FURTHER MECHANICS / CIRCULAR MOTION & OSCILLATIONS

Motion in a Vertical Circle

Questions by topic 15 questions · 206 marks

EXAM BOARDS Edexcel FM2 9FM0/4C · AQA 7367/3M · OCR A Y543 · OCR MEI Mech Major Y421 CIE 9231 Paper 3

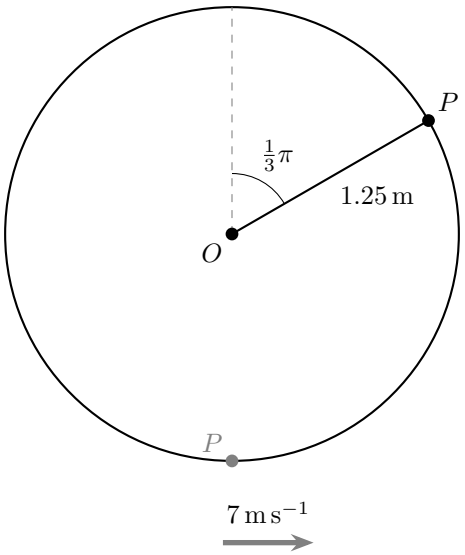
Instructions

Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless a question gives a value. Where a question leaves g unspecified and a final answer depends on it, the solutions also give that answer for $g = 10\,\mathrm{m\,s^{-2}}$.

CIE (9231 Paper 3): papers take $g = 10\,\mathrm{m\,s^{-2}}$; use the $g = 10$ final answers where the solutions give them.

Questions

Question 1	*10 marks*	2
Question 2	*12 marks*	3
Question 3	*12 marks*	4
Question 4	*9 marks*	5
Question 5	*11 marks*	6
Question 6	*13 marks*	7
Question 7	*16 marks*	8
Question 8	*15 marks*	9
Question 9	*13 marks*	10
Question 10	*14 marks*	11
Question 11	*21 marks*	12
Question 12	*17 marks*	13
Question 13	*11 marks*	14
Question 14	*13 marks*	15
Question 15	*19 marks*	16



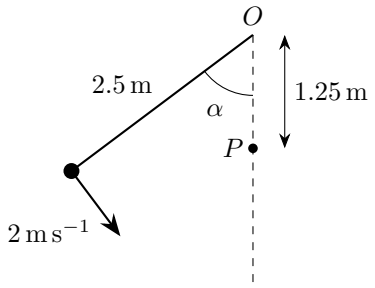
1. One end of a light inextensible string of length 1.25 m is attached to a particle P of mass m kg. The other end of the string is attached to a fixed point O.
Initially P hangs in equilibrium vertically below O. It is then projected horizontally with a speed of $7\,\text{m}\,\text{s}^{-1}$ so that it moves in a vertical circle with centre O, as shown in the diagram above.
At a certain instant, P is above the level of O and first reaches the point where the string makes an angle of $\frac{1}{3}\pi$ radians with the upward vertical through O.

(a) Find the speed of P at this instant. **[3]**

(b) Find the magnitude and direction of the radial acceleration of P at this instant. **[3]**

(c) Find the magnitude of the resultant acceleration of P at this instant. **[4]**

2. In this question use $g = 10\,\text{m}\,\text{s}^{-2}$.
A sphere of mass 0.5 kg is attached to one end of a light inextensible string of length 2.5 m. The other end of the string is attached to a fixed point O. A small smooth peg P is fixed 1.25 m vertically below O. The sphere is held with the string taut and making an angle α with the downward vertical, where $\cos\alpha = \frac{3}{5}$. It is then projected with speed $2\,\text{m}\,\text{s}^{-1}$, perpendicular to the string, so that it moves downwards, as shown in the diagram below. When the string becomes vertical it meets the peg, and the sphere then moves on a circle with centre P.



(a) Show that the speed of the sphere immediately before the string meets the peg is $2\sqrt{6}\,\text{m}\,\text{s}^{-1}$. **[3]**

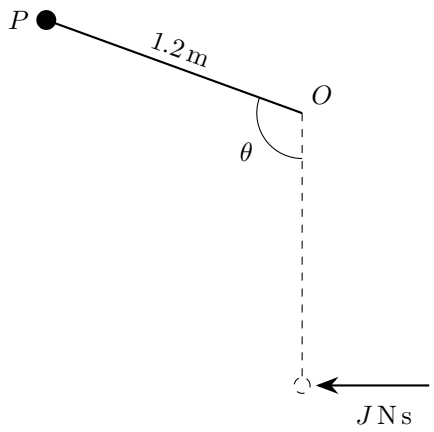
(b) State one modelling assumption that you have made about the sphere in part (a). **[1]**

(c) Find the tension in the string immediately before the string meets the peg and the tension in the string immediately after it meets the peg. **[4]**

(d) In an experiment with this set-up, the speed of the sphere immediately before the string meets the peg is measured as $4.6\,\text{m}\,\text{s}^{-1}$.

(i) Use this measurement to find the work done against air resistance as the sphere moves from its initial position to the point directly below O. **[2]**

(ii) Find the tension in the string immediately after it meets the peg in this experiment. **[2]**

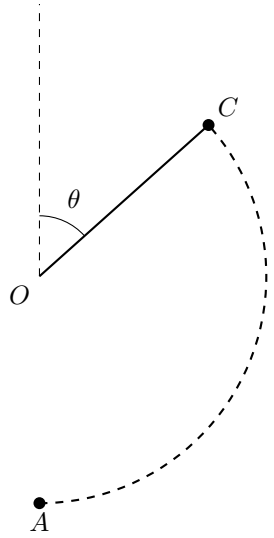


3. A particle P of mass 2.5 kg is attached to one end of a light inextensible string of length 1.2 m. The other end of the string is attached to a fixed point O.
P is hanging at rest vertically below O when it receives a horizontal impulse of magnitude J N s. In the subsequent motion, the angle that the string makes with the downward vertical through O is denoted by θ, as shown in the diagram above.
The string first becomes slack when $\theta = \frac{2}{3}\pi$.

(a) Find the value of J. **[6]**

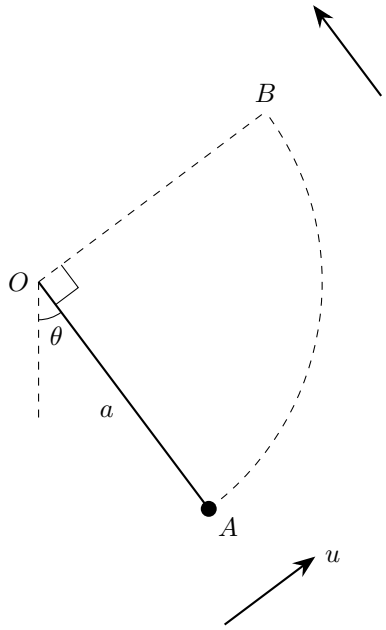
(b) Find the tension in the string at the first instant when $\theta = \frac{1}{3}\pi$. **[6]**

4. A small sphere, of mass m, is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O.
The sphere is at rest at the point A, directly below O, when it is struck, giving it a horizontal impulse of magnitude $2m\sqrt{ag}$, where g is the acceleration due to gravity.
The sphere then moves on a circular path in a vertical plane containing O until the string becomes slack at the point C. At C the string makes an angle θ with the upward vertical through O, as shown in the diagram below.



(a) Find the exact value of $\cos\theta$ and hence find, in terms of a, the height of C above A. **[7]**

(b) With reference to any modelling assumptions that you have made, explain why giving the height of C above A as an inequality would be more appropriate, and state this inequality. **[2]**



5. A particle of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O.
The particle is held with the string taut at the point A, where OA makes an angle θ with the downward vertical through O and $\tan\theta = \frac{3}{4}$. The particle is then projected with speed u perpendicular to OA and begins to move upwards in part of a vertical circle, as shown in the diagram above.
The string goes slack when the particle reaches the point B, where angle AOB is a right angle.

(a) Find the tension in the string when the particle is at A, giving your answer in terms of m and g. **[8]**

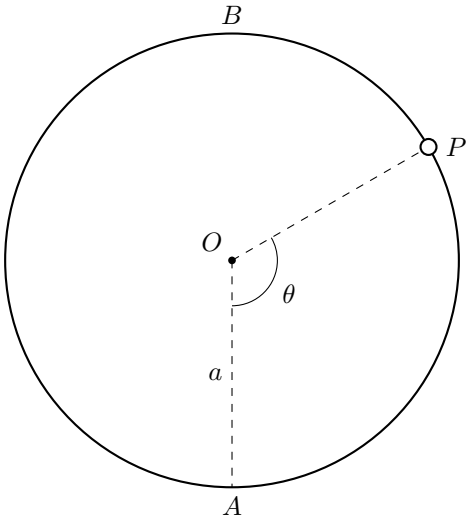
(b) Find, in terms of a, the greatest height above O reached by the particle in its subsequent motion. **[3]**

6. A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle moves in complete vertical circles with centre O. The points A and B are on the path of P, both on the same side of the vertical through O. OA makes an acute angle θ with the downward vertical through O and OB makes an angle θ with the upward vertical through O. The tensions in the string when P is at A and when P is at B are T_A and T_B respectively.

(a) Show that $T_A - T_B = 6mg\cos\theta$. **[8]**

The speed of P at the lowest point of its path is $\sqrt{6ag}$, and it is now given that $5T_A = 11T_B$.

(b) Find the value of θ and find, in terms of a and g, the speed of P when it is at B. **[5]**

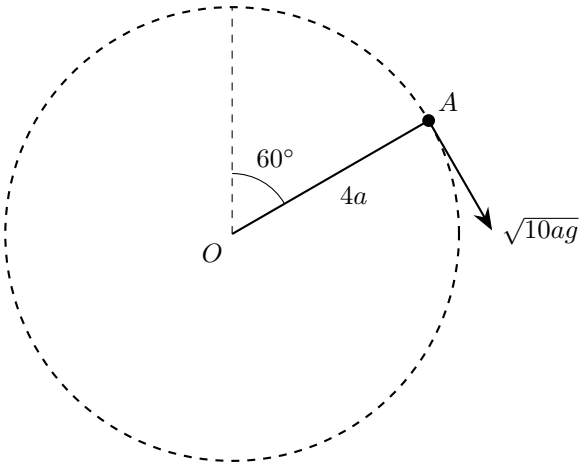


7. The diagram above shows a small bead P of mass m threaded on a smooth thin wire. The wire is in the form of a circle of radius a and centre O, and it is fixed in a vertical plane. The points A and B are the lowest and highest points of the wire respectively.
The bead is at A when it is projected along the wire with speed $\sqrt{5ga}$. While P moves from A to B, the angle AOP is denoted by θ.

(a) Find, in terms of m, g and θ, the magnitude of the reaction of the wire on the bead. Hence find the value of θ when this magnitude is $\frac{3}{2}mg$. **[8]**

(b) Find, in terms of g, a and θ, the angular velocity and the angular acceleration of P. **[4]**

(c) Hence find, in terms of g, the magnitude of the acceleration of P when θ takes the value found in part (a). Give your answer in exact form. **[4]**

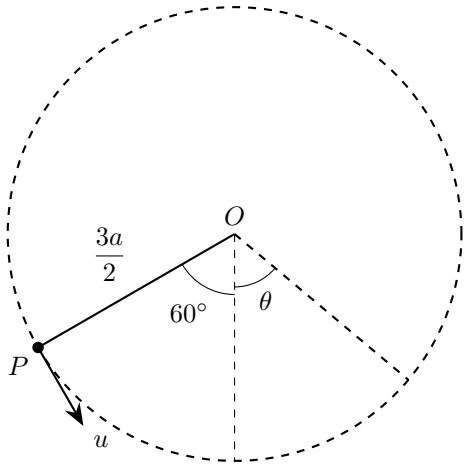


8. A particle of mass m is attached to one end of a light inextensible string of length $4a$. The other end of the string is attached to a fixed point O. The particle is held at the point A, above the level of O, with the string taut and OA making an angle of $60°$ with the upward vertical.
The particle is then projected downwards with speed $\sqrt{10ag}$, perpendicular to OA, in the vertical plane containing OA, as shown in the diagram above.
In an initial model of the motion of the particle, it is assumed that the string does not break.

(a) Using this model, show that the particle performs complete vertical circles. **[8]**

In a refined model it is assumed that the string will break if the tension in it exceeds $5mg$.

(b) Using this refined model, show that the string breaks before the particle first reaches its lowest point, and find the angle between the string and the downward vertical, and the speed of the particle, at the instant the string breaks. **[7]**



9. A small ball P of mass m is attached to one end of a string of length $\frac{3a}{2}$. The other end of the string is attached to a fixed point O.
The ball is held with the string taut and OP at an angle of $60°$ to the downward vertical through O. It is then projected with speed u perpendicular to OP, so that it moves in the vertical plane containing O and initially moves downwards, as shown in the diagram above.
When the string is taut and OP makes an angle θ with the downward vertical through O, the tension in the string is T.
The ball is modelled as a particle and the string as being light and inextensible.

(a) Show that

$$T = 3mg\cos\theta - mg + \frac{2mu^2}{3a}$$ **[6]**

(b) Given that P moves in a complete vertical circle with centre O, find, in terms of a and g, the minimum possible value of u. **[2]**

(c) Given instead that $u = 2\sqrt{ag}$, find, in terms of a and g, the speed of P at the instant when the string becomes slack. **[4]**

(d) State how the modelling assumption that the string is inextensible has been used in your working. **[1]**

10. A particle P of mass m is attached to one end of a light rod of length L. The other end of the rod is attached to a fixed point O so that the rod is free to rotate in a vertical plane about O.
The particle is hanging at rest vertically below O when it is projected horizontally with speed u. The particle first comes to instantaneous rest at the point A.

(a) Explain why the acceleration of P at A is perpendicular to OA. **[1]**

It is given that $u^2 = 3gL$.

(b) (i) Find the magnitude of the acceleration of P at A, giving your answer as an exact multiple of g. **[4]**

(ii) Find the magnitude of the force exerted by the rod on P at A, stating whether the rod is in tension or in thrust. **[3]**

(c) Find, in terms of m and g, the tension in the rod at the instant when OP first makes an angle of $60°$ with the downward vertical. **[6]**

11. A light inextensible string of length a has one end attached to a fixed point O. The other end of the string is attached to a small stone of mass m.
The stone is held with the string taut and horizontal. The stone is then projected vertically upwards with speed $2\sqrt{ag}$.
The stone is modelled as a particle and air resistance is modelled as being negligible.

(a) Assuming that the string does not break, use the model to find, in terms of m and g, the tension in the string when the stone is at the highest point of the circle. **[6]**

The string will break if the tension in it reaches $6mg$.

(b) Use the model to find, in degrees, the total angle through which the string has turned, from the instant the stone is projected to the instant the string breaks. **[8]**

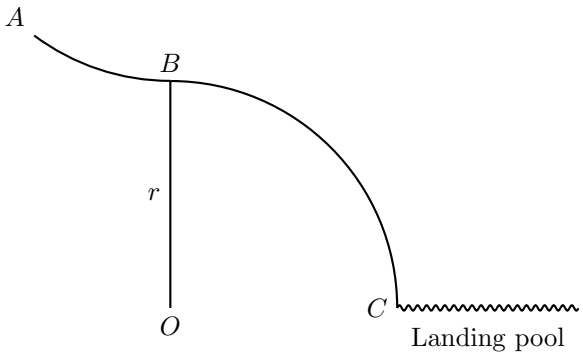
(c) Use the model to find the magnitude of the acceleration of the stone at the instant just before the string breaks, giving your answer as a multiple of g, and the angle between the direction of this acceleration and the string. **[7]**

12. A light inextensible rope, of length 2 m, has one end attached to a fixed point A on the horizontal ceiling of a gym, 5 m above the horizontal ground. The point O on the ground is vertically below A. A child of mass 30 kg holds the other end of the rope, and the child may be modelled as a particle attached to the end of the rope.
Initially the rope is taut and makes an angle α with the downward vertical, where $\tan\alpha = \frac{3}{4}$. The child is given a speed of $2.8\,\mathrm{m\,s^{-1}}$ perpendicular to the rope, moving downwards towards the vertical through A.

(a) Show that, while the rope is taut, the speed $v\,\mathrm{m\,s^{-1}}$ of the child when the rope makes an angle θ with the downward vertical satisfies $v^2 = 7.84 + 4g(\cos\theta - 0.8)$. Hence find the greatest angle that the rope makes with the downward vertical in the subsequent motion. **[6]**

(b) On the other side of the vertical through A, at the instant when the rope again makes an angle α with the downward vertical, the child lets go of the rope and then moves freely under gravity. Find the distance from O to the point where the child lands. **[5]**

(c) The child returns to the initial position and is again given a speed of $2.8\,\mathrm{m\,s^{-1}}$ perpendicular to the rope, moving downwards towards the vertical through A. This time the child holds on to the rope until first coming to instantaneous rest. The rope breaks if the tension in it exceeds 600 N. Find the greatest and the least tension in the rope during this motion, and determine whether the rope breaks. **[6]**



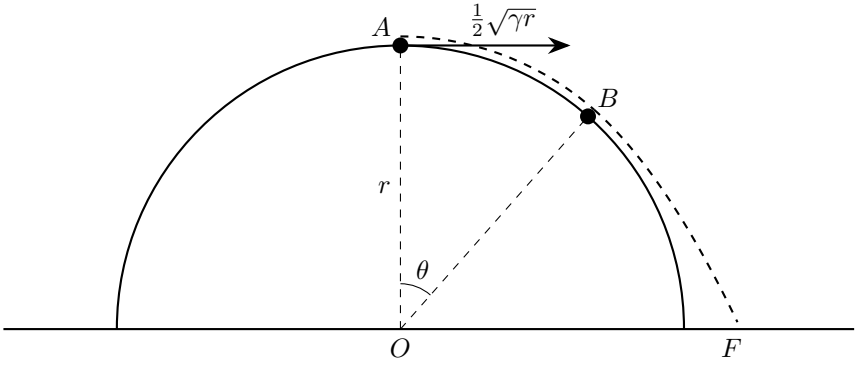
13. The diagram above shows part of a water park slide, ABC, in the shape of two circular arcs, AB and BC, each of radius r.
The point A is at a height of $\frac{r}{5}$ above B. The arc BC has centre O, and B is vertically above O. The two arcs meet at B, where the slide is horizontal.
A child is pushed from A down the slide towards B with speed $\sqrt{\frac{gr}{5}}$, where g is the acceleration due to gravity. The child moves past B and then loses contact with the slide at a point D on the arc BC, where angle BOD is θ.
The slide is modelled as smooth, and the child is modelled as a particle.

(a) Show that $\cos\theta = \frac{13}{15}$. **[6]**

(b) The child lands in a pool whose surface is level with C. Find, in terms of g and r, the speed of the child as it reaches the surface of the pool. **[2]**

(c) A refined model takes air resistance into account. State, with a reason, whether the child would then lose contact with the slide above or below the point D. **[2]**

(d) Explain why, according to the model, the child must lose contact with the slide before reaching C, whatever its speed at A. **[1]**

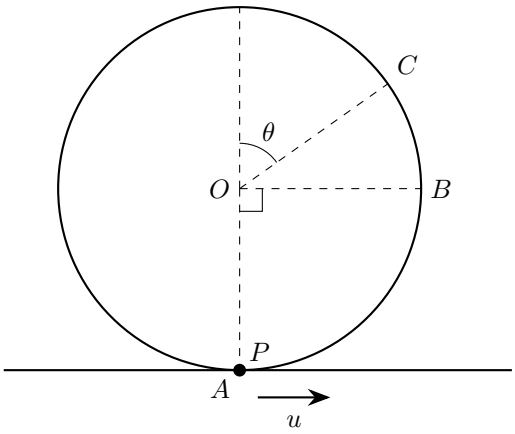


14. The flat surface of a smooth solid hemisphere of radius r is fixed to a horizontal plane on a planet where the acceleration due to gravity is γ. The point O is the centre of the flat surface of the hemisphere, and A is the highest point of its curved surface.
A particle P is placed at A and projected horizontally with speed $\frac{1}{2}\sqrt{\gamma r}$, so that it begins to move on the curved surface. P leaves the surface at the point B, where OB makes an angle θ with the upward vertical through O, and then first hits the plane at the point F, as shown in the diagram above.

(a) Show that $\cos\theta = \frac{3}{4}$. **[6]**

(b) Find an exact expression for the distance OF. **[6]**

(c) Explain why, if P were instead projected horizontally from A with a speed greater than $\sqrt{\gamma r}$, it would leave the surface immediately. **[1]**



15. The diagram above shows a hollow sphere, with centre O and internal radius a, which is fixed to a horizontal surface.
A particle P of mass m is projected horizontally with speed u from the lowest point A of the inner surface of the sphere. The particle moves in a vertical circle with centre O on the smooth inner surface of the sphere, passing through the point B, on the inner surface of the sphere, where OB is horizontal.
The particle leaves the inner surface of the sphere at the point C, where OC makes an angle θ with the upward vertical and $\tan\theta = \sqrt{2}$.

(a) Show that $u^2 = \left(2 + \sqrt{3}\right) ag$. **[6]**

(b) Find, in terms of m and g, the magnitude of the normal reaction exerted on P by the surface of the sphere when P is at B. **[6]**

(c) Show that, after leaving the surface of the sphere at C, the particle passes through O. **[7]**