FURTHER MECHANICS / KINEMATICS & DYNAMICS

Motion Under a Variable Force

Mark scheme 14 questions · 162 marks

EXAM BOARDS Edexcel FM2 9FM0/4C · OCR A Y543 · OCR MEI Mech Major Y421 · CIE 9231 Paper 3

Questions

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	Total	**162**	

Reading the mark scheme

These rules apply throughout, unless a note says otherwise.

Marks

Every mark is worth one, so **M1 A1** on a row is two marks. Only marks coded ft follow through.

Code	Meaning
M1	Method: a correct method, applied to this question. A slip doesn't cost it if the method is clear. A formula must be used, not just quoted.
dM1	Dependent method: as M1, but only if the method mark before it was earned, unless its note names another.
A1	Accuracy: a correct answer or step. It needs the method mark it follows.
A1ft	As A1, following through from an earlier answer that the note names.
A1 AG	The last mark of a part whose answer is printed in the question: complete working, with no errors.
B1	A correct value, fact or statement. It needs no method mark.
B1ft	As B1, following through from an earlier answer that the note names.
E1	A written explanation, interpretation or conclusion. Concluding a proof by induction needs every earlier mark.

Abbreviations

ft and AG qualify a code in the Marks column. The others appear in the notes. A grey line gives the rule where the name alone doesn't.

ft	follow through Correct work from the student's own earlier wrong value earns the mark.	**AG**	answer given The answer is printed in the question, so every step to it must be shown.
cao	correct answer only No follow-through from an earlier error.	**cso**	correct solution only No error anywhere in the part, not just in its answer.
oe	or equivalent	**awrt**	answers which round to
soi	seen or implied Earned if written, or if later working shows it was used.	**isw**	ignore subsequent working A correct answer, once reached, isn't spoiled by later working.
SC	special case Credit, as the note says, for work outside the main method.	**their**	the student's own earlier value
condone	a slip that doesn't cost the mark	[ ]	working the mark doesn't need
s.f.	significant figures	**d.p.**	decimal places

Reading the scheme

- Each row is a line of working, with the marks it earns beside it. Where a row carries two marks, each note starts with its code.
- [$\boldsymbol{n}$] ends each part with its total.
- A shaded *Alternative* is a second method for the same total. Mark an answer by the method it follows.
- Any other valid method earns the equivalent marks, unless the question names a method or says *hence*.
- A sketch earns one B1 for each feature named below its figure.

Accuracy and answers

- Unless a question says otherwise, give answers to 3 significant figures, and angles in degrees to 1 decimal place. Angles in radians take 3 significant figures.
- A final answer with more figures is accepted if it rounds to the scheme's value; one with fewer is not. An answer spoiled by rounding too early loses its accuracy mark.
- Where a question asks for an exact answer, a decimal loses the final accuracy mark. Otherwise either is accepted.
- Wrong or missing units cost nothing unless the question asks for them.
- An omitted $\mathrm{d}\theta$, $\mathrm{d}x$ or $\mathrm{d}u$ in an integral is not penalised where the variable of integration is clear from the integrand and the limits. It is penalised where it is genuinely ambiguous: a substitution part-way through, two variables in play, or limits in the wrong variable.
- A correct answer with no working earns full marks, unless the question needs the working: show that, prove, hence, a method it names, or an exact answer. A wrong answer with no working earns nothing. Calculator answers are treated the same way.
- A correct answer, once reached, isn't spoiled by later working (isw).

Value of g

- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question states a value, which is then the only one accepted. Answers that use $g = 9.8$ may be given to 2 or 3 significant figures.
- Where a question doesn't state g, an answer worked with $g = 10$ throughout earns the same marks, keeping any value printed in the question. The worked solutions list the final answers with $g = 10$.

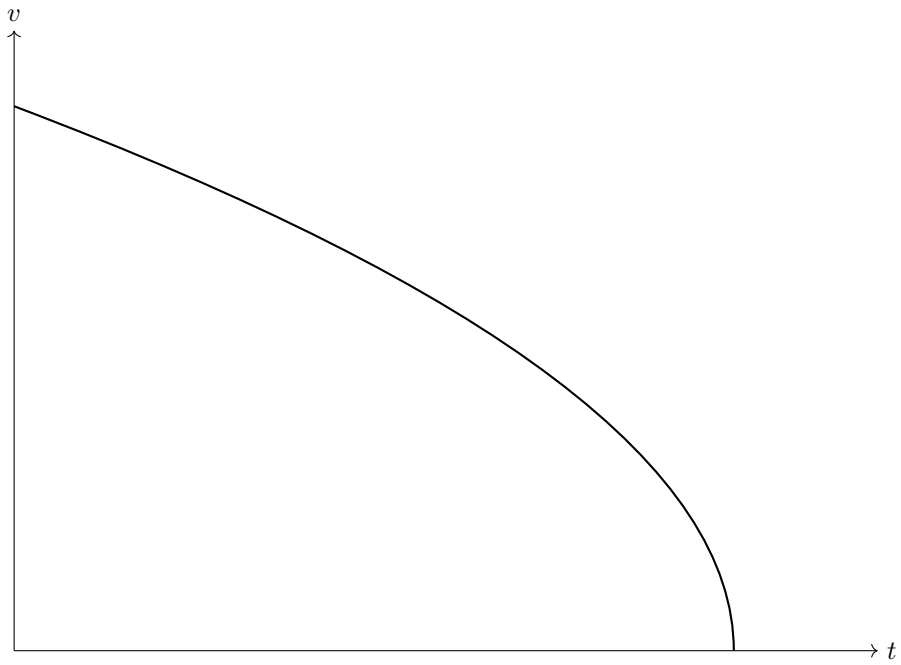
Question	Scheme	Marks	Notes
1(a)	$4v\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{36}{(x+2)^3}$	**M1**	Forms Newton's second law with mass 4, the force and $a = v\frac{\mathrm{d}v}{\mathrm{d}x}$. Condone a sign slip.
	$\int 4v\,\mathrm{d}v = -36\int(x+2)^{-3}\,\mathrm{d}x \Rightarrow \ldots$	**M1**	Separates the variables and integrates both sides, with at least one power increased by 1.
	$2v^2 = \frac{18}{(x+2)^2}\ (+c)$	**A1**	oe
	$2\left(\frac{3}{2}\right)^2 = \frac{18}{2^2} + c \Rightarrow c = \ldots\ [= 0]$	**dM1**	Substitutes $x = 0$, $v = \frac{3}{2}$ into their integrated equation to find the constant, or uses them as limits.
	$v^2 = \frac{9}{(x+2)^2} \Rightarrow v = \frac{3}{x+2}$	**A1**	oe. Positive root only.
		[5]	
1(b)	$\frac{\mathrm{d}x}{\mathrm{d}t} = \text{“}\frac{3}{x+2}\text{”}$ $\int(x+2)\,\mathrm{d}x = \int 3\,\mathrm{d}t \Rightarrow \ldots$	**M1**	Sets $\frac{\mathrm{d}x}{\mathrm{d}t}$ equal to their v in terms of x from (a), separates the variables and integrates, with at least one power increased by 1.
	$\frac{x^2}{2} + 2x = 3t\ (+d)$	**A1**	oe
	$d = 0,\ x^2 + 4x - 6t = 0 \Rightarrow x = \ldots$	**dM1**	Uses $x = 0$ when $t = 0$ for the constant, or as limits, and solves their quadratic in x by formula or completing the square.
	$x = \sqrt{6t+4} - 2$	**A1**	oe. Positive root only.
		[4]	
Total for Question 1		**9**	

Question	Scheme	Marks	Notes
2(a)	$F = \frac{80\,000}{v}$	**B1**	
	$2000\frac{\mathrm{d}v}{\mathrm{d}t} = \text{“}\frac{80\,000}{v}\text{”} - 200v$	**M1**	Forms Newton's second law with mass 2000, their driving force of the form $\frac{k}{v}$ and the resistance, with $a = \frac{\mathrm{d}v}{\mathrm{d}t}$. Condone sign slips.
	$10v\frac{\mathrm{d}v}{\mathrm{d}t} = 400 - v^2$	**A1 AG**	Correct equation of motion leading without error to the printed result.
		[3]	
2(b)	$\int \frac{10v}{400-v^2}\,\mathrm{d}v = \int 1\,\mathrm{d}t$ $\Rightarrow t = k\ln(400 - v^2) + c$	**M1**	Separates the variables and integrates to a logarithm of $400 - v^2$ or of its factors, or differentiates the printed t with respect to v. Condone sign and coefficient slips.
	$t = -5\ln(400 - v^2) + c$	**A1**	oe, with or without modulus signs; c may be absent where limits are used. By differentiation, $\frac{\mathrm{d}t}{\mathrm{d}v} = \frac{10v}{400-v^2}$.
	$0 = -5\ln(400 - 8^2) + c \Rightarrow c = \ldots$	**dM1**	Uses $v = 8$ when $t = 0$ to find their constant, as limits, or to check the printed result.
	$c = 5\ln 336$ $t = 5\ln 336 - 5\ln(400 - v^2)$ $= 5\ln\left(\frac{336}{400-v^2}\right)$	**A1 AG**	Correct constant and the logarithms combined without error to the printed result. By differentiation, needs agreement with (a) and $t = 0$ at $v = 8$.
		[4]	
Total for Question 2		**7**	

Question	Scheme	Marks	Notes
3(a)	$\frac{1}{2}v\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{k}{x^3}$	**M1**	Uses Newton's second law with acceleration $v\frac{\mathrm{d}v}{\mathrm{d}x}$, or equates $\int \frac{k}{x^3}\,\mathrm{d}x$ to a change in KE. Condone a slip in the mass.
	$\int 2v\,\mathrm{d}v = \int 4kx^{-3}\,\mathrm{d}x$ $\Rightarrow v^2 = -\frac{2k}{x^2} + c$	**M1 A1**	**M1** Separates the variables and integrates both sides to the form $pv^2 = \frac{q}{x^2}$. Condone coefficient and sign slips. **A1** oe, with a constant of integration or correct limits.
	$1 = -\frac{k}{2} + c, \quad 4 = -\frac{k}{8} + c$ $\Rightarrow k = \ldots,\ c = \ldots$	**dM1**	Substitutes both given conditions into their equation of the form $pv^2 = \frac{qk}{x^2} + c$, or uses them as limits, and solves for the unknown constants.
	$k = 8,\ c = 5 \Rightarrow v^2 = 5 - \frac{16}{x^2}$	**A1 AG**	Correct equations solved without error to $k = 8$, $c = 5$ and the printed result. A0 for only checking the printed result against the conditions.
		[5]	
3(b)	$\frac{\mathrm{d}x}{\mathrm{d}t} = \sqrt{5 - \frac{16}{x^2}} = \frac{\sqrt{5x^2-16}}{x}$ $\Rightarrow \int \mathrm{d}t = \int \frac{x}{\sqrt{5x^2-16}}\,\mathrm{d}x$	**M1**	Uses $v = \frac{\mathrm{d}x}{\mathrm{d}t}$ with the positive root of the printed v^2, separates the variables and integrates, e.g. to the form $p\sqrt{5x^2-16}$. Condone a wrong p.
	$t = \frac{1}{5}\sqrt{5x^2-16}\ (+d)$	**A1**	oe, with or without a constant.
	$T = \frac{1}{5}\sqrt{80-16} - \frac{1}{5}\sqrt{20-16}$	**dM1**	Applies $x = 4$ and $x = 2$, or the equivalent limits in another variable, to their integrated expression and subtracts.
	$= \frac{8}{5} - \frac{2}{5} = \frac{6}{5}$	**A1 AG**	Correct substituted expression leading without error to $T = \frac{6}{5}$.
		[4]	
	Total for Question 3	**9**	

Question	Scheme	Marks	Notes
4(a)	$3v\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{6}{v}$	**M1**	Uses $F = ma$ with $a = v\frac{\mathrm{d}v}{\mathrm{d}x}$ and the force $\frac{6}{v}$. Condone a sign slip.
	$\int v^2\,\mathrm{d}v = -2\int 1\,\mathrm{d}x$ $\frac{v^3}{3} = -2x + c$	**M1 A1**	**M1** Separates the variables and integrates, raising the power of v by 1. **A1** oe. The constant, or limits, may first appear when $x = 0$, $v = 4$ are used.
	$x = 0,\ v = 4 \Rightarrow c = \ldots\ [= \frac{64}{3}]$	**dM1**	Uses $x = 0$, $v = 4$ to find their constant, or as limits.
	$v = \sqrt[3]{64 - 6x}$	**A1**	oe, with v the subject.
		[5]	
4(b)	$2^3 =$ "$64 - 6x$" $\Rightarrow x = \ldots$	**M1**	Substitutes $v = 2$ into their non-constant expression for v in terms of x from (a) and solves, or integrates with limits 4 and 2.
	$x = \frac{28}{3}$ m	**A1**	$\frac{28}{3}$ or awrt 9.33.
		[2]	
4(c)	$3a = -\frac{3}{2} \Rightarrow a = -\frac{1}{2}$ $0 = 4 - \frac{1}{2}t \Rightarrow t = \ldots$ $x = 4t + \frac{1}{2}\left(-\frac{1}{2}\right)t^2 = \ldots$	**M1**	Finds the acceleration of Q from $F = ma$ and uses constant-acceleration formulae for its stopping time or distance. Condone a sign slip.
	$t = 8$ s, $x = 16$ m	**A1**	Both, from correct working; not for quoting the printed values.
	$0 < v < 4 \Rightarrow \frac{6}{v} > \frac{3}{2}$, so P decelerates more $\Rightarrow P$ stops with $t < 8$ s, $x < 16$ m	**E1**	States that the resistance on P exceeds $\frac{3}{2}$ once $v < 4$, so P decelerates more than Q throughout, and concludes both inequalities.
		[3]	

Question 4 continued

Question	Scheme	Marks	Notes
4(d)			
	Shape	**B1**	Decreasing curve, concave and getting steeper, from a positive intercept on the v-axis down to the t-axis.
		[1]	
4(e)	$0 = \text{“}64 - 6x\text{”} \Rightarrow x = \ldots$	**M1**	Sets $v = 0$ in their non-constant expression for v in terms of x from (a) and solves.
	$x_{\max} = \frac{32}{3}$ m	**A1**	$\frac{32}{3}$ or awrt 10.7.
		[2]	
	Total for Question 4	**13**	

Question	Scheme	Marks	Notes
5(a)	$54 = k(3^3) \Rightarrow k = 2$	**B1**	
	$2v\frac{\mathrm{d}v}{\mathrm{d}x} = -2v^3$	**M1**	Uses Newton's second law with $a = v\frac{\mathrm{d}v}{\mathrm{d}x}$ and a resistance kv^3 opposing the motion. Condone a slip in the mass or their k.
	$\frac{1}{v^2}\frac{\mathrm{d}v}{\mathrm{d}x} = -1$	**A1 AG**	Correct equation with $k = 2$, divided by $2v^3$ to the printed result without error. Condone no mention of $v > 0$.
		[3]	
5(b)(i)	$\int v^{-2}\,\mathrm{d}v = -\int 1\,\mathrm{d}x$	**M1**	Separates the variables and integrates, raising the power of v by 1. Condone sign slips.
	$-\frac{1}{v} = -x + c$	**A1**	oe, with a constant, or as definite integrals with limits u, v and 0, x.
	$x = 0,\ v = u \Rightarrow c = \ldots\ [= -\frac{1}{u}]$	**dM1**	Uses $v = u$ at $x = 0$ to find their constant, or as limits.
	$v = \frac{u}{1+ux}$	**A1**	Any equivalent with v the subject.
		[4]	
5(b)(ii)	$v > 0$ for all x, so B never comes to rest and travels an unlimited distance	**E1**	States that B never comes to rest, or travels an unlimited distance. Not for $v \to 0$ alone.
		[1]	
5(c)(i)	$W = -2\int_0^X \text{“}\frac{u}{1+ux}\text{”}^3\,\mathrm{d}x$ $= -2u^3\int_0^X (1+ux)^{-3}\,\mathrm{d}x = \ldots$	**M1**	Integrates $-2v^3$ from 0 to X, with their v from (b)(i) in terms of x, raising the power of $(1+ux)$ by 1. Condone sign and coefficient slips.
	$-2u^3\left[\frac{(1+ux)^{-2}}{-2u}\right]_0^X$	**A1**	Correct integral, oe. Need not be simplified.
	$W = u^2\left[(1+uX)^{-2} - 1\right]$	**A1**	oe in X and u.
		[3]	
5(c)(ii)	$W < 0$: the resistance opposes the displacement, so it reduces the kinetic energy	**E1**	Relates the negative sign to the resistance opposing the motion, or to B losing kinetic energy. Not for "W is negative" alone.
		[1]	

Question 5 continued

Question	Scheme	Marks	Notes
5(c)(iii)	$V = \frac{u}{1+uX} \Rightarrow W = \frac{u^2}{(1+uX)^2} - u^2 = \ldots$	**M1**	Eliminates X from their W in (c)(i), using V equal to their v from (b)(i) at $x = X$.
	$W = V^2 - u^2$ $= \frac{1}{2}(2)V^2 - \frac{1}{2}(2)u^2$, the change in KE	**A1**	cso. Needs the change in KE as $\frac{1}{2}(2)V^2 - \frac{1}{2}(2)u^2$ and a statement that it equals W.
		[2]	
	Total for Question 5	**14**	

Question	Scheme	Marks	Notes
6(a)	$4k = 10 \Rightarrow k = \frac{5}{2}$	**B1**	Resistance $\frac{5}{2}v$, soi.
	$5\frac{\mathrm{d}v}{\mathrm{d}t} = 5(9.8) - 14 - \frac{5}{2}v$	**M1 A1**	**M1** Forms Newton's second law with weight, upthrust and resistance their kv, and acceleration $\frac{\mathrm{d}v}{\mathrm{d}t}$. Condone sign slips. **A1** Correct substituted equation, oe.
	$\frac{\mathrm{d}v}{\mathrm{d}t} = 7 - \frac{1}{2}v$	**A1 AG**	Correct substituted equation leading without error to the printed result; combined steps accepted.
		[4]	
6(b)(i)	$\int \frac{1}{v-14}\,\mathrm{d}v = -\frac{1}{2}\int 1\,\mathrm{d}t$	**M1**	Separates the variables and integrates the v side to the form $p\ln\lvert v-14\rvert$ or $p\ln\lvert 14-v\rvert$, or uses the integrating factor $e^{t/2}$ and integrates. Condone sign and coefficient slips.
	$\ln\lvert v-14\rvert = -\frac{t}{2} + c$	**A1**	oe, such as $-2\ln\lvert 14-v\rvert = t + c$ or $ve^{t/2} = 14e^{t/2} + c$. Condone missing modulus signs.
	$t = 0,\ v = u \Rightarrow c = \ln\lvert u-14\rvert$	**dM1**	Uses $v = u$ at $t = 0$ to find their constant, before or after exponentiating.
	$\lvert v-14\rvert = \lvert u-14\rvert e^{-t/2}$	**A1**	oe, with the constant found. Condone missing modulus signs.
	$[e^{-t/2} \neq 0$, so $v-14$ has the sign of $u-14$; $u = 14$ gives $v = 14]$ $\Rightarrow v = 14 + (u-14)e^{-t/2}$	**A1 AG**	Printed result reached without error from the correct constant; condone missing modulus signs. A sign argument and separate treatment of $u = 14$ are not required.
		[5]	
6(b)(ii)	$0 < u < 14$: speed increases towards $14\,\mathrm{m\,s^{-1}}$	**E1**	States that the speed increases and tends to 14. A limit of 14 stated once for all cases counts for each.
	$u = 14$: speed stays constant at $14\,\mathrm{m\,s^{-1}}$	**E1**	States that the speed is constant at 14.
	$u > 14$: speed decreases towards $14\,\mathrm{m\,s^{-1}}$	**E1**	States that the speed decreases and tends to 14.
		[3]	

Question 6 continued

Question	Scheme	Marks	Notes
6(c)	$x = \int \left(14 - 5e^{-t/2}\right) \mathrm{d}t$ $= 14t + 10e^{-t/2} + d$	**M1 A1**	**M1** Integrates v with $u = 9$ to the form $pt + qe^{-t/2}$. Condone sign and coefficient slips. **A1** Correct, oe; d need not be shown here.
	$t = 0,\ x = 0 \Rightarrow d = -10$	**dM1**	Uses $x = 0$ at $t = 0$ to find the constant, or applies limits 0 and 6 to their integral.
	$t = 6 \Rightarrow x = \left(74 + 10e^{-3}\right)$ m	**A1**	Any exact equivalent.
		[4]	
	Total for Question 6	**16**	

Question	Scheme	Marks	Notes
7(a)	$900\frac{\mathrm{d}v}{\mathrm{d}t} = 576 - (v^2 + 14v)$ $\Rightarrow \int \frac{900}{(18-v)(v+32)}\,\mathrm{d}v = \int 1\,\mathrm{d}t$	**M1**	Forms an equation of motion with $900\frac{\mathrm{d}v}{\mathrm{d}t}$, the driving force and the resistance, and separates the variables. Condone sign slips.
	$\frac{900}{(18-v)(v+32)} = \frac{18}{18-v} + \frac{18}{v+32}$ $\Rightarrow -18\ln(18-v) + 18\ln(v+32) = t\ (+c)$	**M1 A1**	**M1** Splits into partial fractions of the form $\frac{A}{18-v} + \frac{B}{v+32}$ and integrates both to logarithms. Condone sign and coefficient slips. **A1** Correct partial fractions, oe such as $\frac{1}{50}$ each with the 900 kept outside.
	$18\ln\left(\frac{v+32}{18-v}\right) = t + c$	**A1**	oe, uncombined logarithms accepted. No constant needed where limits are used.
	$T = 18\ln\left(\frac{13+32}{18-13}\right) - 18\ln\left(\frac{3+32}{18-3}\right)$	**dM1**	Substitutes $v = 13$ and $v = 3$ into their integrated expression and subtracts, or uses them as limits.
	$T = 18\ln 9 - 18\ln\frac{7}{3} = 18\ln\left(\frac{27}{7}\right)$	**A1 AG**	Correct substituted expression leading without error to the printed result, with the logarithms combined.
		[6]	
7(b)	$18\ln\left(\frac{8+32}{18-8}\right) = 0 + c \Rightarrow c = 18\ln 4$ $\Rightarrow \frac{v+32}{18-v} = e^{(t+\text{“}18\ln 4\text{”})/18} = \ldots$	**M1**	Uses $v = 8$ when $t = 0$ to find the constant in their relation of the form $k\ln\left(\frac{v+32}{18-v}\right) = t + c$, then exponentiates both sides.
	$\frac{v+32}{18-v} = 4e^{t/18}$ $p = 4,\ q = \frac{1}{18}$	**A1**	Both values, or the equation with them; q exact.
		[2]	
7(c)	$18 - v = \frac{50}{1+4e^{t/18}}$ $1 + 4e^{t/18} > 0 \Rightarrow 18 - v > 0 \Rightarrow v < 18$	**E1**	Deduces $18 - v > 0$ from (b), by making v or $18 - v$ the subject, or from $4e^{t/18} > 0$ with $v + 32 > 0$, and concludes.
		[1]	
	Total for Question 7	**9**	

Question	Scheme	Marks	Notes
8(a)	$2v\frac{\mathrm{d}v}{\mathrm{d}x} = -kv(v+2)$	**M1**	Forms Newton's second law with acceleration $v\frac{\mathrm{d}v}{\mathrm{d}x}$, mass 2 and the force in the negative direction.
	$\int \frac{1}{v+2}\,\mathrm{d}v = -\frac{k}{2}\int 1\,\mathrm{d}x$	**M1**	Cancels v, separates the variables and integrates to the form $p\ln(v+2) = qx\ (+c)$. Condone sign and coefficient slips.
	$\ln(v+2) = -\frac{kx}{2} + c$	**A1**	oe, including the constant of integration.
	$\ln 9 = -\frac{k}{2} + c$ $\ln 3 = -\frac{3k}{2} + c \Rightarrow k = \ldots$	**dM1**	Substitutes both pairs of values, or uses them as limits, and solves for k.
	$k = \ln 3$	**A1**	oe, or awrt 1.10.
	$\ln\left(\frac{v+2}{9}\right) = \frac{1-x}{2}\ln 3$ $\Rightarrow v = 9\times 3^{(1-x)/2} - 2$	**A1 AG**	Correct relation with $k = \ln 3$ and c found or eliminated, leading without error to the printed result.
		[6]	
8(b)	$2\frac{\mathrm{d}v}{\mathrm{d}t} = -kv(v+2)$ $\Rightarrow \frac{2}{v(v+2)}\,\mathrm{d}v = -k\,\mathrm{d}t$	**M1**	Forms Newton's second law with acceleration $\frac{\mathrm{d}v}{\mathrm{d}t}$, mass 2 and the force negative, and separates the variables.
	$\int\left(\frac{1}{v} - \frac{1}{v+2}\right)\mathrm{d}v = -k\int 1\,\mathrm{d}t$	**M1**	Uses partial fractions of the form $\frac{A}{v} + \frac{B}{v+2}$ and integrates to logarithms of v and $v+2$. Condone coefficient slips.
	$\ln v - \ln(v+2) = -kt + d$	**A1**	oe, with k or $\ln 3$.
	$\ln\frac{7}{9} = -kt_0 + d,\ \ln\frac{1}{3} = -k(t_0+T) + d$ $\Rightarrow \ln\frac{1}{3} - \ln\frac{7}{9} = -T\ln 3$	**dM1**	Substitutes $v=7$ and $v=1$ at times T apart, or uses them as limits, eliminates the constant and uses their k from (a).
	$\ln\frac{3}{7} = -T\ln 3 \Rightarrow T = \frac{\ln(7/3)}{\ln 3}$	**A1 AG**	Correct substituted equation with $k = \ln 3$, leading without error to the printed T.
		[5]	
	Total for Question 8	**11**	

Question	Scheme	Marks	Notes
9(a)	$4v\frac{\mathrm{d}v}{\mathrm{d}x} = 4(9.8) - 6v$	**M1**	Uses Newton's second law with $a = v\frac{\mathrm{d}v}{\mathrm{d}x}$, weight and resistance. Condone sign slips.
	$10v\frac{\mathrm{d}v}{\mathrm{d}x} = 98 - 15v$ $\Rightarrow \left(\frac{10v}{98-15v}\right)\frac{\mathrm{d}v}{\mathrm{d}x} = 1$	**A1 AG**	Correct equation of motion leading without error to the printed result; combined rearrangement accepted.
		[2]	
9(b)	$\int \frac{10v}{98-15v}\,\mathrm{d}v = \int 1\,\mathrm{d}x$	**M1**	Separates the variables in the equation from (a) and attempts to integrate both sides, by division, a substitution or otherwise; or differentiates the printed relation with respect to v.
	$\frac{10v}{98-15v} = -\frac{2}{3} + \frac{196}{3(98-15v)}$	**B1**	oe, or the equivalent integrand from a substitution such as $u = 98 - 15v$. By differentiation, the logarithm term's derivative $\frac{2940}{98-15v}$ oe.
	$x = -\frac{2v}{3} - \frac{196}{45}\ln(98 - 15v) + c$	**A1**	oe. The constant may first appear in the next step. By differentiation, $45\frac{\mathrm{d}x}{\mathrm{d}v} = \frac{450v}{98-15v}$ oe.
	$0 = -\frac{196}{45}\ln 98 + c \Rightarrow c = \ldots$	**dM1**	Substitutes $x = 0$, $v = 0$ into their integrated equation to find the constant, or uses them as limits; or shows the printed relation gives $x = 0$ at $v = 0$.
	$45x = -30v - 196\ln(98 - 15v) + 196\ln 98$ $= 196\ln\left(\frac{98}{98-15v}\right) - 30v$	**A1 AG**	Correct constant and logarithms combined, leading without error to the printed relation. By differentiation, agreement with (a), $x = 0$ at $v = 0$ and a conclusion.
		[5]	
9(c)	$9.8 - 1.5v = 2.45 \Rightarrow v = \ldots$	**M1**	Sets their acceleration from Newton's second law, with weight and resistance, equal to 2.45 and solves for v. Condone sign slips.
	$v = 4.9$	**A1**	
	$45x = 196\ln\left(\frac{98}{98-15(\text{“}4.9\text{”})}\right) - 30(\text{“}4.9\text{”})$ $\Rightarrow x = \ldots$ $4(9.8)x - W = \frac{1}{2}(4)(\text{“}4.9\text{”})^2$	**M1**	Forms a work-energy equation with KE, GPE and the work against resistance, using x from (b) at their v. Condone sign slips.

Question 9 continued

Question	Scheme	Marks	Notes
	$39.2\left(\frac{196\ln 4-147}{45}\right)-W=2(4.9)^2$	**A1**	Correct equation, with x exact or awrt 2.77.
	$W=60.6\,\text{J}$	**A1**	awrt 60.6.
		[5]	
	Total for Question 9	**12**	

Question	Scheme	Marks	Notes
10(a)	$R = mg\cos\alpha$	**B1**	May be seen in the friction term.
	$mv\frac{\mathrm{d}v}{\mathrm{d}x}$ $= -mg\sin\alpha - \mu mg\cos\alpha - \frac{kmg}{x+a}$	**M1 A1**	**M1** Forms Newton's second law along the track with acceleration $v\frac{\mathrm{d}v}{\mathrm{d}x}$, the weight component, friction μR and the variable force. Condone sign slips. **A1** Correct equation, oe.
	$\frac{\mathrm{d}}{\mathrm{d}x}(v^2) = 2v\frac{\mathrm{d}v}{\mathrm{d}x}$ $\Rightarrow \frac{\mathrm{d}}{\mathrm{d}x}(v^2) + \frac{2gk}{x+a}$ $+ 2g(\sin\alpha + \mu\cos\alpha) = 0$	**A1 AG**	Uses $\frac{\mathrm{d}}{\mathrm{d}x}(v^2) = 2v\frac{\mathrm{d}v}{\mathrm{d}x}$ on a correct equation of motion and reaches the printed equation without error.
		[4]	
10(b)	$v^2 = -2gk\ln(x+a) - 2g(\sin\alpha + \mu\cos\alpha)x + c$	**M1 A1**	**M1** Integrates the equation in (a) with respect to x, at least one term correctly; or differentiates the printed v^2 with respect to x. **A1** Correct, including a constant; or a correct derivative compared with (a).
	$0 = -2gk\ln(3a) - 4ga(\sin\alpha + \mu\cos\alpha) + c$ $\Rightarrow c = \ldots$	**dM1**	Substitutes $v = 0$, $x = 2a$ into their integral with a constant to find it; or shows the printed v^2 is zero at $x = 2a$.
	$c = 2gk\ln(3a) + 4ga(\sin\alpha + \mu\cos\alpha)$ $\Rightarrow v^2 = 2gk\ln\left(\frac{3a}{x+a}\right)$ $+ 2g(\sin\alpha + \mu\cos\alpha)(2a - x)$	**A1 AG**	Correct constant and the printed v^2 reached without error, logarithms combined correctly. By verification, both checks and a conclusion.
		[4]	
10(c)	$mg\sin\alpha + \frac{kmg}{3a} \leq \mu mg\cos\alpha$	**M1**	Resolves along the track at $x = 2a$ with friction up the track, and compares the friction needed with $\mu mg\cos\alpha$. Condone equality.
	$\mu \geq \tan\alpha + \frac{k}{3a\cos\alpha}$	**A1**	oe, such as $\mu \geq \frac{3a\sin\alpha + k}{3a\cos\alpha}$. Condone $>$.
		[2]	
	Total for Question 10	**10**	

Question	Scheme	Marks	Notes
11(a)	$m\frac{\mathrm{d}v}{\mathrm{d}t} = mg - \frac{m}{20}v^2$	**M1**	Forms Newton's second law with weight, resistance and acceleration $\frac{\mathrm{d}v}{\mathrm{d}t}$. Condone sign slips.
	$\int \frac{1}{196-v^2}\,\mathrm{d}v = \int \frac{1}{20}\,\mathrm{d}t$ $\Rightarrow \frac{1}{28}\int\left(\frac{1}{14-v}+\frac{1}{14+v}\right)\mathrm{d}v = \ldots$	**M1**	Separates and integrates $\frac{1}{20g-v^2}$ with their g, by partial fractions or a standard result, reaching logarithms or an artanh. Condone sign and coefficient slips.
	$\frac{1}{28}\ln\left(\frac{14+v}{14-v}\right) = \frac{t}{20} + c$	**A1**	Correct, oe such as $\frac{1}{14}\mathrm{artanh}\frac{v}{14}$. The constant need not be shown here.
	$\frac{1}{28}\ln\left(\frac{21}{7}\right) = c \Rightarrow c = \ldots\ [= \frac{\ln 3}{28}]$	**dM1**	Uses $v = 7$ when $t = 0$ to find their constant, or as limits.
	$\frac{14+v}{14-v} = 3e^{7t/5}$ $\Rightarrow v = \frac{14\left(3e^{7t/5}-1\right)}{3e^{7t/5}+1}$	**A1**	Correct v, oe such as $14\tanh\left(\frac{7t}{10}+\frac{1}{2}\ln 3\right)$.
		[5]	
11(b)	$\frac{\mathrm{d}x}{\mathrm{d}t} = -14 + \frac{84e^{7t/5}}{3e^{7t/5}+1} \Rightarrow x = \ldots$	**M1**	Uses $v = \frac{\mathrm{d}x}{\mathrm{d}t}$ and integrates their v: $\frac{Ae^{kt}+B}{Ce^{kt}+D}$ to a term $p\ln\left(Ce^{kt}+D\right)$ oe, or a tanh form to $p\ln\cosh(\ldots)$. Condone coefficient slips.
	$x = 20\ln\left(3e^{7t/5}+1\right) - 14t\ (+d)$	**A1**	Correct, oe such as $20\ln\cosh\left(\frac{7t}{10}+\frac{1}{2}\ln 3\right)$. The constant need not be shown here.
	$0 = 20\ln 4 + d \Rightarrow d = \ldots\ [= -20\ln 4]$	**dM1**	Uses $x = 0$ when $t = 0$ to find their constant, or as limits.
	$x = 20\ln\left(\frac{3e^{7t/5}+1}{4}\right) - 14t$	**A1**	oe.
		[4]	
11(c)	$v\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{196-v^2}{20}$ $\Rightarrow \int 1\,\mathrm{d}x = \int \frac{20v}{196-v^2}\,\mathrm{d}v = \ldots$	**M1**	Uses $v\frac{\mathrm{d}v}{\mathrm{d}x}$ with weight and resistance, separates and integrates to a form $p\ln\left(20g-v^2\right)$ with their g; or sets $v = \frac{21}{2}$ in their v from (a) and solves for t. Condone sign and coefficient slips.
	$x = -10\ln\left(196-v^2\right)\ (+h)$	**A1**	Correct, oe. The constant need not be shown here. Through (a), $t = \frac{5}{7}\ln\frac{7}{3}$ oe.

Question 11 continued

Question	Scheme	Marks	Notes
	$h = 10\ln 147$ $x = 10\ln 147 - 10\ln\left(196 - \left(\frac{21}{2}\right)^2\right) = \ldots$	**dM1**	Uses $x = 0$, $v = 7$ to find their constant, or as limits, and substitutes $v = \frac{21}{2}$; or substitutes their t in their x from (b).
	$x = 10\ln\left(\frac{12}{7}\right)$	**A1**	Any exact form, such as $10\ln\frac{588}{343}$.
		[4]	
	Total for Question 11	**13**	

Question	Scheme	Marks	Notes
12(a)	$m\frac{\mathrm{d}v}{\mathrm{d}t} = mg - mkv^2$	**M1**	Forms an equation of motion with weight, resistance and $\frac{\mathrm{d}v}{\mathrm{d}t}$. Condone sign slips and m omitted throughout.
	$\int \frac{1}{g-kv^2}\,\mathrm{d}v = \int 1\,\mathrm{d}t$ $\frac{1}{2\sqrt{g}}\int\left(\frac{1}{\sqrt{g}+\sqrt{k}\,v} + \frac{1}{\sqrt{g}-\sqrt{k}\,v}\right)\mathrm{d}v = \ldots$	**M1**	Separates the variables and integrates to a logarithmic or $\tanh^{-1}$ form, by partial fractions or a standard result. Condone coefficient slips.
	$\frac{1}{2\sqrt{gk}}\ln\left(\frac{\sqrt{g}+\sqrt{k}\,v}{\sqrt{g}-\sqrt{k}\,v}\right) = t + c$	**A1**	oe, including $\frac{1}{\sqrt{gk}}\tanh^{-1}\left(\sqrt{\frac{k}{g}}\,v\right) = t + c$.
	$t = 0,\ v = 0 \Rightarrow c = 0$ $\frac{\sqrt{g}+\sqrt{k}\,v}{\sqrt{g}-\sqrt{k}\,v} = e^{2\sqrt{gk}\,t} \Rightarrow v = \ldots$	**dM1**	Uses $v = 0$ at $t = 0$ for the constant, removes the logarithm or $\tanh^{-1}$ and makes v the subject. Condone rearranging slips.
	$v\left(1 + e^{2\sqrt{gk}\,t}\right) = \sqrt{\frac{g}{k}}\left(e^{2\sqrt{gk}\,t} - 1\right)$ $v = \sqrt{\frac{g}{k}}\left(\frac{e^{2\sqrt{gk}\,t}-1}{e^{2\sqrt{gk}\,t}+1}\right)$	**A1 AG**	Correct equation without logarithms leading without error to the printed result; a tanh form must be written in exponentials.
		[5]	
12(b)	$s = \int 28\left(\frac{e^{0.7t}-1}{e^{0.7t}+1}\right)\mathrm{d}t$ $= \int\left(\frac{56e^{0.7t}}{e^{0.7t}+1} - 28\right)\mathrm{d}t$	**M1**	Substitutes for g and k and integrates v with respect to t to a form $p\ln\left(e^{2\sqrt{gk}\,t}+1\right) + qt$ or $p\ln\cosh\left(\sqrt{gk}\,t\right)$, with their g.
	$= 80\ln\left(e^{0.7t}+1\right) - 28t\ (+c)$	**A1**	oe, such as $80\ln\cosh(0.35t)$.
	$t = 0,\ s = 0 \Rightarrow c = -80\ln 2$ $s(2) = 80\ln\left(\frac{e^{1.4}+1}{2}\right) - 56$	**dM1**	Uses $s = 0$ at $t = 0$ for the constant and substitutes $t = 2$, or applies limits 0 and 2 to their integral.
	$\approx 18.2\,\mathrm{m}$	**A1**	awrt 18.2.
		[4]	
	Total for Question 12	**9**	

Question	Scheme	Marks	Notes
13(a)	$6 = k\sqrt{25-4(2)^2} \Rightarrow k = 2$	**B1**	
	$16\frac{\mathrm{d}v}{\mathrm{d}t} = -\text{“2”}\sqrt{25-4v^2}$	**M1**	Forms Newton's second law with mass 16, a force of the form $k\sqrt{25-4v^2}$ and acceleration $\frac{\mathrm{d}v}{\mathrm{d}t}$. Condone sign slips.
	$\Rightarrow \frac{1}{\sqrt{25-4v^2}}\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{1}{8}$	**A1 AG**	Correct equation with $k = 2$ and the force opposing the motion, leading without error to the printed result.
		[3]	
13(b)	$\int \frac{1}{\sqrt{25-4v^2}}\,\mathrm{d}v = -\frac{1}{8}\int 1\,\mathrm{d}t$ $\Rightarrow \ldots \arcsin\left(\frac{2v}{5}\right) = \ldots$	**M1**	Separates the variables and integrates to the form $p\arcsin(qv)$. Condone slips in p and q.
	$\frac{1}{2}\arcsin\left(\frac{2v}{5}\right) = -\frac{t}{8} + c$	**A1**	oe
	$t = 0,\ v = \frac{5\sqrt{3}}{4} \Rightarrow c = \ldots\ [= \frac{\pi}{6}]$	**dM1**	Substitutes $t = 0$ and $v = \frac{5\sqrt{3}}{4}$ to find the constant, or uses them as lower limits.
	$v = \frac{5}{4} \Rightarrow \frac{\pi}{6} = \frac{\pi}{3} - \frac{t}{4} \Rightarrow t = \frac{2\pi}{3}$	**A1**	Substitutes $v = \frac{5}{4}$ into a correct equation to reach $t = \frac{2\pi}{3}$. Stating the printed interval alone scores A0.
	$v = \frac{5}{2}\sin\left(\frac{\pi}{3} - \frac{t}{4}\right)$	**A1**	oe, such as $\frac{5}{2}\cos\left(\frac{t}{4} + \frac{\pi}{6}\right)$.
		[5]	
13(c)	$x = \int \frac{5}{2}\sin\left(\frac{\pi}{3} - \frac{t}{4}\right)\mathrm{d}t$ $= \ldots\cos\left(\frac{\pi}{3} - \frac{t}{4}\right)$	**M1**	Integrates their v from (b), of the form $p\sin\left(\frac{\pi}{3} - \frac{t}{4}\right)$ oe, to a cosine of the same argument. Condone sign and coefficient slips.
	$x = 10\cos\left(\frac{\pi}{3} - \frac{t}{4}\right) + d$	**A1**	oe
	$t = 0,\ x = 0 \Rightarrow d = \ldots\ [= -5]$	**dM1**	Uses $x = 0$ at $t = 0$ to find the constant, or applies limits 0 and $\frac{2\pi}{3}$.
	$x\left(\frac{2\pi}{3}\right) = 10\cos\frac{\pi}{6} - 5 = 5\sqrt{3} - 5$	**A1**	oe, or awrt 3.66
	$\text{“}5\sqrt{3} - 5\text{”} + \frac{5}{4}\left(8 - \frac{2\pi}{3}\right)$	**M1**	Adds $\frac{5}{4}$ times the remaining time $8 - \frac{2\pi}{3}$ to their distance from integrating v while the force acts.

Question 13 continued

Question	Scheme	Marks	Notes
	$5\sqrt{3}+5-\frac{5\pi}{6}$ (m)	**A1**	Any exact equivalent.
		[6]	
	Total for Question 13	**14**	

Question	Scheme	Marks	Notes
14(a)	$F = \frac{2kmg}{v}$	**B1**	
	$mv\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{2kmg}{v} - \frac{mgv}{k}$	**M1**	Forms Newton's second law with their driving force, the resistance and $a = v\frac{\mathrm{d}v}{\mathrm{d}x}$. Condone sign slips.
	$v^2\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{g}{k}\left(2k^2 - v^2\right)$	**A1 AG**	Correct equation of motion leading without error to the printed result; combined steps accepted.
		[3]	
14(b)	$\int g\,\mathrm{d}x = \int \frac{kv^2}{2k^2-v^2}\,\mathrm{d}v$	**M1**	Separates the variables from (a) and attempts to integrate both sides, or differentiates the printed relation with respect to v. Condone slips in constants.
	$\frac{v^2}{2k^2-v^2} = -1 + \frac{2k^2}{2k^2-v^2}$	**B1**	Or the equivalent partial fractions with the -1 term. By differentiation, the logarithm term's derivative $\frac{2k^3}{2k^2-v^2}$ oe.
	$gx = -kv + \frac{k^2}{\sqrt{2}}\left(\ln(\sqrt{2}k+v) - \ln(\sqrt{2}k-v)\right) + c$	**A1**	oe, such as an artanh form. By differentiation, $g\frac{\mathrm{d}x}{\mathrm{d}v} = \frac{kv^2}{2k^2-v^2}$ oe.
	$x = 0,\ v = 0 \Rightarrow c = \ldots\ [= 0]$	**dM1**	Substitutes $x = 0$, $v = 0$ into their integrated relation with a constant, or uses these as limits; or shows the printed relation gives $x = 0$ at $v = 0$.
	$gx = -kv + \frac{k^2}{\sqrt{2}}\ln\left(\frac{\sqrt{2}k+v}{\sqrt{2}k-v}\right)$	**A1 AG**	$c = 0$ shown and the logarithms combined without error. By differentiation, agreement with (a), $x = 0$ at $v = 0$ and a conclusion.
		[5]	
14(c)	$W = -\int \frac{mgv}{k}\,\mathrm{d}x$ $\mathrm{d}x = \frac{kv^2}{g(2k^2-v^2)}\,\mathrm{d}v$	**M1**	Writes the work by resistance as an integral of $\frac{mgv}{k}$ in x and changes to v using (a). Condone sign slips.
	$\frac{\mathrm{d}W}{\mathrm{d}v} = -\frac{mv^3}{2k^2-v^2}$	**A1**	oe, or the work against resistance with the opposite sign used consistently.
	$-\frac{mv^3}{2k^2-v^2} = mv - \frac{2mk^2v}{2k^2-v^2}$ $\Rightarrow W(V) = \left[\frac{1}{2}mv^2 + mk^2\ln(2k^2-v^2)\right]_0^V$	**dM1**	Integrates to the form $pv^2 + q\ln(2k^2-v^2)$ and applies limits 0 and V, or $W(0) = 0$. Condone sign and coefficient slips.

Question 14 continued

Question	Scheme	Marks	Notes
	$W(V) = \frac{1}{2}mV^2 - mk^2 \ln\left(\frac{2k^2}{2k^2 - V^2}\right)$	**A1**	oe, with the sign matching their convention.
	Engine work $= 2kmg\,t$	**B1**	
	$2kmg\,t +$ "$W(V)$" $= \frac{1}{2}mV^2$	**M1**	Forms a work-energy equation with the engine work Pt, their work by resistance and KE $\frac{1}{2}mV^2$. Condone sign slips.
	$2kmg\,t + \frac{1}{2}mV^2 - mk^2 \ln\left(\frac{2k^2}{2k^2 - V^2}\right)$ $= \frac{1}{2}mV^2$	**A1**	
	$t = \frac{k}{2g} \ln\left(\frac{2k^2}{2k^2 - V^2}\right)$	**A1 AG**	Correct work-energy equation leading without error to the printed time.
		[8]	
	Total for Question 14	**16**	