FURTHER MECHANICS / CIRCULAR MOTION & OSCILLATIONS

Simple Harmonic Motion

Questions by topic 10 questions · 139 marks

EXAM BOARDS Edexcel FM2 9FM0/4C · OCR MEI Mech Major Y421

Instructions

Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question gives a value. Where a question leaves g unspecified and a final answer depends on it, the solutions also give that answer for $g = 10\,\mathrm{m\,s^{-2}}$.

Questions

Question 1	*15 marks*	2
Question 2	*14 marks*	3
Question 3	*14 marks*	4
Question 4	*10 marks*	5
Question 5	*11 marks*	6
Question 6	*9 marks*	7
Question 7	*15 marks*	8
Question 8	*14 marks*	9
Question 9	*17 marks*	10
Question 10	*20 marks*	11

1. A particle P is moving along the x-axis. At time t seconds, the displacement of P from the origin O is x metres, where

$$x = 3\cos\left(\tfrac{1}{4}\pi t\right) + 3\sqrt{3}\sin\left(\tfrac{1}{4}\pi t\right)$$

(a) Prove that P is moving with simple harmonic motion. **[3]**

(b) Find the period of the motion. **[2]**

(c) Find the amplitude of the motion. **[2]**

(d) Find, in terms of π, the maximum speed of P. **[2]**

(e) Find the total time during each complete oscillation for which the speed of P is greater than $\frac{3\sqrt{3}}{4}\pi\,\mathrm{m\,s^{-1}}$. **[6]**

2. A particle P of mass $0.5\,\text{kg}$ moves in a straight line with simple harmonic motion about a centre O. The time taken for one complete oscillation is $\frac{\pi}{2}$ seconds.
When P is $0.3\,\text{m}$ from O, its speed is $1.6\,\text{m}\,\text{s}^{-1}$.

(a) Show that the amplitude of the motion is $0.5\,\text{m}$. **[4]**

(b) Find the length of time within each complete oscillation for which the speed of P is greater than $1\,\text{m}\,\text{s}^{-1}$, giving your answer as an exact multiple of π. **[6]**

(c) Find the magnitude of the resultant force acting on P at an instant when the kinetic energy of P is $0.36\,\text{J}$. **[4]**

3. A particle P moves in a straight line with simple harmonic motion about a fixed point O.
When P is 0.5 m from O, its speed is $3.6\,\mathrm{m\,s^{-1}}$, and when P is 1.2 m from O, its speed is $1.5\,\mathrm{m\,s^{-1}}$.
The amplitude of the motion is a metres.

(a) Show that $a = 1.3$. **[5]**

(b) Find the exact period of the motion and the greatest magnitude of the acceleration of P. **[3]**

During one complete oscillation, the magnitude of the acceleration of P is at least $6.3\,\mathrm{m\,s^{-2}}$ for a total of T seconds.

(c) Find the value of T. **[6]**

4. The depth of water at the entrance to a harbour varies with the tides, and the water level moves with simple harmonic motion.
On a particular day, high tide occurs at 05:40, when the depth of water is 11.2 m, and the next low tide occurs at 11:55, when the depth of water is 3.6 m.

(a) Between 05:40 and 11:55 there are two moments at which the water level is falling at a rate of $1.5\,\mathrm{m\,h^{-1}}$. Find the depth of water at each of these moments. **[6]**

A ship can only use the harbour when the depth of water at the entrance is at least 5.5 m.

(b) Find the length of time between 05:40 and the following high tide during which the ship cannot use the harbour. **[4]**

5. A particle P moves along the x-axis with simple harmonic motion. At time t seconds, its displacement from the origin O is x metres, where

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 9(2-x)$$

and P is instantaneously at rest when $x = 5$.

(a) Show that v, the velocity of P in $\mathrm{m\,s^{-1}}$ at time t, satisfies

$$v^2 = 9(5-x)(x+1)$$ **[5]**

(b) When $t = 0$, P is at the point where $x = \frac{1}{2}$ and is moving in the positive x-direction. Hence, or otherwise, show that

$$x = 2 + 3\sin\left(3t - \frac{\pi}{6}\right)$$ **[6]**

6. A box of mass m kg rests on a horizontal platform. The platform moves vertically with simple harmonic motion of angular frequency ω rad s^{-1} and amplitude a m about a mean position O. At time t seconds, the upward displacement of the platform from O is x m.
The box remains in contact with the platform throughout the motion, and the greatest magnitude of the force exerted by the platform on the box is $\frac{7}{3}$ times the least magnitude of this force.

(a) Show that $\omega^2 = \frac{2g}{5a}$. **[3]**

The angular frequency is unchanged, but the amplitude of the motion is increased to $3a$ m.

(b) Show that the box leaves the platform, and find, in terms of a, the height above O at which this first happens. **[3]**

(c) Find, in terms of a and g, the speed of the box at the instant it leaves the platform. **[3]**

7. Throughout this question, use $g = 10\,\mathrm{m\,s^{-2}}$.
A light elastic string has natural length 0.9 m and modulus of elasticity 36 N.
A particle P of mass 0.4 kg is attached to one end of the string. The other end of the string is attached to a fixed point A. Particle P hangs freely in equilibrium with P vertically below A.
The particle is then pulled vertically down to a point B and released from rest.

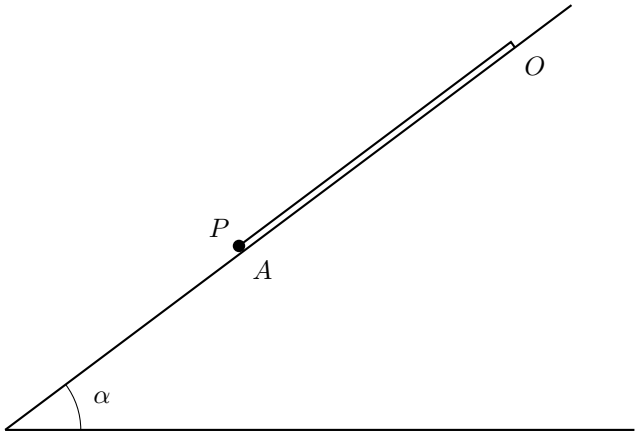
(a) Show that, while the string is taut, P moves with simple harmonic motion with period $\frac{\pi}{5}$ seconds. **[6]**

When the string first becomes slack, the speed of P is $\sqrt{3}\,\mathrm{m\,s^{-1}}$.

(b) Find the distance AB. **[3]**

After the string becomes slack, P continues to move vertically upwards until it first comes to instantaneous rest at the point C.

(c) Find the distance BC and the exact time taken for P to move from B to C. **[6]**



8. A particle P of mass m is attached to one end of a light elastic string of natural length $5a$ and modulus of elasticity $3mg$. The other end of the string is fixed to a point O on a smooth plane, which is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{3}{5}$. The string lies along a line of greatest slope of the plane and P rests in equilibrium on the plane at a point A below O, as shown in the diagram above.

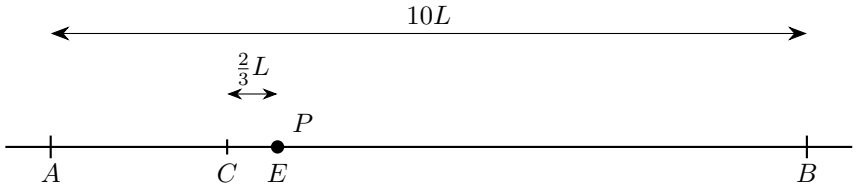
At time $t = 0$, P is projected from A with speed $2\sqrt{\frac{3ag}{5}}$ up the line of greatest slope towards O. At time t, the displacement of P from A is x, where the positive direction of x is up the plane.

(a) Show that, while the string is taut, x satisfies the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \frac{3gx}{5a} = 0 \qquad \textbf{[6]}$$

(b) Find, in terms of a and g, the time at which the string first becomes slack. **[5]**

(c) Find, in terms of a, the distance from A of the highest point on the plane reached by P. **[3]**



9. A particle P of mass m lies on a smooth horizontal table.
One end of a light elastic string, of natural length $2L$ and modulus of elasticity $3mg$, is attached to P, and the other end is attached to a fixed point A on the table.
One end of a second light elastic string, of natural length $4L$ and modulus of elasticity $2mg$, is also attached to P, and the other end is attached to a fixed point B on the table, where $AB = 10L$.
The particle P rests in equilibrium at the point E, where AEB is a straight line, as shown in the diagram above.

(a) Show that $AE = 3L$. **[2]**

The particle P is now held at the point C on AE, where $CE = \frac{2}{3}L$, and released from rest.

(b) Show that P moves with simple harmonic motion with centre E and period $\pi\sqrt{\frac{2L}{g}}$. **[6]**

(c) Find, in terms of L and g, the speed of P at the instant when $AP = \frac{5}{2}L$. **[3]**

(d) Find, in terms of L and g, the exact length of time in each complete oscillation for which the tension in the string AP is greater than $2mg$. **[6]**

10. A particle P of mass m is attached to one end of a light elastic string of natural length l and modulus of elasticity mg. The other end of the string is attached to a fixed point O.
Particle P is projected vertically downwards from O with speed $\sqrt{gl}$.

(a) Show that the greatest length of the string in the subsequent motion is $4l$. **[5]**

The string first becomes taut at time $t = 0$. At time $t \geq 0$, the length of the string is $l + x$, where x is the extension in the string.

(b) Show that, while the string is taut, x satisfies the differential equation

$$\ddot{x} + \omega^2 x = g, \qquad \text{where } \omega^2 = \frac{g}{l}$$ **[3]**

(c) By using the substitution $x = y + \frac{g}{\omega^2}$, solve the differential equation and hence find, in terms of l and g, the exact length of time for which the string is taut before it first becomes slack. **[12]**