x = u\cos\theta,\ v_y = u\sin\theta - 60$ $(u\cos\theta)^2 + (u\sin\theta - 60)^2 = (\sqrt{5}u)^2$	**M1**	Finds both velocity components at $t = 6$ and equates the sum of their squares to $5u^2$. Condone a sign slip in v_y.
	$u^2 - 120u\sin\theta + 3600 = 5u^2$	**A1**	Correct, with $\cos^2\theta + \sin^2\theta$ replaced by 1, oe.
	$u^2 + 30u\sin\theta - 900 = 0$	**A1 AG**	Correct speed equation leading without error to the printed result; combined steps accepted.
		[3]	
3(b)	$x = 6u\cos\theta,\ y = 6u\sin\theta - 180$ $6u^2\cos^2\theta + (6u\sin\theta - 180)u\sin\theta = 0$	**M1**	Finds both displacement components at $t = 6$ and uses perpendicularity to the initial velocity, by zero scalar product or gradients with product -1. Condone sign slips.
	$6u^2 - 180u\sin\theta = 0 \Rightarrow u = 30\sin\theta$	**A1**	oe, such as $\sin\theta = \frac{u}{30}$.
	Substitutes into (a): $u^2 + 30u \cdot \text{“}\frac{u}{30}\text{”} - 900 = 0 \Rightarrow u = \ldots$	**dM1**	Eliminates u or $\sin\theta$ between their $u = k\sin\theta$ and the equation from (a), and solves for u or $\sin\theta$.
	$u = 15\sqrt{2},\ \sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$	**A1**	Both, u exact (oe such as $\sqrt{450}$), with no negative u offered as an answer.
	$x = 90,\ y = -90$ $OA = \sqrt{\text{“}90\text{”}^2 + \text{“}90\text{”}^2}$	**M1**	Uses their u and θ in both displacement components at $t = 6$ and finds the magnitude, or another valid method for OA.
	$OA = 90\sqrt{2}$	**A1**	Exact, oe such as $\sqrt{16200}$.
		[6]	
Total for Question 3		**9**	

Question	Scheme	Marks	Notes
4(a)	$10t = 40 \Rightarrow t = \ldots$ $h = 24\text{“}t\text{”} - \frac{1}{2}(9.8)\text{“}t\text{”}^2$	**M1**	Finds t from the horizontal motion with resolved velocity and substitutes it into a two-term vertical displacement. Condone sign slips.
	$t = 4\,\text{s}$, height $17.6\,\text{m}$	**A1**	Both values.
		[2]	
4(b)	$\text{“}17.6\text{”} = 22(4-T) - 4.9(4-T)^2$	**M1**	Equates Q's two-term vertical displacement over time their t from (a) minus T to their height from (a). Condone sign slips.
	$49T^2 - 172T + 80 = 0$	**A1**	Correct three-term quadratic oe, including one in the time $4-T$.
	$T = \ldots \left[= \frac{86 \pm 2\sqrt{869}}{49}\right]$	**dM1**	Solves their three-term quadratic by formula, completing the square or calculator, reaching $T = \ldots$ or $4 - T = \ldots$
	$22 - 9.8(4-T) > 0 \Rightarrow T > \frac{86}{49}$	**M1**	Uses Q's velocity at the collision, or its time to greatest height, to choose the root with Q rising.
	$T = \frac{86+2\sqrt{869}}{49}$	**A1**	Any exact equivalent, with the other root rejected.
		[5]	
4(c)	$0 = (v^2 - 10^2) - 2(9.8)(H - \text{“}17.6\text{”})$	**M1**	Uses $v_y^2 = v_{y0}^2 + 2as$ vertically, with $v_{y0}^2 = v^2 - \text{“}10\text{”}^2$ and $v_y = 0$ at the top, or an equivalent energy equation. Condone sign slips.
	$H = 17.6 + \frac{v^2-100}{19.6}$	**A1**	Height above the ground, oe. The rise $\frac{v^2-100}{19.6}$ alone scores A0.
	$\text{“}17.6 + \frac{v^2-100}{19.6}\text{”} \geq 40 \Rightarrow v = \ldots$	**M1**	Sets their greatest height in terms of v equal to or at least 40 and solves for v.
	$v = \frac{2\sqrt{3369}}{5}$ $[\approx 23.2]$	**A1**	oe, or awrt 23.2.
		[4]	
	Total for Question 4	**11**	

Question	Scheme	Marks	Notes
5(a)	$\ddot{\mathbf{r}} = -g\mathbf{j}$ $\Rightarrow \dot{\mathbf{r}} = c_1\mathbf{i} + (-gt + c_2)\mathbf{j}$	**M1**	Integrates the acceleration with respect to t, with a constant or limits, as a vector or by components. Not for constant acceleration formulae quoted.
	$\dot{\mathbf{r}} = 4u\mathbf{i} + (3u - gt)\mathbf{j}$	**A1**	
	$\mathbf{r} = \text{“}4u\text{”}t\,\mathbf{i} + \left(\text{“}3u\text{”}t - \frac{1}{2}gt^2\right)\mathbf{j} + \mathbf{c}$ $\mathbf{r} = h\mathbf{j}$ at $t = 0 \Rightarrow \mathbf{c} = \ldots$	**dM1**	Integrates their velocity, each power of t increased by 1, and finds the constant from $\mathbf{r} = h\mathbf{j}$ at $t = 0$, or uses limits.
	$\mathbf{r} = 4ut\,\mathbf{i} + \left(h + 3ut - \frac{1}{2}gt^2\right)\mathbf{j}$	**A1**	
		[4]	
5(b)	$t = \frac{x}{4u}$ $\Rightarrow y = h + 3u\left(\frac{x}{4u}\right) - \frac{1}{2}g\left(\frac{x}{4u}\right)^2$	**M1 A1**	**M1** Makes t the subject of their x, a multiple of t, and substitutes into their y. **A1** Correct, unsimplified.
	$y = h + \frac{3x}{4} - \frac{gx^2}{32u^2}$	**A1**	Simplified, oe, such as $32u^2y = 32u^2h + 24u^2x - gx^2$.
		[3]	
5(c)	$y = h - \frac{g}{32u^2}\left[\left(x - \frac{12u^2}{g}\right)^2 - \ldots\right]$	**M1**	Completes the square on their equation from (b), of the form $y = h + px - qx^2$, or sets their $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. Condone sign and coefficient slips.
	$x = \frac{12u^2}{g}$	**A1**	soi by a correct maximum height.
	Maximum height $h + \frac{9u^2}{2g}$	**A1**	oe. Height above the ground, so must include h.
		[3]	
5(d)	$(4u\mathbf{i} + 3u\mathbf{j}) \cdot (4u\mathbf{i} + v_y\mathbf{j}) = 0$ $\Rightarrow 16u^2 + 3uv_y = 0$	**M1**	Sets the scalar product of the initial velocity and an impact velocity with horizontal component $4u$ to zero, or the product of the gradients to -1.
	$v_y = -\frac{16u}{3}$	**A1**	oe, such as a downward component $\frac{16u}{3}$.
	$v^2 = v_0^2 + 2as$: $\left(\text{“}-\frac{16u}{3}\text{”}\right)^2 = (3u)^2 + 2(-g)(-h)$	**M1**	Uses $v^2 = v_0^2 + 2as$ vertically with their v_y, $v_0 = 3u$ and displacement $-h$, or any equivalent vertical route, such as energy. Condone sign slips.

Question 5 continued

Question	Scheme	Marks	Notes
	$h = \frac{175u^2}{18g}$	**A1**	cso, oe.
		[4]	
	Total for Question 5	**14**	

Question	Scheme	Marks	Notes
6(a)	$v_{x0} = u\cos(\alpha+\theta) = \frac{9u}{5\sqrt{10}}$ $v_{y0} = u\sin(\alpha+\theta) = \frac{13u}{5\sqrt{10}}$	**B1**	Both, oe; or $\frac{4}{5}u$ and $\frac{3}{5}u$ along and normal to the plane in a solution working in those directions.
	$45 = \frac{9u}{5\sqrt{10}}t$ $15 = \frac{13u}{5\sqrt{10}}t - \frac{1}{2}gt^2$	**M1 A1**	**M1** Uses $s = v_0t + \frac{1}{2}at^2$ in two perpendicular directions, with the displacement to A and g resolved in those directions. Condone sign slips. **A1** Both equations correct.
	$ut = 25\sqrt{10}$ $15 = 65 - \frac{1}{2}gt^2 \Rightarrow t = \ldots$ $[= \frac{10}{\sqrt{g}}]$ $u = \frac{25\sqrt{10}}{\text{“}t\text{”}} = \ldots$	**dM1**	Eliminates u or t between their equations and solves for both.
	$u = \frac{35\sqrt{2}}{2} \approx 24.7\,\text{m s}^{-1}$	**A1**	
	$v_x = \frac{9}{2}\sqrt{g}$ $v_y = \frac{13}{2}\sqrt{g} - g\text{“}\frac{10}{\sqrt{g}}\text{”} = -\frac{7}{2}\sqrt{g}$ $v_\parallel = v_x\cos\alpha + v_y\sin\alpha = \ldots$ $v_\perp = -v_x\sin\alpha + v_y\cos\alpha = \ldots$	**M1**	Uses $v = v_0 + at$ with their positive flight time and resolves both components parallel and perpendicular to the plane, or works in those directions. Condone sign slips.
	Parallel: $7\sqrt{2} \approx 9.90\,\text{m s}^{-1}$ up the plane	**A1**	Needs the direction, in words or as a sign with its positive direction stated.
	Perpendicular: $\frac{21\sqrt{2}}{2} \approx 14.8\,\text{m s}^{-1}$ towards the plane	**A1**	Needs the direction, in words or as a sign with its positive direction stated.
		[8]	
6(b)	$v_\parallel = \sqrt{10g}$ up the plane $v_\perp = \frac{3e}{2}\sqrt{10g}$ away from the plane	**B1ft**	ft their (a) components: parallel unchanged, perpendicular e times their approach component, away from the plane.
	$0 = \frac{3e}{2}\sqrt{10g}\,t - \frac{3g}{2\sqrt{10}}t^2 \Rightarrow t = \ldots$	**M1**	Uses $s = v_0t + \frac{1}{2}at^2$ normal to the plane with zero displacement, their $v_\perp$ in terms of e and acceleration $g\cos\alpha$. Condone sign slips.
	$t = \frac{10e}{\sqrt{g}}$	**A1**	
	$AB = \sqrt{10g}\text{“}\frac{10e}{\sqrt{g}}\text{”}$ $\quad - \frac{g}{2\sqrt{10}}\text{“}\frac{10e}{\sqrt{g}}\text{”}^2$	**M1**	Uses $s = v_0t + \frac{1}{2}at^2$ along the plane with their $v_\parallel$, acceleration $g\sin\alpha$ and their t in terms of e. Condone sign slips.
	$AB = 5\sqrt{10}(2e - e^2)$	**A1**	oe.

Question 6 continued

Question	Scheme	Marks	Notes
	$\frac{15\sqrt{10}}{4} = "5\sqrt{10}(2e - e^2)"$ $\Rightarrow 4e^2 - 8e + 3 = 0 \Rightarrow e = \ldots$	**dM1**	Equates their quadratic in e to $\frac{15\sqrt{10}}{4}$ and solves it by factorising, the formula or completing the square; a calculator is allowed.
	$e = \frac{1}{2}$ or $e = \frac{3}{2}$	**A1**	
	Rejects $e = \frac{3}{2}$ since $0 \leq e \leq 1$, so $e = \frac{1}{2}$	**E1**	States that e cannot exceed 1 to reject $\frac{3}{2}$, and concludes $e = \frac{1}{2}$.
		[8]	
	Total for Question 6	**16**	

Question	Scheme	Marks	Notes
7(a)	$x = 20t\cos\theta$ $y = 20t\sin\theta - 4.9t^2$	**M1 A1**	**M1** Uses $s = v_0t + \frac{1}{2}at^2$ for both components, with $20\cos\theta$, $20\sin\theta$ and acceleration $\pm g$ vertically only. Condone sign slips. **A1** Both equations correct.
	$5 = 20\tan\theta - 4.9\left(1 + \tan^2\theta\right)$	**dM1**	Eliminates t and substitutes $x = 20$, $y = 5$, in either order, reaching an equation in one trigonometric ratio of θ, or a harmonic form in 2θ. Condone sign and coefficient slips.
	$49\tan^2\theta - 200\tan\theta + 99 = 0$	**A1**	oe, such as $4.9\tan^2\theta - 20\tan\theta + 9.9 = 0$ or $10\sin 2\theta - 2.5\cos 2\theta = 7.4$.
	$\tan\theta = \frac{100 \pm \sqrt{5149}}{49}$ $\Rightarrow \theta = 30.0°$ or $74.1°$	**A1**	Both, awrt $30.0°$ and $74.1°$.
		[5]	
7(b)	$\frac{49x^2}{4000}\tan^2\theta - x\tan\theta + y + \frac{49x^2}{4000} = 0$ $x^2 - 4\left(\frac{49x^2}{4000}\right)\left(y + \frac{49x^2}{4000}\right) \geq 0$	**M1**	Writes the trajectory as a quadratic in $\tan\theta$ and sets its discriminant ≥ 0 or $= 0$. Condone coefficient slips.
	$y \leq \frac{1000}{49} - \frac{49x^2}{4000}$	**A1**	Greatest height $\frac{1000}{49} - \frac{49x^2}{4000}$ oe, coefficients exact or to 3 s.f.
	$y = \frac{1000}{49} - \frac{49x^2}{4000}$ $(x > 0)$	**B1ft**	ft their greatest height, written as $y = \ldots$; neither $x > 0$ nor the upper limit on x is required.
		[3]	
7(c)	$-15 = \frac{1000}{49} - \frac{49R^2}{4000} \Rightarrow R = \ldots$	**M1**	Sets $y = -15$ in their boundary of the form $y = a - bx^2$ and solves for x.
	$R = \frac{100\sqrt{694}}{49}$	**A1**	Any exact form.
	$\tan\theta = \frac{2000}{49R} = \frac{20}{\sqrt{694}} \Rightarrow \theta = 37.2°$	**A1**	awrt $37.2°$.
		[3]	
	Total for Question 7	**11**	

Question	Scheme	Marks	Notes
8	$0 = u\cos(\theta - \alpha) - gt\sin\alpha$ $0 = ut\sin(\theta - \alpha) - \frac{1}{2}gt^2\cos\alpha$	**M1 A1**	**M1** Uses both impact conditions, in plane or horizontal and vertical components, each equation in u, t and α. Condone sign slips and sin/cos interchanged. **A1** Both correct, in any consistent components.
	$\tan(\theta - \alpha) = \frac{\cos\alpha}{2\sin\alpha}$ $\Rightarrow 2\tan^2\alpha - 3\tan\alpha + 1 = 0 \Rightarrow \tan\alpha = \ldots$	**dM1**	Eliminates u and t, uses $\tan\theta = 3$ to form a quadratic in $\tan\alpha$ (or another single ratio) and solves it. Condone expansion slips.
	$\tan\alpha = \frac{1}{2}$ or 1 $[\alpha = 26.6°$ or $45°]$	**A1**	Both values, as $\tan\alpha$ or α.
	$18 = ut\cos(\theta - \alpha) - \frac{1}{2}gt^2\sin\alpha$ $\Rightarrow 18 = \frac{1}{2}gt^2\sin\alpha \Rightarrow t^2 = \ldots$	**M1**	Sets the displacement along the plane to 18, or horizontal displacement to $18\cos\alpha$, and eliminates u or t to reach t^2 or u^2. Condone sign slips.
	$u^2 = \frac{36g\sin\alpha}{\cos^2(\theta-\alpha)}$	**A1**	Correct, oe. Both correct u^2 values, $\frac{72g}{\sqrt{5}}$ and $\frac{45g}{\sqrt{2}}$, imply it.
	$\alpha = 26.6°,\ u = 17.8$ $\alpha = 45.0°,\ u = 17.7$	**A1**	Both pairs, correctly matched, awrt, and no others.
	Total for Question 8	**7**	

Question	Scheme	Marks	Notes
9(a)	Components $u\cos(\theta-\alpha)$ and $u\sin(\theta-\alpha)$	**B1**	Both, horizontal and vertical; or $u\cos\theta$ and $u\sin\theta$ along and perpendicular to the plane.
	$x = ut\cos(\theta-\alpha)$ $y = ut\sin(\theta-\alpha) - \frac{1}{2}gt^2$	**M1 A1**	**M1** Uses $s = v_0t + \frac{1}{2}at^2$ in both directions with their components, and g resolved on plane axes. Condone sign slips. **A1** Both correct, on either axes.
	$ut\sin(\theta-\alpha) - \frac{1}{2}gt^2 = -ut\cos(\theta-\alpha)\tan\alpha$	**M1**	Uses the condition at R with their displacements, $y = -x\tan\alpha$ oe, or eliminates d from $x = d\cos\alpha$, $y = -d\sin\alpha$, or zero displacement perpendicular to the plane.
	$\frac{1}{2}gt\cos\alpha$ $= u\left[\sin(\theta-\alpha)\cos\alpha + \cos(\theta-\alpha)\sin\alpha\right]$ $= u\sin\theta$	**dM1**	Divides by t and makes t the subject, in terms of u, θ, α and g. Condone sign slips.
	$t = \frac{2u\sin\theta}{g\cos\alpha}$	**A1**	oe, unsimplified accepted; the $\sin(A+B)$ step may come in the u^2 working.
	$d\cos\alpha = u\cos(\theta-\alpha) \times$ "$\frac{2u\sin\theta}{g\cos\alpha}$"	**M1**	Equates a displacement to R to its value in d, using their time in terms of u, θ, α and g.
	$gd\cos^2\alpha = 2u^2\sin\theta\cos(\theta-\alpha)$ $\Rightarrow u^2 = \frac{gd\cos^2\alpha}{2\sin\theta\cos(\theta-\alpha)}$	**A1 AG**	Correct substituted equation from a correct time, rearranged without error to the printed result; on plane axes, by the $\cos(A-B)$ identity.
		[8]	
9(b)	$2\sin\theta\cos(\theta-\alpha) = \sin(2\theta-\alpha) + \sin\alpha$	**M1**	Writes the denominator as a sine or cosine of $2\theta-\alpha$ plus a constant, by product-to-sum or compound- and double-angle identities. Condone sign slips.
	Denominator greatest when $\sin(2\theta-\alpha) = 1$ $\Rightarrow \theta = \ldots$	**dM1**	Sets their sine (or cosine) of $2\theta-\alpha$ to its maximum to minimise u, and solves for θ.
	$\theta = 45° + \frac{1}{2}\alpha$	**A1**	oe, including $\frac{\pi}{4} + \frac{\alpha}{2}$.
	$u^2 = \frac{gd\cos^2\alpha}{1+\sin\alpha} \Rightarrow u_{\min} = \sqrt{gd(1-\sin\alpha)}$	**A1**	oe, such as $\cos\alpha\sqrt{\frac{gd}{1+\sin\alpha}}$; need not be simplified.
		[4]	
9(c)	$(5\sqrt{6})^2 = \frac{10\times20\cos^2 30°}{2\sin\theta\cos(\theta-30°)}$	**M1**	Substitutes the given u, d, g and α into the result of (a).

Question 9 continued

Question	Scheme	Marks	Notes
	$\Rightarrow 2\sin\theta\cos(\theta - 30°) = 1$ $\Rightarrow \sin(2\theta - 30°) = \frac{1}{2}$	**A1**	Or any correct equation from which θ follows directly, such as $\cos\theta(\sqrt{3}\sin\theta - \cos\theta) = 0$.
	$\theta = 30°$ or $90°$	**A1**	Both, and no others in $0 < \theta < 120°$.
	$t = \frac{2(5\sqrt{6})\sin\theta}{10\cos 30°}$ $[= 2\sqrt{2}\sin\theta]$	**M1**	Substitutes the values and one of their angles into their time of flight from (a), or into $x = d\cos\alpha$ with their horizontal velocity.
	$\theta = 30°$: $t = \sqrt{2}$; $\theta = 90°$: $t = 2\sqrt{2}$	**A1**	Both exact, each paired with its angle.
		[5]	
	Total for Question 9	**17**	